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Government Expenditure Composition, Economic Growth, and Welfare under Aging Democracy

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Abstract

This paper examines endogenous fiscal policy with two types of expenditures under an aging democracy in a two-sided altruistic overlapping generations model. Intergenerational altruism leads young people to be future-biased, as they wish to postpone receiving their benefits; the elected parliament members are also future-biased. Hence, the democratic government inefficiently provides two types of public services financed by income tax, leading to flow and stock effects on the economy-wide productivity. Increasing the young people's political power tends to make the fiscal policy more biased. The biased parliament chooses to increase public investment rather than current public expenditure. Since population aging weakens the young's political power, it causes an expenditure shift from public investment to current expenditure, leading to a slowdown in economic growth. However, regarding welfare, the democratic economy might be superior to the non-biased social planner, who could derive the optimal fiscal policy.

Keywords: Future bias; Intergenerational altruism; Population aging; Public investment

JEL Classifications: D72; H54; E62; O41

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1. Introduction

In recent years, population aging has progressed in major economies, substantially transforming the composition of government expenditures. Under democracy, population aging tends to strengthen the political influence of the elderly relative to younger generations, thereby shaping fiscal policies according to their preferences. Against this backdrop, understanding how changes in the composition of public spending affect economic performance has become an important policy concern.

Excluding the dimension of aging, the relationship between the composition of public expenditures and economic growth has long been examined. For instance, Devarajan et al. (1996) and Ghosh and Roy (2004) show that a misallocation of public spending can hinder growth. Several studies have also investigated the implications of aging, suggesting that the effects depend on whether expenditures are productive, redistributive, or directed toward improving living standards (Yakita, 2008; Maebayashi, 2013; Dioikitopoulos, 2014; Kamiguchi and Tamai, 2019).

In contrast, Gonzalez et al. (2018) theoretically demonstrate that a future bias emerges in overlapping generations (OLG) models with two-sided altruism. Previous studies have largely neglected this bias, leaving its policy implications unexplored. To address this gap, the present study develops an overlapping generations model that incorporates two features: two-sided altruism between generations and public expenditure components with distinct productivity characteristics. We investigate how the composition of government spending affects economic growth and welfare in such a setting.

In the model, the political power of the young corresponds to the strength of future bias. A decline in the political influence of younger generations enables the parliament to lower tax rates devoted to public investment and to reallocate fiscal resources toward current expenditures. Consequently, under democratic decision-making, economic growth is positively correlated with the political power of the young—or equivalently, with the degree of future orientation—and negatively correlated with population aging.

When future bias is sufficiently small (i.e., in an aging society), the socially optimal growth rate is highest, while the growth rate under a growth-maximizing policy is second highest. However, when future bias is large (i.e., under youth dominance), the growth rate achieved by a growth-maximizing policy exceeds that of the social optimum. This result implies that pursuing growth maximization may yield higher intergenerational welfare than pursuing the social optimum under certain democratic conditions. Quantitative analysis further suggests that there exists an inverted-U relationship between population aging and intergenerational welfare.

In relation to our study, Tamai (2022, 2023) examines the relationships between public goods/public investment and economic growth. The contribution of the present study lies in extending this line of research by distinguishing between flow-type and stock-type productive expenditures. Therefore, we successfully elucidate how the timing mismatch of policy effects, which are mediated through future bias, shapes both economic growth and welfare outcomes.

The remainder of this paper is organized as follows. The next section explains the setup of our theoretical framework. Section 3 derives the competitive equilibrium and the social optimum as the analytical benchmark. Section 4 characterizes the two different equilibria by qualitative and quantitative analyses. Finally, Section 5 delivers the conclusion of this paper.

2. The model

We consider a closed economy that consists of identical individuals and firms. Each individual has a two-period lifetime: youth and old age. There is no uncertainty for their lifetime. The population of each generation is normalized to unity; the total population equals two.

The individuals have the following intergenerationally altruistic preference:[1]

$$U_t = \sum_{s=1}^{\infty} \theta^s \left[u^y\left(c_{t-s}^y\right) + u^o\left(c_{t-s+1}^o\right)\right] + u^y\left(c_t^y\right) + u^o\left(c_{t+1}^o\right) + \sum_{s=1}^{\infty} \delta^s \left[u^y\left(c_{t+s}^y\right) + u^o\left(c_{t+s+1}^o\right)\right], \quad (1)$$

where

$$\theta \equiv \frac{1-\sqrt{1-4\mu\lambda}}{2\lambda} \in (0,1), \delta \equiv \frac{1-\sqrt{1-4\mu\lambda}}{2\mu} \in (0,1), \mu + \lambda \in (0,1).$$

Note that μ and λ represent the degree of filial altruism toward their parents, U_{t-1}, and that of parental altruism toward their children, U_{t+1}, respectively.

Intergenerational altruism is a key factor in explaining the observed upstream or downstream transfers in families in the real world (Arrondel and Masson, 2006). In particular, two-sided altruism provides the rationale for the dual stream transfers (Sloan et al., 2002; Kohli and Künemund, 2003).[2] We specify the utility functions as $u^y\left(c_t^y\right) = \log c_t^y$ and $u^o\left(c_t^o\right) = \rho \log c_t^o$, where ρ is the weight of the elderly's utility ($\rho > 0$). Ignoring the dead ancestors, Eq. (1) can be reduced to

$$U_t \simeq -\rho(\delta^{-1} - \theta) \log c_t^o + \sum_{s=0}^{\infty} \delta^s \left[\log c_{t+s}^y + \delta^{-1} \rho \log c_{t+s}^o\right],$$

$$U_{t-1} \simeq \delta \sum_{s=0}^{\infty} \delta^s \left[\log c_{t+s}^y + \delta^{-1} \rho \log c_{t+s}^o\right].$$

Within the household, the young have one unit of fixed labor and work to earn, whereas the old retire from the labor market. Furthermore, we assume that the inter-vivos transfer is available. Hence, the budget constraint for the household in period t is

$$k_{t+1} = (1-\tau_t)(r_t k_t + w_t l_t) - c_t, \quad (2)$$

where k_{t+1} is the private capital in the next period, τ_t is the income tax rate, r_t is the interest factor, k_t is the private capital in the period t, w_t is the wage rate, l_t is the labor supply, and $c_t \equiv c_t^y + c_t^o$.

The household's objective function is formulated as the weighted sum of the members' utility functions:

$$W_t = U_{t-1} + \eta U_t, \quad (3)$$

where η denotes the young's bargaining/political power (relative to the senior people).[3] Inserting the reduced form of the utility function into Eq. (3) yields

$$W_t \simeq (\delta + \eta) \left\{\log c_t^y + \psi \log c_t^o + \sum_{s=1}^{\infty} \delta^s \left[\log c_{t+s}^y + \phi \log c_{t+s}^o\right]\right\}, \quad (4)$$

where

$$\psi \equiv \left(\frac{1+\eta\theta}{\delta+\eta}\right)\rho, \phi \equiv \frac{\rho}{\delta}.$$

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[1] Kimball (1987) and Hori and Kanaya (1989) show that the functional form (1) can be derived from $U_t = u^y(c_t^y) + u^o(c_{t+1}^o) + \mu U_{t-1} + \lambda U_{t+1}$.

[2] See Laitner (1997) for the early literature on this issue. Some studies model two-sided altruism to illustrate realistic situations for analyzing social security issues, including long-term health care (e.g., Fuster et al. 2003, 2007; Barczyk and Kredler, 2018; İmrohoroğlu and Zhao, 2018).

[3] This functional form is identical to that presented by Hori (1997) and Aoki and Nishimura (2017).

The utility functions U_t and U_{t-1} imply that the young individual has both filial and parental altruism (U_t) whereas the old-age individual has only parental altruism (U_{t-1}). Each household consists of the young and the old, who have different preferences, leading to endogenous *future bias*.[4] Therefore, decision-making within the household might generate a time-inconsistency problem due to this endogenous bias. Based on the utility function U_t, the elderly's utility $\rho \log c_{t+s}^{o}$ is discounted by the discount factor subject to the sequence $\{\theta, 1, \delta, \delta^2, \cdots\}$ $(s = 0,1,2,\cdots)$, which differs from the sequence $\{1, \delta, \delta^2, \cdots\}$ for that of the young $\log c_{t+s}^{y}$.

The people born at period t discount their ancestors' utility but do not discount their own utility. However, they discount the utility of young people relative to that of the coexisting elderly people in the future. Therefore, the young people are willing to transfer their resources to the future themselves to compensate for consumption loss in the future. In other words, such preferences generate future bias. This biased utility affects the household's objective function, leading to biased preferences. Note that a rise in the young's political power decreases ψ. In other words, increasing the young's political power strengthens future bias.

Following Devarajan et al. (1996) and Ghosh and Roy (2004), each firm has the identical production technology specified as

$$y_t = A(g_t^{\alpha} z_t^{1-\alpha})^{1-\beta} k_t^{\beta} l_t^{1-\beta}, \tag{5}$$

where A is the total factor productivity, g_t is the public capital as the stock of public good, z_t is the productive public good as the flow of public good, k_t is the private capital, l_t is the labor supply, $0 < \alpha < 1$, and $0 < \beta < 1$. The factor prices are

$$r_t = \beta A(g_t^{\alpha} z_t^{1-\alpha})^{1-\beta} k_t^{\beta-1} l_t^{1-\beta}, \tag{6a}$$

$$w_t = (1-\beta) A(g_t^{\alpha} z_t^{1-\alpha})^{1-\beta} k_t^{\beta} l_t^{-\beta}. \tag{6b}$$

The government in this economy imposes an income tax to finance two types of government expenditures, as determined by the parliament in fiscal policy. The government expenditures are used for providing two different types of public goods service: the "stock" public good g_t (i.e., public capital) and the "flow" public good z_t (i.e., productive public good).

The government budget constraint equals the sum of investment in public capital and the current expenditure for a productive public good:

$$g_{t+1} + z_t = \tau_t y_t. \tag{7}$$

Moreover, expenditure rules follow

$$g_{t+1} = \sigma_t \tau_t y_t, \tag{8a}$$

$$z_t = (1-\sigma_t)\tau_t y_t. \tag{8b}$$

In the decentralized economy, we consider that the representatives of the parliament decide the fiscal policy at the beginning of every period (e.g., Austen-Smith and Banks, 1988; Baron and Ferejohn, 1989). Following Marsiliani and Renström (2007), the objective function of the parliament is equivalent to Eq. (3).[5] Furthermore, we also examine the growth-maximizing as the parliament's alternative objective to compare the results in the previous studies.

Since the population of the working generation is normalized to unity and the labor supply of each worker is unity, we have $l_t = 1$. Using Eqs. (2), (5), (6a), (6b), (7), and $l_t = 1$, we obtain the following resource constraint:

$$y_t = c_t + z_t + g_{t+1} + k_{t+1}. \tag{9}$$

Following the conventional notations in this research field, we hereafter use x' as x_{t+1} and x as x_t for any t.

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[4] See Gonzalez et al. (2018) for the detail.

[5] We can adopt the other formulation developed by Lindbeck and Weibull (1987) and Grossman and Helpman (1998).

3. Competitive equilibrium and endogenous fiscal policy

This section examines the nature of competitive equilibrium to derive the endogenous time-consistent fiscal policy. The time-consistent policy is derived as a sub-optimal policy under endogenous bias. The comparison between the decentralized and social planner's outcomes characterizes the properties of time-consistent fiscal policy.

3.1. Competitive equilibrium

We consider the structure of the household's optimization problem and the concept of equilibrium. First, we examine the consumption allocation among household members. In each period, households face the same problem to solve. Such an optimization problem is defined as follows:

$$\max_{\pi}[\log \pi + \psi \log(1-\pi)], \tag{SP}$$

where $\pi \equiv c^y/c$ and $1-\pi \equiv c^o/c$.

Solving the static optimization problem (SP) yields

$$\pi^* = \frac{1}{1+\psi}. \tag{10}$$

Eq. (10) implies that the consumption share for the young is decreasing in the welfare weight for the old ψ. Since ψ is decreasing in η, an increase in the young's political power increases the consumption share for the young.

Solving the dynamic optimization for households requires the expectation values of some future variables for π and fiscal policy. Let $\bar{k}$ and $\bar{g}$ be the aggregate private and public capital. The optimization problem is formulated as

$$V_0(k,\bar{k},\bar{g}) = \max_{k'}\{(1+\psi)\log c + \log \pi + \psi \log(1-\pi) + \delta V(k',\bar{k}',\bar{g}')\}, \tag{HP}$$

with some constraints, the anticipated values of future young's consumption share $\hat{\pi}$, income tax rate $\hat{\tau}$, and public investment share $\sigma = \hat{\sigma}$, and

$$V(k,\bar{k},\bar{g}) = (1+\phi)\log c + \log \pi + \phi \log(1-\pi) + \delta V(k',\bar{k}',\bar{g}'). \tag{11}$$

In Eq. (11), $\bar{k}$ and $\bar{g}$ are out of control for the households because these motions are governed by aggregate behavior or the public sector.

The formal definition of competitive equilibrium is given as follows.

Definition 1. *A recursive competitive equilibrium satisfies the following conditions:*
(i) π^* and $k'(k,\bar{k},\bar{g})$ are solutions of (SP) and (HP), respectively;
(ii) $V(k,\bar{k},\bar{g})$ is the functional equation in (HP);
(iii) pricing functions $r(\bar{k},\bar{g})$ and $w(\bar{k},\bar{g})$ are consistent with (4a) and (4b);
(iv) and the government budget constraint and expenditure rules are (7), (8a), and (8b).

A recursive competitive equilibrium consists of the following policy and value function (see Appendix A):

Proposition 1. *The policy and value functions in a recursive competitive equilibrium are*

$$k'(k,\bar{k},\bar{g}) = \frac{(1+\phi)\delta}{(1+\phi)\delta + (1+\psi)(1-\delta)}(1-\tau)r(\bar{k},\bar{g})k$$

$$V(k,\bar{k},\bar{g}) = \Omega + \frac{1+\phi}{1-\delta}\left[\log(k+\omega\bar{k}) + \frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}(\log\bar{g} - \log\bar{k})\right],$$

where

$$r(\bar{k},\bar{g}) = \beta[(1-\sigma)\tau]^{\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}}\bar{k}^{-\frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}}\bar{g}^{\frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}}A^{\frac{1}{\beta+\alpha(1-\beta)}},$$

$$(1-\delta)\Omega \asymp \left[\frac{\beta+\alpha(1-\beta)(1-\delta)}{\beta+\alpha(1-\beta)}\right]\frac{1+\phi}{1-\delta}\log(1-\tau) + \frac{[1-(1-\delta)\alpha](1-\beta)(1+\phi)}{[\beta+\alpha(1-\beta)](1-\delta)}\log\tau$$
$$+\frac{(1-\alpha)(1-\beta)(1+\phi)}{[\beta+\alpha(1-\beta)](1-\delta)}\log(1-\sigma) + \frac{\alpha\delta(1-\beta)(1+\phi)}{[\beta+\alpha(1-\beta)](1-\delta)}\log\sigma,$$

$$\omega = \frac{(1-\beta)[(1+\phi)\delta + (1+\psi)(1-\delta)]}{(1+\psi)(1-\delta)\beta}.$$

If fiscal variables are determined, using Proposition 1, we obtain the saving rate (the private saving as a fraction of disposable income), the public-to-private capital ratio, and the economic growth rate in a recursive equilibrium as follows.

$$s^* \equiv \frac{k'}{(1-\tau)y} = \frac{\beta\delta}{1-(1-\delta)\Delta}, \tag{12}$$

$$x \equiv \frac{g}{k} = \frac{\sigma\tau}{(1-\tau)s^*}, \tag{13}$$

$$\gamma = \sigma^{\frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}}(1-\sigma)^{\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}}\tau^{\frac{1-\beta}{\beta+\alpha(1-\beta)}}[(1-\tau)s^*]^{\frac{\beta}{\beta+\alpha(1-\beta)}}A^{\frac{1}{\beta+\alpha(1-\beta)}}. \tag{14}$$

where

$$\Delta \equiv 1 - \left(\frac{1+\psi}{1+\phi}\right) \in (0,1).$$

We now analyze the effects of future bias and fiscal policy on key economic variables, including the saving rate, the ratio of public to private capital, and the equilibrium growth rate. Logarithmic partial differentiation of Eqs. (12)–(14) with respect to each of τ and σ derives the following result:

Lemma 1. *Suppose that the government expenditure share and the income tax rate are constant over time. (i) Then, the stronger young's political power has the following effects:*

$$\frac{\partial\log s^*}{\partial\eta} = -\frac{1-\delta}{(1+\phi)\delta + (1+\psi)(1-\delta)}\frac{\partial\psi}{\partial\eta} > 0,$$
$$\frac{\partial\log x}{\partial\eta} = -\frac{\partial\log s^*}{\partial\eta} < 0,$$
$$\frac{\partial\log\gamma}{\partial\eta} = \frac{\beta}{\beta+\alpha(1-\beta)}\frac{\partial\log s^*}{\partial\eta} > 0.$$

(ii) The macroeconomic effects of fiscal policies are given by

$$\frac{\partial\log s^*}{\partial\tau} = \frac{\partial\log s^*}{\partial\sigma} = 0, \frac{\partial\log x}{\partial\tau} = \frac{1}{(1-\tau)\tau} > 0, \frac{\partial\log x}{\partial\sigma} = \frac{1}{\sigma} > 0,$$
$$\frac{\partial\log\gamma}{\partial\tau} = \frac{1-\beta}{\beta+\alpha(1-\beta)}\frac{1}{\tau} - \frac{\beta}{\beta+\alpha(1-\beta)}\frac{1}{1-\tau} \gtreqless 0 \Leftrightarrow \tau \lesseqgtr 1-\beta,$$
$$\frac{\partial\log\gamma}{\partial\sigma} = \frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}\frac{1}{\sigma} - \frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}\frac{1}{1-\sigma} \gtreqless 0 \Leftrightarrow \sigma \lesseqgtr \alpha.$$

The result (i) in Lemma 1 is explained as follows. Eq. (12) implies that the saving rate is increasing in Δ. Note that Δ measures the degree of future bias. If there is no future bias ($\psi = \phi$), Δ is zero. Hence, future bias has a positive effect on private saving/investment. As shown in Gonzalez et al. (2018), consumption allocation between the young and old is distorted by discounting the consumption benefits for them at different rates. The young people cover their consumption loss by making more investments to increase their future consumption resources. Therefore, strengthening the degree of future bias enhances private investment.

For a given fiscal policy, Eq. (12) shows that the ratio of public to private capital decreases with an increase in s^*. Hence, an increase in Δ decreases x, and future bias has a negative effect on the ratio of public to private capital. Eq. (14) is the equilibrium growth rate. For a given fiscal policy, future bias affects the growth rate through the saving rate. Naturally, future bias stimulates economic growth through an increase in private investment. Since the young's political power is positively associated with the degree of future bias, the discussion explained above derives the result (i).

We now move on to the interpretation of the result (ii) in Lemma 1. Eq. (12) indicates that the saving rate is independent of fiscal policy. The relationship between the ratio and fiscal policy is straightforwardly derived from Eq. (13). More public investment raises x while more private capital reduces x. Based on Eq. (14), an increase in the tax rate has two effects: the productivity effects through flow and stock of public goods and the distortionary tax effects (Barro, 1990; Futagami et al., 1993). Hence, the equilibrium growth rate is maximized at a certain level of the tax rate. Similarly, there exists a growth-maximizing share of public investment to tax revenue because more (less) public investment results in less (more) productive expenditure, leading to a reduced (enhanced) flow effect of public goods by decreasing (increasing) the marginal productivity.

In the decentralized economy, the parliament representative determines the fiscal policy. If the parliament's political objective is the equilibrium growth rate, they naturally choose the growth-maximizing policy. Under the growth-maximizing policy, the income tax rate and the expenditure share of public investment to total government expenditure are $\tau^g = 1 - \beta$ and $\sigma^g = \alpha$, derived from the result (ii) of Lemma 1. The political power (i.e., future bias) does not influence the growth-maximizing policy.

The growth-maximizing tax rate coincides with that presented by Barro (1990) and Futagami et al. (1993). Moreover, the growth-maximizing share of public investment is consistent with that derived by Agénor (2008). With the growth-maximizing policy, Eqs. (12)–(14) become $s^g = s^*$,

$$x^g = \frac{(1-\beta)\alpha}{\beta s^*}, \tag{15}$$

$$\gamma^g = \alpha^{\frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}}(1-\alpha)^{\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}}\beta^{\frac{\beta}{\beta+\alpha(1-\beta)}}(1-\beta)^{\frac{1-\beta}{\beta+\alpha(1-\beta)}}(s^*)^{\frac{\beta}{\beta+\alpha(1-\beta)}}A^{\frac{1}{\beta+\alpha(1-\beta)}}. \tag{16}$$

The political power affects the equilibrium growth rate through the saving rate, even though the growth-maximizing policy is independent of political power.

3.2. Sub-optimal fiscal policy

Sub-optimal fiscal policy as a time-consistent policy must be the pair (τ^*, σ^*), satisfying a recursive competitive equilibrium with $\tau = \tau'$ and $\sigma = \sigma'$ to maximize the parliament's objective function. In other words, the parliament representative in each period chooses the same fiscal policy. Regarding the sub-optimal fiscal policy, we have the following result (see Appendix B for the proof of Proposition 2):

Proposition 2. *Sub-optimal fiscal policy consists of*

$$(\tau^*, \sigma^*) = \left(\frac{[(1-\alpha)(1-\delta)\beta + s^*](1-\beta)}{(1-\delta)\beta + s^*}, \frac{\alpha s^*}{s^* + (1-\delta)(1-\alpha)\beta}\right).$$

Inserting $\tau = \tau^*$ and $\sigma = \sigma^*$ into Eqs. (13) and (14), we obtain

$$x^* = \frac{\sigma^* \tau^*}{(1-\tau^*)s^*}, \tag{17}$$

$$\gamma^* = A^{\frac{1}{\beta+\alpha(1-\beta)}} (\sigma^*)^{\frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}} (1-\sigma^*)^{\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}} (\tau^*)^{\frac{1-\beta}{\beta+\alpha(1-\beta)}} [(1-\tau^*)s^*]^{\frac{\beta}{\beta+\alpha(1-\beta)}}. \tag{18}$$

Eqs. (17) and (18) denote the ratio of public to private capital and the growth rate in the decentralized equilibrium.

The characteristics of the key economic variables concerning the young's political power are summarized as follows:

Lemma 2. *An increase in the young's political power has the following effects:*

$$\frac{\partial \log \tau^*}{\partial \eta} = \frac{(1-\delta)\alpha\beta}{[(1-\alpha)(1-\delta)\beta + s^*][(1-\delta)\beta + s^*]} \frac{\partial s^*}{\partial \eta} > 0,$$

$$\frac{\partial \log \sigma^*}{\partial \eta} = \frac{(1-\delta)(1-\alpha)\beta}{[(1-\delta)(1-\alpha)\beta + s^*]s^*} \frac{\partial s^*}{\partial \eta} > 0,$$

$$\frac{\partial \log x^*}{\partial \eta} = -\frac{1}{\{[\beta+\alpha(1-\beta)](1-\delta) + s^*\}} \frac{\partial s^*}{\partial \eta} < 0,$$

$$\frac{\partial \log \gamma^*}{\partial \eta} = \frac{\beta}{\beta+\alpha(1-\beta)} \frac{1}{[\beta+\alpha(1-\beta)](1-\delta) + s^*} \frac{\partial s^*}{\partial \eta} > 0.$$

We now consider the interpretation and policy implications of Lemma 2. The young people have an incentive to increase investment in private and public capital. They wish to raise the tax rate and allocate the fiscal resources to public investment. Increasing the young's political power strengthens this tendency. The ratio of public to private capital decreases because public capital is more accumulated than private capital. As a result of enhancing the accumulation of private and public capital, the equilibrium growth rate increases with an increase in the young's political power. In reality, population aging weakens the young's political power. On reflection, the adverse mechanisms mentioned above work.

3.3. The optimal fiscal policy

The equilibrium outcomes in the decentralized economy involve some distortions, such as distortionary taxes and future bias. To evaluate the results, we consider the optimal policy in the sense that a non-biased planner can determine the fiscal policy financing by a lump-sum tax. The non-biased planner's optimization problem is formulated as follows:[6]

$$\tilde{V}(k,g) = \max_{\pi,k',g',z} \{(1+\phi)\log c + \log \pi + \phi \log(1-\pi) + \delta \tilde{V}(k',g')\}. \tag{PP}$$

Solving the problem (PP), the policy function and the fiscal policy are determined in Proposition 3 (see Appendix C for the proof of Proposition 3).

Proposition 3. *The policy functions in the optimal equilibrium are*

$$k'(k,g) = \delta\beta y^\dagger(k,g),$$
$$g'(k,g) = \delta\alpha(1-\beta) y^\dagger(k,g),$$
$$z(k,g) = (1-\alpha)(1-\beta) y^\dagger(k,g),$$

with

[6] The elderly people have no bias. Hence, the social planner who has the elderly's utility function is a non-biased planner.

$$\pi^{\dagger} = \frac{1}{1+\phi}, y^{\dagger}(k,g) = [(1-\alpha)(1-\beta)]^{\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}} A^{\frac{1}{\beta+\alpha(1-\beta)}} g^{\frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}} k^{\frac{\beta}{\beta+\alpha(1-\beta)}}.$$

Then, the optimal fiscal policy is given by

$$\left(\tau^{\dagger}, \sigma^{\dagger}\right) = \left([1-(1-\delta)\alpha](1-\beta), \frac{\alpha\delta}{1-(1-\delta)\alpha}\right).$$

Using Proposition 3, we can derive the saving rate, the public-to-private capital ratio, and the economic growth rate in the optimal as follows.

$$s^{\dagger} = \frac{\beta\delta}{\beta+(1-\delta)(1-\beta)\alpha}, \tag{19}$$

$$x^{\dagger} = \frac{\alpha(1-\beta)}{\beta}, \tag{20}$$

$$\gamma^{\dagger} = \delta\alpha^{\frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}}(1-\alpha)^{\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}}\beta^{\frac{\beta}{\beta+\alpha(1-\beta)}}(1-\beta)^{\frac{1-\beta}{\beta+\alpha(1-\beta)}}A^{\frac{1}{\beta+\alpha(1-\beta)}}. \tag{21}$$

Proposition 3 and Eqs. (19)–(21) are essentially the same as those of Gosh and Roy (2004) because the non-biased planner is an infinitely lived agent without any bias used in the analysis of the existing studies.

4. Characteristics of competitive equilibria

This section aims to characterize the effects of future bias on the equilibrium outcomes, including social welfare. Hence, we examine the dynamic properties of competitive equilibrium under sub-optimal fiscal policy by comparing equilibria with different fiscal policies.

4.1. The effects of future bias on equilibrium outcomes

We consider four cases of equilibrium outcomes to clarify the effects of future bias: sub-optimal, optimal, growth-maximizing, and minimum bias equilibria. The benchmark of measuring the future bias is the optimal economy determined by the non-biased social planner. The growth-maximizing equilibrium serves as a benchmark to evaluate differences from previous studies, including Barro (1990) and Futagami et al. (1993). Finally, the minimum bias equilibrium is defined as the sub-optimal equilibrium with $\psi = \phi$ ($\psi \to \phi$ as $\eta \to 0$); the variables are indicated by the superscript "o". The minimum bias equilibrium is helpful in solely evaluating the effects of future bias.

Using Propositions 1–3, Lemmas 1 and 2, Eqs. (10) and (12)–(21), we establish the following results:

Proposition 4. *The performance of the competitive equilibria with different policies exhibits the following inequalities: (i)* $\pi^g = \pi^* > \pi^o = \pi^{\dagger}$*; (ii)* $\tau^g > \tau^* > \tau^o = \tau^{\dagger}$*; (iii)* $\sigma^g > \sigma^* > \sigma^o = \sigma^{\dagger}$*; (iv)* $s^g = s^* > s^o$*,* $s^{\dagger} > s^o$*, and* $s^* \gtreqless s^{\dagger} \Leftrightarrow \Delta \gtreqless \frac{(1-\beta)\delta}{1-\delta}$*; (v)* $x^* < x^o < x^g$*,* $x^{\dagger} < x^o$*, and* $x^* \gtreqless x^{\dagger} \Leftrightarrow \Delta \lesseqgtr \frac{[1-(1-\delta)\alpha](1-\beta)}{(1-\delta)\{\beta\delta \ [1-(1-\delta)\alpha](1-\beta)\}}$*; (vi)* $\gamma^g > \gamma^* > \gamma^o$*,* $\gamma^{\dagger} > \gamma^o$*, and* $\gamma^{\dagger} \gtreqless \gamma^g \Leftrightarrow \Delta \lesseqgtr \frac{1-\beta\delta^{-\alpha(1-\beta)/\beta}}{1-\delta}$.

(Proof) Almost all of the inequalities can be derived from the direct comparison between the two values in the key variables. In this proof, we focus on the necessary and sufficient conditions in (iv) and (v). Taking the difference between s^* and $s^\dagger$, we have

$$\mathrm{sgn}(s^* - s^\dagger) = \mathrm{sgn}\left[(1-\beta)\delta - \left(\frac{\phi-\psi}{1+\phi}\right)(1-\delta)\right].$$

Therefore, we arrive at

$$s^* \gtreqless s^\dagger \Leftrightarrow (1-\beta)\delta - \left(\frac{\phi-\psi}{1+\phi}\right)(1-\delta) \gtreqless 0 \Leftrightarrow \Delta \lesseqgtr \frac{(1-\beta)\delta}{1-\delta}.$$

Similarly, the comparison between Eqs. (17) and (20) yields

$$x^* \gtreqless x^\dagger \Leftrightarrow (1-\delta)\beta\delta\left[\frac{\Delta}{1-(1-\delta)\Delta}\right] - [1-(1-\delta)\alpha](1-\beta) \gtreqless 0$$

$$\Leftrightarrow \Delta \lesseqgtr \frac{[1-(1-\delta)\alpha](1-\beta)}{(1-\delta)\{\beta\delta + [1-(1-\delta)\alpha](1-\beta)\}}.$$

Finally, using Eqs. (16) and (21), one can obtain

$$\frac{\gamma^\dagger}{\gamma^g} - 1 = \frac{\delta}{\left[\frac{\beta\delta}{1-(1-\delta)\Delta}\right]^{\frac{\beta}{\beta+\alpha(1-\beta)}}} - 1 \gtreqless 0 \Leftrightarrow \Delta \lesseqgtr \frac{1-\beta\delta^{-\frac{\alpha(1-\beta)}{\beta}}}{1-\delta}.$$

Proposition 4 characterizes the effects of distortions, including future bias, on the equilibrium outcomes. There are three sources of inefficiency: distortionary tax, flow and stock effects of public goods, and future bias. Considering these inefficiencies, the results of Proposition 4 can be explained as follows.

First, the consumption allocation between the existing generations is distorted only by future bias. Increasing the degree of future bias decreases the elderly's consumption share because the elderly's benefits are more discounted compared with the young's utility. On the other hand, young people wish to compensate for future consumption losses when they are old by investing in private and public capital. Hence, future bias stimulates the private saving rate and public investment (tax rate and public investment share). At the same time, productive public goods are undersupplied due to the mutual effects of externality and future bias.

The private savings in the decentralized economy could be larger or smaller than the optimal, depending on the degree of future bias. If the future bias is sufficiently strong, then it stimulates private saving. With a distortionary tax, private capital accumulation is less than public capital accumulation. In addition to this, future bias stimulates private investment more, while it raises the equilibrium tax rate. Hence, the ratio of public to private capital could be more or less than the optimal, depending on the degree of future bias. Note that the saving rate and the ratio of public to private capital in the decentralized economy do not coincide with those in the optimal because of distortionary taxes and the presence of productive public goods.

Productivity and the accumulation of private and public capital are essential to determine the equilibrium growth rate. In the decentralized economy, the growth-maximizing policy naturally generates the maximized growth rate. The productivity and distortionary tax effects may reduce the equilibrium growth rate compared with the optimal. However, future bias induces investment in private and public capital. Therefore, depending on the degree of future bias, the growth rate in the decentralized economy could be larger than the optimal outcome. If $\beta > \delta$ and Δ is sufficiently large, then the equilibrium growth rate under the growth-maximizing policy might be larger than that under the optimal. This result implies that the decentralized economy (especially, that with growth-maximizing policy) improves some generations' welfare compared with the non-biased planner's economy.

4.2. Numerical analysis

We provide numerical examples to illustrate the quantitative evaluation of the equilibrium outcomes. The baseline parameters of numerical analysis are set to the values shown in Table 1.[7] Under these parameter values (especially, $\lambda > \mu$), we have $\psi = 0.372$ and $\phi = 0.510$, leading to $\Delta = 0.091$. First, we consider the comparison among different equilibrium outcomes within the baseline case. Then, the results of some key economic indicators and fiscal policy variables are summarized in Table 2.

The consumption allocation and public-to-private capital ratio vary over the equilibria. However, the differences in fiscal policy variables are small, except in the case of growth-maximizing, due to a small bias (i.e., $\Delta = 0.091$). Focusing on the equilibrium growth rate at an annual rate, they are 1.95% for sub-optimum, 1.89% for non-biased, 1.96% for growth-maximizing, and 2.96% for social optimum.[8] The social optimal growth rate is the highest one of them because there is a small bias under the baseline parameter and no distortionary effect on taxes.

We next consider one of the extreme cases with a large Δ. $\lambda = 0.2$ and $\mu = 0.7$ ($\lambda < \mu$) are set; $\Delta = 0.336$ holds. Table 3 reports some key economic indicators and fiscal policy variables. This case shows slightly larger differences in the values of key variables compared to the baseline case. For example, the equilibrium tax rates are 25.8% for sub-optimum, 25.4% for non-biased, 0.300 for growth-maximizing, and 0.254 for social optimum. More interestingly, the equilibrium growth rates are 2.08% for sub-optimum, 1.16% for non-biased, 2.22% for growth-maximizing, and 2.03% for social optimum, at annual rate. Therefore, the equilibrium growth rate under the decentralized economy exceeds that under the social optimum. The results are summarized as follows:

Remark 1. *If $\lambda > \mu$, $\gamma^{\dagger} > \gamma^{g} > \gamma^{*} > \gamma^{o}$ holds. In contrast, $\gamma^{g} > \gamma^{*} > \gamma^{\dagger} > \gamma^{o}$ when $\lambda < \mu$.*

We now characterize the generational welfare levels in different equilibria. The key determinants of welfare levels are initial consumption and equilibrium growth rate. In particular, growth effects are cumulated over periods. Hence, a larger growth rate leads to higher welfare. Tables 3 and 4 indicate that a sub-optimal economy or a growth-maximizing economy can achieve higher welfare than an optimal economy if the effect of future bias is strong. The numerical results are summarized as follows:

Remark 2. *If $\lambda > \mu$, $U_{t-1}^{\dagger} > U_{t-1}^{g} > U_{t-1}^{*} > U_{t-1}^{o}$ and $U_{t}^{\dagger} > U_{t}^{g} > U_{t}^{*} > U_{t}^{o}$, while $U_{t-1}^{o} > U_{t-1}^{\dagger} > U_{t-1}^{g} > U_{t-1}^{*}$ and $U_{t}^{g} > U_{t}^{*} > U_{t}^{\dagger} > U_{t}^{o}$ if $\lambda < \mu$.*

Finally, we analyze the welfare effect of population aging under endogenous fiscal policy in the baseline scenario. Figure 1 illustrates the relationships between the young's political power and the key variables related to welfare. The graph of initial consumption sharply contrasts with that of the growth rate; initial consumption is positively (negatively) associated with population aging (the young's political power), while the growth rate is negatively (positively) associated with population aging (the young's political power). This result implies that the maximum value of the generational utility exists with respect to η. Based on this observation, we have the following result:

Remark 3. *With $\lambda > \mu$, population aging improves generational welfare to a moderate extent.*

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[7] Based on Barro (1990) and Futagami et al. (1993), we set $\beta = 0.7$. Bom and Ligthart (2014) found that the output elasticity of public capital is about 0.12. Then, it should be $\alpha = 0.2$. Following Tamai (2023), $\mu = 0.3$, $\lambda = 0.6$, and $\rho = 0.4$ are assumed. Finally, we set $A = 5$, leading to the realistic values of the equilibrium growth rate.

[8] The annual rate is calculated as one period corresponds to thirty years.

However, excess population aging decreases generational welfare levels.

4.3. Discussion

We consider the implications of our analysis to compare with the existing literature. Some studies examined the relationship between population aging and long-run economic growth in the OLG models with various types of public expenditures (e.g., Yakita, 2008; Maebayashi, 2013; Dioikitopoulos, 2014; Kamiguchi and Tamai, 2019). Moreover, some of them also analyzed the welfare effects of fiscal policy under population aging (Maebayashi, 2013; Kamiguchi and Tamai, 2019). These models characterize population aging by incorporating the mortality rate from young to old.

For instance, Yakita (2008) developed a two-period OLG model with public capital and its maintenance expenditure. The paper shows that population aging raises the income tax rate and the expenditure share of maintenance to maximize the equilibrium growth rate. As a result, under the growth-maximizing policy, population aging increases the equilibrium growth rate.

Maebayashi (2013) constructed a two-period OLG model with public capital accumulation and a pay-as-you-go pension program. Increasing the expenditure share of public pension decreases the equilibrium growth rate. Hence, if population aging tends to increase public pension expenditure, it will lead to a decline in economic growth. The negative growth effect is based on the usual negative impact of unproductive expenditure on capital accumulation and the shift from productive expenditure (public investment) to non-productive expenditure (pension expenditure).

Dioikitopoulos (2014) considered a two-period OLG economy with public education and health, based on an endogenous growth model of human capital accumulation. Under population aging, the government can improve the equilibrium growth rate without increasing the tax rate by reallocating public expenditures, as the higher growth rate generates a larger tax base.[9] Therefore, the positive relationship between population aging and economic growth is established.

Kamiguchi and Tamai (2019) developed a perpetual youth OLG model with debt-financed public investment. They showed that population aging increases the equilibrium growth rate due to an increase in households' savings for their expanded lifetime. These previous studies revealed that population aging has a positive growth effect by enhancing saving and a negative effect of unproductive expenditure or distortionary taxes on capital accumulation.

In contrast, we show that the equilibrium growth rate is negatively associated with population aging. Expanding a personal lifetime naturally stimulates personal savings for old-age consumption. However, such a positive effect on capital accumulation is bounded by the natural limit of expectancy. More importantly, we find that population aging negatively affects aggregate saving/investment through the political power balance between the young and old and endogenously determined fiscal policy. The essential factor is an endogenous future bias originating from two-sided altruism.

Regarding the welfare effects of population aging without intergenerational altruism, Maebayashi (2013) demonstrated that an increase in social security expenditure rather than an increase in public investment improves social welfare under population aging. More recently, Kamiguchi and Tamai (2019) showed that population aging might increase welfare through an increase in the equilibrium growth rate. The results derived from these two studies are based on the fact that the old only benefit from public pension, and public investment only benefits the next generation. Without altruism, these

[9] In the model, population aging is triggered by an increase in the technology of the health stock accumulation through an increase in life expectancy and a decline in the fertility rate.

two studies revealed a monotonic relationship between population aging and welfare.

In contrast to previous studies, each generation with two-sided altruism considers the benefits to other generations, even though they discount their own benefits at a different rate. As shown in the previous part, population aging reallocates the government's financial resources from investment to current expenditure. It increases current private consumption by expanding consumption possibilities through the flow effect of public expenditure. Moreover, population aging decreases total investment because it weakens future bias. Hence, the former productive and latter bias effects mutually generate an inverted-U relationship between population aging and generational welfare.

5. Conclusion

This paper developed an endogenous growth model with two types of public goods and two-sided altruistic overlapping generations. The two-sided altruism generates endogenous future bias. In the democratic economy, the parliament is also future-biased through voting. As found by numerous empirical studies, demographics affect the parliament's decision about fiscal policy through the political power of the young/elderly. Therefore, we investigated the macroeconomic effects of endogenous fiscal policy in the aging democracy with future bias.

We first derived equilibrium fiscal policy in the democratic economy. The equilibrium income tax rate and expenditure allocation depend on the output elasticities of public services, the subjective discount rate, and the private saving rate. Since future bias affects the saving rate, endogenous fiscal policy is also influenced by its degree. Increasing the young's power (i.e., the degree of future bias) incentivizes the parliament to raise the tax rate and reallocate financial resources for public investment. Hence, economic growth under democracy is positively associated with the young's political power or the degree of future bias.

Regarding the fiscal policy variables, the equilibrium tax rate and the expenditure share of public investment under growth-maximizing are the largest, respectively; those under democracy are the second largest, and those of the first-best are the smallest. If future bias is sufficiently small, the equilibrium growth rate in the social optimum is the highest, and the growth-maximizing is the second highest. However, if future bias is sufficiently large, the equilibrium growth rate under growth-maximizing exceeds that of the social optimum. This result implies that growth-maximizing target or democracy is superior to the social optimum, concerning generational welfare.

Welfare analysis of population aging is also conducted analytically and numerically. Welfare effects of population aging are theoretically decomposed into initial consumption and growth effects. Population aging decreases public investment through decreasing tax revenue, leading to a lower economic growth rate. This negative growth effect of population aging generates a negative welfare effect by decreasing future income growth. On the other hand, decreased tax rates increase current consumption resources; an increase in initial consumption improves welfare. The former growth effect is dominated by the latter initial consumption effect according to aging population. Therefore, an inverted-U relationship under democracy exists between population aging and generational welfare.

Finally, we would like to indicate future directions of our research. We considered two types of productive effects of public expenditures. The flow effect is the result derived from health and medical expenditures. We treated the expenditures as a direct productive factor because these improve workers' physical and psychological conditions, leading to higher labor productivity. However, if we incorporate the household's choices of education, fertility, and long-term health care, these choices mutually affect private consumption and investment. Future studies should address this topic for comprehensive fiscal policy, including various welfare and social security programs.

Appendix

A. Proof of Proposition 1

The optimality conditions corresponding to (HP) are

$$\frac{1+\psi}{c} = \delta \frac{\partial V\left(k', \bar{k}', \bar{g}'\right)}{\partial k'}, \tag{A1}$$

$$\frac{\partial V\left(k, \bar{k}, \bar{g}\right)}{\partial k} = \left(\frac{1+\phi}{c}\right)\frac{\partial c}{\partial k} + \delta \frac{\partial V\left(k', \bar{k}', \bar{g}'\right)}{\partial k'}\frac{\partial k'}{\partial k}, \tag{A2}$$

To identify the policy function, we specify the value function as

$$V\left(k, \bar{k}, g\right) = \Omega + L \log \bar{k} + M \log g + N \log\left(k + \omega \bar{k}\right), \tag{A3}$$

Ω, ω, L, M, and N are the undetermined intercept and coefficients. Using Eq. (A3), Eqs. (A1) and (A2) become

$$\frac{1+\psi}{c} = \frac{\delta N}{k' + \omega \bar{k}'}, \tag{A4}$$

$$\frac{N}{k + \omega \bar{k}} = \left(\frac{1+\phi}{c}\right)\frac{\partial c}{\partial k} + \frac{\delta N}{k' + \omega \bar{k}'}\frac{\partial k'}{\partial k}. \tag{A5}$$

We suppose that the policy functions have the forms of

$$k'\left(k, \bar{k}, \bar{g}\right) = (1-\tau) s_r r\left(\bar{k}, \bar{g}\right) k, \tag{A6}$$

$$\bar{k}'\left(\bar{k}, \bar{g}\right) = (1-\tau) s_r r\left(\bar{k}, \bar{g}\right) \bar{k}, \tag{A7}$$

where s_r is the undetermined coefficient. Note that we abstract the functional forms (i.e., $r\left(\bar{k}, \bar{g}\right)$ is described as r hereafter). Eqs. (2) and (A6) provide

$$c = (1-\tau)[(1-s_r) r k + w]. \tag{A8}$$

Using (A4), (A5), (A6), and (A8), we have

$$\frac{k' + \omega \bar{k}'}{k + \omega \bar{k}} = \left[\left(\frac{1+\phi}{1+\psi}\right)(1-\tau)(1-s_r) r + (1-\tau) s_r r\right]\delta. \tag{A9}$$

Eqs. (A6), (A7), and (A9) lead to

$$\frac{\bar{k}'}{k'} = \frac{\bar{k}}{k} \Leftrightarrow \frac{k'}{k} = \frac{\bar{k}'}{\bar{k}} \tag{A10}$$

Using (A6) and (A10), Eq. (A9) becomes

$$(1-\tau) s_r r = \left[\left(\frac{1+\phi}{1+\psi}\right)(1-\tau)(1-s_r) r + (1-\tau) s_r r\right]\delta.$$

Solving the above equation, we obtain

$$s_r^* = \frac{(1+\phi)\delta}{(1+\phi)\delta + (1+\psi)(1-\delta)}. \tag{A11}$$

Eqs. (A4), (A5), (A6), and (A8) lead to

$$\frac{N}{1 + \omega \frac{\bar{k}}{k}} = \frac{1 + \phi + (1+\psi)\left(\frac{s_r^*}{1-s_r^*}\right)}{\left[1 + \frac{(1-\beta)}{(1-s_r^*)\beta}\frac{\bar{k}}{k}\right]}.$$

Therefore, it must be

$$N = 1 + \phi + (1+\psi)\left(\frac{s_r^*}{1-s_r^*}\right) = \frac{1+\phi}{1-\delta},$$

$$\omega = \frac{(1-\beta)}{(1-s_r^*)\beta} = \frac{(1-\beta)[(1+\phi)\delta + (1+\psi)(1-\delta)]}{(1+\psi)(1-\delta)\beta}.$$

We next consider the pricing functions. Using Eqs. (5), (8a), and (8b), we obtain

$$y(k,g) = A^{\frac{1}{\beta+\alpha(1-\beta)}}[(1-\sigma)\tau]^{\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}} k^{\frac{\beta}{\beta+\alpha(1-\beta)}} g^{\frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}}. \tag{A12}$$

Eqs. (6a), (6b), and (A12) yield

$$r(k,g) = \beta A^{\frac{1}{\beta+\alpha(1-\beta)}}[(1-\sigma)\tau]^{\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}} k^{-\frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}} g^{\frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}},$$

$$w(k,g) = \beta A^{\frac{1}{\beta+\alpha(1-\beta)}}[(1-\sigma)\tau]^{\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}} k^{\frac{\beta}{\beta+\alpha(1-\beta)}} g^{\frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}}.$$

Substituting $\bar{k}$ and $\bar{g}$ for k and g, we obtain the pricing functions in Proposition 1.

We now move to determination of the form of the value function. Using the pricing function, Eqs. (A6), (A7), and (8a) can be written as

$$\begin{aligned}\log k' = \log \bar{k}' &= \log(1-\tau) + \frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}[\log\tau + \log(1-\sigma)] + \log s_r^* + \log\beta \\ &\quad + \frac{1}{\beta+\alpha(1-\beta)}\log A + \frac{\beta}{\beta+\alpha(1-\beta)}\log\bar{k} + \frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}\log g,\\ \log g' &= \log\sigma + \log\tau + \frac{1}{\beta+\alpha(1-\beta)}\log A + \frac{\beta}{\beta+\alpha(1-\beta)}\log\bar{k} + \frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}\log g \\ &\quad + \frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}[\log\tau + \log(1-\sigma)].\end{aligned}$$

Using these equations, (11), and (A3), we have $L = -M$ and

$$M = \frac{\alpha(1-\beta)(1+\phi)}{[\beta+\alpha(1-\beta)](1-\delta)}.$$

Inserting the determined coefficients into (A3) and comparison between the equation and (11), we obtain

$$\begin{aligned}(1-\delta)\Omega &= \left[\frac{\beta+\alpha(1-\beta)(1-\delta)}{\beta+\alpha(1-\beta)}\right]\frac{1+\phi}{1-\delta}\log(1-\tau) + \frac{[1-(1-\delta)\alpha](1-\beta)(1+\phi)}{[\beta+\alpha(1-\beta)](1-\delta)}\log\tau \\ &\quad + \frac{(1-\alpha)(1-\beta)(1+\phi)}{[\beta+\alpha(1-\beta)](1-\delta)}\log(1-\sigma) + \frac{\alpha\delta(1-\beta)(1+\phi)}{[\beta+\alpha(1-\beta)](1-\delta)}\log\sigma + \cdots.\end{aligned}$$

B. Proof of Proposition 2

The objective function of the parliament is calculated as

$$\begin{aligned}V_0(k,\bar{k},g) &\simeq (1+\psi)\left\{\log(1-\tau) + \frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}[\log(1-\sigma) + \log\tau]\right\} \\ &\quad + \delta\left\{\Omega + \frac{\alpha(1-\beta)(1+\phi)}{[\beta+\alpha(1-\beta)](1-\delta)}[\log\sigma + \log\tau - \log(1-\tau)]\right\} \\ &\quad + \delta\frac{1+\phi}{1-\delta}\left\{\log(1-\tau) + \frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}[\log\tau + \log(1-\sigma)]\right\}.\end{aligned}$$

Note that all the predetermined values are removed in the above equations.

The partial derivative of V_0 with respect to τ is

$$\begin{aligned}\frac{\partial V_0}{\partial\tau} &= (1+\psi)\left[\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}\frac{1}{\tau} - \frac{1}{1-\tau}\right] + \delta\left\{\frac{\partial\Omega}{\partial\tau} + \frac{\alpha(1-\beta)(1+\phi)}{[\beta+\alpha(1-\beta)](1-\delta)}\left(\frac{1}{\tau} + \frac{1}{1-\tau}\right)\right\} \\ &\quad + \delta\frac{1+\phi}{1-\delta}\left[\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}\frac{1}{\tau} - \frac{1}{1-\tau}\right].\end{aligned}$$

We have

$$\frac{\partial\Omega}{\partial\tau} = -\left[\frac{\beta+\alpha(1-\beta)(1-\delta)}{\beta+\alpha(1-\beta)}\right]\frac{1+\phi}{(1-\delta)^2}\frac{1}{1-\tau} + \frac{[1-(1-\delta)\alpha](1-\beta)(1+\phi)}{[\beta+\alpha(1-\beta)](1-\delta)^2}\frac{1}{\tau}.$$

Hence, we obtain

$$\frac{\partial V_0}{\partial \tau} = \frac{(1+\psi)(1-\delta)+(1+\phi)\delta}{1-\delta}\left[\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}\frac{1}{\tau}-\frac{1}{1-\tau}\right] + \frac{(1+\phi)\delta}{[\beta+\alpha(1-\beta)](1-\delta)^2}\left[\frac{1-\beta}{\tau}-\frac{\beta}{1-\tau}\right]. \text{(B1)}$$

Note that

$$\left.\frac{\partial V_0}{\partial \tau}\right|_{\tau=1-\beta} = -\frac{(1+\psi)(1-\delta)+(1+\phi)\delta}{1-\delta}\frac{\alpha}{[\beta+\alpha(1-\beta)]\beta} < 0.$$

The equilibrium tax rate is derived from Eq. (B1) with $\partial V_0/\partial \tau = 0$:

$$\tau^* = \frac{[(1-\alpha)(1-\delta)\beta+s^*](1-\beta)}{(1-\delta)\beta+s^*}.$$

The partial derivative of V_0 with respect to σ is

$$\frac{\partial V_0}{\partial \sigma} = \delta\left\{\frac{\partial \Omega}{\partial \sigma}+\frac{\alpha(1-\beta)(1+\phi)}{[\beta+\alpha(1-\beta)](1-\delta)}\frac{1}{\sigma}\right\}-(1+\psi)\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}\frac{1}{1-\sigma} - \delta\frac{1+\phi}{1-\delta}\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}\frac{1}{1-\sigma}.$$

We have

$$\frac{\partial \Omega}{\partial \sigma} = \frac{\alpha\delta(1-\beta)(1+\phi)}{[\beta+\alpha(1-\beta)](1-\delta)^2}\frac{1}{\sigma}-\frac{(1-\alpha)(1-\beta)(1+\phi)}{[\beta+\alpha(1-\beta)](1-\delta)^2}\frac{1}{1-\sigma}$$

Then, we obtain

$$\frac{\partial V_0}{\partial \sigma} = \frac{(1-\beta)(1+\phi)\delta}{[\beta+\alpha(1-\beta)](1-\delta)^2}\left[\frac{\alpha}{\sigma}-\frac{1-\alpha}{1-\sigma}\right] - \frac{(1+\psi)(1-\delta)+(1+\phi)\delta}{1-\delta}\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}\frac{1}{1-\sigma}. \text{(B2)}$$

Note that

$$\left.\frac{\partial V_0}{\partial \sigma}\right|_{\sigma=\alpha} = -\frac{(1+\psi)(1-\delta)+(1+\phi)\delta}{1-\delta}\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}\frac{1}{1-\sigma} < 0.$$

The equilibrium value of σ is determined by Eq. (B2) with $\partial V_0/\partial \sigma = 0$:

$$\sigma^* = \frac{\alpha s^*}{s^*+(1-\delta)(1-\alpha)\beta}.$$

C. Proof of Proposition 3

The first-order condition for the optimization problem are

$$\frac{1+\phi}{c} = \delta\frac{\partial \tilde{V}(k',g')}{\partial k'}, \tag{C1}$$

$$\frac{1+\phi}{c} = \delta\frac{\partial \tilde{V}(k',g')}{\partial g'}, \tag{C2}$$

$$\frac{\partial y}{\partial z} = 1. \tag{C3}$$

Moreover, we have

$$\frac{\partial \tilde{V}(k,g)}{\partial k} = \left(\frac{1+\phi}{c}\right)\frac{\partial c}{\partial k}+\delta\frac{\partial \tilde{V}(k',g')}{\partial k'}\left(\frac{\partial k'}{\partial k}+\frac{\partial g'}{\partial k}\right), \tag{C4}$$

$$\frac{\partial \tilde{V}(k,g)}{\partial g} = \left(\frac{1+\phi}{c}\right)\frac{\partial c}{\partial g}+\delta\frac{\partial \tilde{V}(k',g')}{\partial g'}\left(\frac{\partial k'}{\partial g}+\frac{\partial g'}{\partial g}\right). \tag{C5}$$

A guess of the value function is

$$\tilde{V}(k,g) = \tilde{\Omega} + \tilde{L}\log(k+g). \tag{C6}$$

Note that $\tilde{\Omega}$ and $\tilde{L}$ are the undetermined intercept and coefficients.

Eqs. (C1)–(C6) lead to

$$\frac{1+\phi}{c} = \frac{\delta\tilde{L}}{k'+g'}. \tag{C7}$$

$$\frac{\partial y}{\partial k} = \frac{\partial y}{\partial g}. \tag{C8}$$

Using Eq. (5), we have

$$\frac{\partial y}{\partial k} = \beta A g^{\alpha(1-\beta)} z^{(1-\alpha)(1-\beta)} k^{\beta-1},$$
$$\frac{\partial y}{\partial g} = \alpha(1-\beta) A g^{\alpha(1-\beta)-1} z^{(1-\alpha)(1-\beta)} k^{\beta}.$$

Hence, combined these equations with Eq. (C8), we obtain

$$x^{\dagger} = \frac{\alpha(1-\beta)}{\beta}.$$

Eqs. (C4), (C5), (C7), and (C8) yield

$$\frac{1}{k+g} = \frac{\delta}{k'+g'}\left(\frac{\partial c}{\partial k} + \frac{\partial k'}{\partial k} + \frac{\partial g'}{\partial k}\right) \Rightarrow \frac{k'+g'}{k+g} = \delta\frac{\partial y}{\partial k}. \tag{C9}$$

Using Eqs. (C3) and (C5), we obtain

$$z = [(1-\alpha)(1-\beta)]^{\frac{1}{\beta+\alpha(1-\beta)}} A^{\frac{1}{\beta+\alpha(1-\beta)}} k^{\frac{\beta}{\beta+\alpha(1-\beta)}} g^{\frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}} \tag{C10}$$

Inserting Eq. (10) and $x^{\dagger}$ into the marginal product of private capital, we have

$$\frac{\partial y}{\partial k} = \alpha^{\frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}}(1-\alpha)^{\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}}\beta^{\frac{\beta}{\beta+\alpha(1-\beta)}}(1-\beta)^{\frac{1-\beta}{\beta+\alpha(1-\beta)}}A^{\frac{1}{\beta+\alpha(1-\beta)}}. \tag{C11}$$

Eqs. (C9) and (C11) provide

$$\gamma^{\dagger} = \delta\alpha^{\frac{\alpha(1-\beta)}{\beta+\alpha(1-\beta)}}(1-\alpha)^{\frac{(1-\alpha)(1-\beta)}{\beta+\alpha(1-\beta)}}\beta^{\frac{\beta}{\beta+\alpha(1-\beta)}}(1-\beta)^{\frac{1-\beta}{\beta+\alpha(1-\beta)}}A^{\frac{1}{\beta+\alpha(1-\beta)}}.$$

Using Eqs. (C3), (C8), (C9), and (C11), we have

$$k' = \delta\frac{\partial y}{\partial k}k = \delta\beta y, g' = \delta\frac{\partial y}{\partial g}g = \delta\alpha(1-\beta)y, z = (1-\alpha)(1-\beta)y.$$

By the definition of σ, τ, and s, we obtain

$$\sigma^{\dagger} = \frac{g'}{g'+z} = \frac{\alpha\delta}{1-(1-\delta)\alpha},$$
$$\tau^{\dagger} = \frac{g'+z}{y} = \delta\alpha(1-\beta) + (1-\alpha)(1-\beta) = [1-(1-\delta)\alpha](1-\beta),$$
$$s^{\dagger} = \frac{k'}{(1-\tau^{\dagger})y} = \frac{\beta\delta}{1-[1-(1-\delta)\alpha](1-\beta)} = \frac{\beta\delta}{\beta+(1-\delta)(1-\beta)\alpha}.$$

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Figures

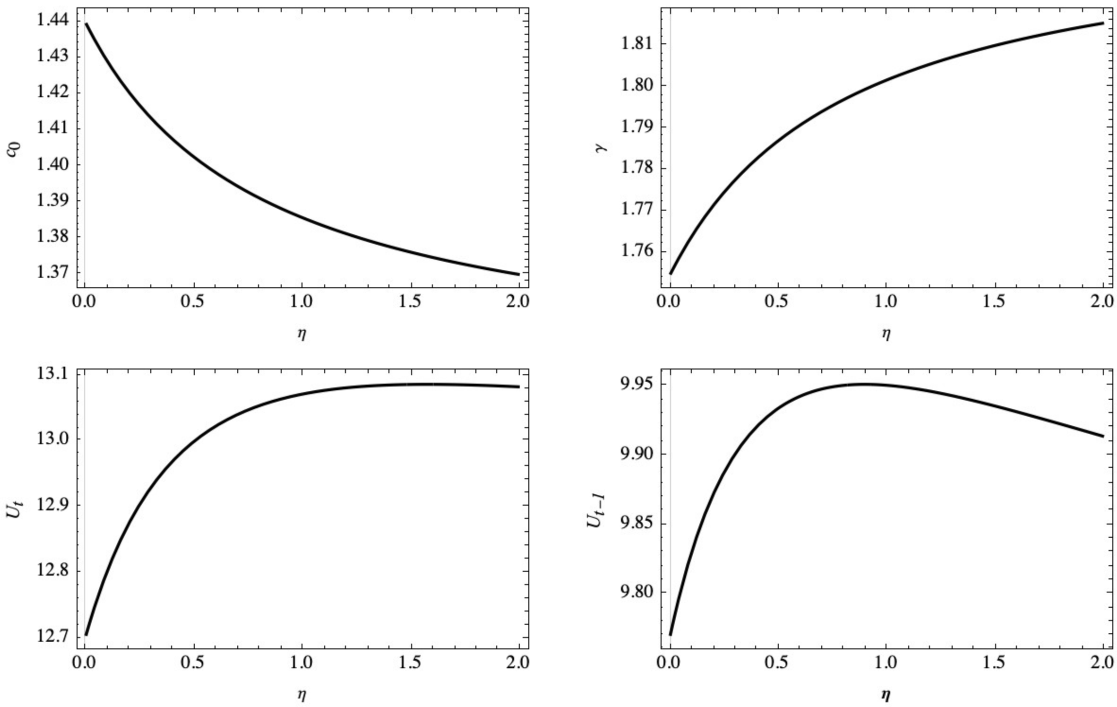

Figure 1. The relationship between young's political power, key variables, and welfare

Tables

Table 1. Parameters

α	0.200
β	0.700
μ	0.300
λ	0.600
ρ	0.400
A	5.000

Table 2. Key economic indicators and fiscal policy variables ($\lambda > \mu$)

	Sub-optimum ($\eta > 0$)	Non-biased ($\eta \to 0$)	Growth-maximizing	Optimum
π	0.729	0.662	0.729	0.662
s	0.560	0.549	0.560	0.771
x	0.118	0.120	0.153	0.086
τ	0.287	0.287	0.300	0.287
σ	0.165	0.164	0.200	0.164
γ	1.787	1.755	1.791	2.396

Note: $\eta > 0$ in the case of growth-maximizing

Table 3. Key economic indicators and fiscal policy variables ($\lambda < \mu$)

	Sub-optimum ($\eta > 0$)	Non-biased ($\eta \to 0$)	Growth-maximizing	Optimum
π	0.566	0.375	0.566	0.375
s	0.226	0.168	0.226	0.226
x	0.107	0.115	0.379	0.086
τ	0.258	0.254	0.300	0.254
σ	0.069	0.057	0.200	0.057
γ	1.853	1.395	1.933	1.828

Note: $\eta > 0$ in the case of growth-maximizing

Table 4. Welfare implications ($\lambda > \mu$)

	Sub-optimum ($\eta > 0$)	Non-biased ($\eta \to 0$)	Growth-maximizing	Optimum
c_0	1.403	1.440	1.406	0.714
U_t	13.00	12.52	13.07	16.00
U_{t-1}	9.934	9.770	9.991	12.16

Note: $\eta > 0$ in the case of growth-maximizing

Table 5. Welfare implications ($\lambda < \mu$)

	Sub-optimum ($\eta > 0$)	Non-biased ($\eta \to 0$)	Growth-maximizing	Optimum
c_0	6.346	6.892	6.616	6.269
U_t	3.243	2.592	3.381	2.975
U_{t-1}	1.103	1.159	1.150	1.151

Note: $\eta > 0$ in the case of growth-maximizing