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Government Expenditure Composition and Long-Run Economic Growth in the Aging Democracy

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Abstract

This paper analyzes how population aging affects government expenditure composition, economic growth, and social welfare under democracy using an overlapping generations model. As public investment and welfare expenditures are financed by income tax, intergenerational conflicts remain in the aged societies. Retired generations, comprising elderly citizens, favor increased income tax rates through increased current welfare expenditures relative to public investment. Working generations, comprising young citizens, favor increased public investment but not increased income tax. Population aging strengthens elderly citizens' political power, leading to increased taxes and budgets shifting from public investment to current welfare. Hence, population aging deteriorates economic growth and social welfare through democracies. On the other hand, population aging by an increase in longevity enhances capital accumulation by increasing savings for old-age consumption. Furthermore, an increase in longevity also brings survival benefits for young and old generations, allowing them to enjoy their retired lives and returns on public investment. Population aging brings positive direct effects on growth and welfare. Finally, we numerically find an inverted-U-shaped relationship between population aging and growth and welfare because the direct and indirect effects of population aging coexist.

Keywords: Democracy; Economic growth; Expenditure composition; Public investment; Welfare expenditure
JEL Classifications: D70; H53; H54; O41

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1. Introduction

This study aims to examine the effects of population aging under democracy on government expenditure allocation, economic growth, and social welfare. Population aging occurs alongside social and economic changes related to public finance and public policy issues around the world. Old-age dependency ratios in 2019 were 35.8%, 38.0%, 51.9%, 32.3%, and 27.8% for France, Germany, Japan, the United Kingdom, and the United States, respectively (OECD, 2022). All G5 Members are regarded as highly aged societies.[1] In the major countries, population aging has changed government expenditure composition and, especially, increased general government health expenditure. From 2005 to 2019, G5 countries faced about 17.6–53.8% of increased general government health expenditure percent of GDP (Figure 1).[2]

Naturally, healthcare and social welfare programs supported by health expenditures should be nonrival and nonexcludable. All existing generations benefit from this public service regardless of paying the supply cost. Furthermore, the retired generation generally obtains more benefits than the working generation because the senior people are more vulnerable to diseases and less taxed. Hence, elderly citizens favor greater health expenditures rather than public investment.[3] Conversely, the young favor public investment at the expense of healthcare and social welfare expenditures because they can obtain more benefits from public investment, and their tax burden mainly covers the government budget. Empirical evidence supports these views.

For instance, using OECD countries from 1970 to 1997, Sanz and Velazquez (2007) found that the elderly share is positively associated with public expenditure mainly benefiting their groups, such as social welfare and health. Jäger and Schmidt (2016) empirically indicated that population aging leads to a cutback in public investment for a panel date of 19 OECD countries from 1971 to 2007. Furthermore, Katsimi and Sarantides (2012) found that elections reallocate government budgets to current expenditures at the cost of public investment; however, the effects of demographic factors are statistically insignificant in most cases. Empirically, population aging and elections affect public expenditure composition and levels.

Besides population aging, the macroeconomic effects of public expenditures have long been

[1] These are based on the WHO's definition. If the aging rate exceeds 14%, it is called as an "aged society"; if over 21%, it is categorized as a "super-aged society".
[2] We used the data of 2019 not the latest one to avoid counting the increased heath expenditure by the COVID-19.
[3] For instance, Brunner and Johnson (2016) empirically showed that older voters are significantly less likely than younger voters to support a tax increase to fund higher education.

studied in the literature on macroeconomic analysis of fiscal policy. Numerous studies originating from Aschauer (1989) provide empirical evidence to support a positive productivity effect of public investment.[4] Moreover, many theoretical studies have clarified the growth effects of public investment and current service expenditures (e.g., Barro, 1990; Futagami et al., 1993; Turnovsky, 1996).[5] Furthermore, empirical evidence between government expenditure allocation and economic growth shows that increased allocation of unproductive expenditure impedes economic growth, whereas increased allocation of public investment enhances economic growth (e.g., Kneller et al., 1999; Bleaney et al., 2001).[6]

Regarding this issue, non-monotonicity or nonlinear effects (e.g., reverse action to population aging) provide one plausible interpretation of such a counter-intuitive relationship between public investment and population aging. Population aging affects our economy through lifetime consumption behaviors under uncertain lifetimes, leading to increasing saving behaviors (Yaari, 1965; Blanchard, 1985; Yakita, 2001). Under democratic governments, tax and expenditure policies are influenced by this direct effect and demographic changes. Therefore, we should examine not only each of the direct and indirect effects of population aging under democracies but also the total effects, including both effects.

This paper considers the relationship among government expenditure allocation, population aging, and economic growth under democracy using an overlapping generations (OLG) model of Diamond (1965) with public service and investment expenditures financed by labor income tax. Specifically, we introduce uncertain lifetimes into the OLG model to illustrate population aging as extending the lifetime of the same generation. Moreover, this study incorporates the democratic determination of public policy, which is a bargaining in parliament. Under balanced budget constraints for the government, income tax rates, public service, and investment expenditures are mutually determined under democracy.

The downward trend of the fertility rate is one of the key factors affecting the population aging. However, we do not explicitly treat the issue of fertility rate in the model for analytical tractability. The existing studies on the relationship between fertility and old-age survival rate show that a higher

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4 For surveys on the literature, see Ligthart and Suárez (2011), Pereira and Andraz (2013), Bom and Ligthart (2014), and Välilä (2020). Using data from 80 countries, Morozumi and Veiga (2016) found that public investment enhances economic growth irrespective of the financial sources if governments are to be accountable.

5 Irmen and Kuehnel (2009) provide an excellent survey of the theoretical literature, including various extensions.

6 The similar results are empirically clarified by Romero-Ávila and Strauch (2008) and Afonso and Alegre (2011). More recently, Arin et al. (2019) also found that the result of productive public goods is similar to that of Kneller et al. (1999), even though the result of aggregate government expenditure is not statistically significant. Chu et al. (2020) empirically show that the allocation shift from unproductive government expenditure to productive expenditure has a positive growth effect. Using data of 151 countries from 1960 to 2014, An et al. (2019) find substitutability between private and public capital in production.

survival rate decreases fertility (e.g., Yakita, 2001). The mechanism implies that a rise in the old-age survival rate speeds up population aging by extending lifetime and decreasing the young population. This effect obviously strengthens the effect of extending lifetime on population aging compared with that without an endogenous fertility rate.

The qualitative analysis of this paper shows that the politico-economic equilibrium exists as a Markov perfect equilibrium, which is a set of a sequence of the Markov perfect strategy. Therefore, demographic changes, including population aging, affect government expenditure composition and economic growth by impacting each individual's political power balance and consumption behaviors. Government policy variables depend on the survival rate representing the population aging index. Analyzing the unique equilibrium, we find the following characteristics of the relationship among government policies, economic growth, and population aging.

First, by focusing on changes in the political power balance between the young and elderly, this paper shows that strengthening elderly citizens' political power increases the income tax rate and the current welfare expenditures to GDP ratio. In contrast, it decreases the public investment to GDP ratio. Elderly citizens favor more current welfare expenditures to improve their felicity rather than public investment for future generations. Furthermore, increasing the tax burden is irrelevant for senior citizens because they have no taxable income. This mechanism is straightforward and consistent with empirical evidence on government expenditure and tax policies. Naturally, increased tax and decreased public investment decrease the equilibrium growth rate.

Second, we demonstrate that population aging with an increase in survival rate increases income tax rate, current welfare expenditures to GDP ratio, and public investment to GDP ratio if the political power balance does not change. Extension of longevity strengthens the benefits of public investment by increasing the net return on savings; moreover, population aging directly affects public investment. However, an increase in longevity has an ambiguous effect on the equilibrium growth rate because of the negative growth effect of increased tax and the positive effect of increased public investment. Therefore, the total effects of population aging on government expenditure policy, tax policy, and economic growth depend on the elasticity of the degree of population aging with respect to elderly citizens' political power.

We also develop welfare analyses in this study. Under certain conditions, the direct welfare effect tends to be positive as extending longevity provides benefits of increasing survival rate and cumulative income effects through a high growth rate. In contrast, the indirect effect of population aging under democracies is negatively associated with social welfare. Strengthening elderly citizens' political power forces democratic governments to increase income tax rates and cut budgets for public investment expenditure, imposing additional tax burden on young and future generations and low growth rates in exchange for increasing current welfare expenditure. Therefore, the total welfare effects of population aging could be either positive or negative, depending on the reactivity of the old age's political power with respect to population aging.

Finally, our numerical analyses within realistic parameters imply that inverted U-shaped relationships exist between population aging and key economic variables (e.g., public investment share to tax revenue, economic growth rate, and social welfare) if the elasticity of elderly citizens' political power with respect to the longevity is increasing in longevity. As elderly citizens' political power increases with increased longevity, the direct positive effects of population aging are gradually overtaken by indirect negative effects. Hence, population aging in highly aged societies impedes not only economic growth but also social welfare. Conversely, population aging in a lesser-aged economy might enhance both economic growth and social welfare.

The remainder of this study is organized as follows: Section 2 reviews the relevant literature. Section 3 presents a basic setup of our analytical framework. Section 4 develops qualitative analyses of the politico-economic equilibrium to examine the relationship among population aging, government policy, and economic growth. Section 5 contains a welfare analysis of population aging and a quantitative evaluation of equilibrium outcomes. Finally, Section 6 provides the conclusion of this study.

2. Literature review

Our study contributes to the literature on fiscal policy and economic growth based on the theoretical models developed by Arrow and Kurz (1970), Barro (1990), and Futagami et al. (1993).[7] The literature has several study branches related to our work concerning government expenditure composition. In particular, earlier studies focus on the growth effects of changing government expenditure composition and optimal (second-best) composition of government expenditure.[8]

[7] These studies focus on pure public goods. Turnovsky (1996) and Fisher and Turnovsky (1998) consider the case where public goods are subject to congestion.
[8] Several studies examine the relationship between intergenerational transfer, such as social security benefits and population aging (e.g., Razin et al., 2002; Galasso et al., 2004; Conde-Ruiz and Galasso, 2005). Galasso and Profeta (2002) provide an excellent survey

The first category's literature includes theoretical and empirical approaches (e.g., Devarajan et al., 1996; Glomm and Ravikumar, 1997; Ghosh and Roy, 2004; Angelopoulos et al., 2007; Felice, 2016). Devarajan et al. (1996) pioneered research on the relationship between government expenditure composition and economic growth. They find that a change in government expenditure composition raises the equilibrium growth rate. Productive government expenditure could be unproductive if the government excessively devotes the revenue to the productive one. Their empirical results imply that capital expenditures may have been excessive in developing countries, indicating that developing countries have misallocated their resources.

Succeeding studies have developed models with various types of productive and unproductive government expenditures. Glomm and Ravikumar (1997) consider infrastructure investment and expenditure on education, which are inputs in investment technologies of public and human capital. Ghosh and Roy (2004) present a stock-flow model of public goods by the mix of Barro (1990) and Futagami et al. (1993). Angelopoulos et al. (2007) extend Barro's (1990) model by incorporating productive and unproductive government expenditure. Felice (2016) analyzes the effects of productive government expenditure and infrastructure investment through the government size and expenditure composition using an endogenous growth model with two private sectors.

These studies provide details on the effects of government expenditure composition on economic growth. Larger shares of productive expenditure to total expenditure lead to higher growth rates if productive expenditure is not excessive. On the other hand, they do not cover the effects of population aging on economic outcomes in democracies. Our analysis demonstrates that population aging affects government expenditure composition and revenue through democracy and equilibrium growth rate. This new channel of policy effects implies that policymakers in aged societies should modify conventional views on the relationship between expenditure composition and growth.

Literature on optimal government expenditure composition utilizes a purely theoretical analysis rather than the approach used for the first one (e.g., Lee, 1992; Chen, 2006; Economides et al., 2011; Zhang et al., 2016). Lee (1992) considers welfare-maximizing policies such that the government chooses the income tax rate, public service (unproductive government) expenditure to GDP ratio, and productive government expenditure to GDP ratio, keeping them time-invariant. It is shown in the earlier study that the optimal policy may have two distinct types: one is characterized by a high tax rate on income, a large income transfer, and a low growth rate. The other is characterized by a low tax

of the literature.

rate, large public investment, and a high growth rate.

Other studies analyze the second-best optimal policy, which is characterized by the dynamic paths of tax rates and government expenditure composition. Along with models presented by Barro (1990) and Lee (1992), Chen (2006) considers the second-best policy and clarifies how differences in preferences affect macroeconomic performance. Economides et al. (2011) examine the second-best policy using an endogenous growth model with utility- and productivity-enhancing public capitals. Moreover, they characterize the second-best optimal policy and show its macroeconomic implications depending on whether public goods are subject to congestion. Zhang et al. (2016) focus on not only expenditure composition but also optimal tax structure.[9]

Existing literature clarify how tax revenue should be allocated between different public expenditures. Theoretical findings are naturally based on normative perspectives of government policy decisions. Focusing on the democratic determination of public policy through intergenerational conflicts, we highlight how government policy—government expenditure composition and revenue––is distorted by population aging. In particular, the degree of deviation from benevolent government decisions increases with the response of elderly citizens' political power to population aging.

Several studies have tackled public investment issues in democracies (e.g., Kaas, 2003; Azzimonti, 2015; Kamiguchi and Tamai, 2019). Kaas (2003) develops an OLG model of endogenous growth with productive government spending wherein people vote sequentially on tax policy. With sequential majority voting, self-fulfilling policy expectations generate endogenous cycles and politico-economic equilibria, which are Pareto-inefficient. Constitutional rules effectively remove the inefficiencies and cycles because such rules provide partial commitment. On the other hand, government size becomes too high relative to growth-maximizing size.

Azzimonti (2015) examines the effects of asymmetries in reelection probabilities across parties on public policy and their subsequent propagation to the economy using the two regions model with local public goods and public capital. Political conflicts between groups evoke distorted composition of government expenditure: underinvestment in public capital and overspending on local public goods. Furthermore, political turnover causes economic fluctuations because the party with an electoral advantage becomes less short-sighted and allocates larger government resources to public investment. Therefore, output increases with electoral advantage.

Kaas (2003) and Azzimonti (2015), respectively, shed light on the existence of Pareto-inefficient

[9] Our model uses solely income tax to finance two types of public expenditure because we illuminate the relationship between expenditure composition, macroeconomic performance, and population aging.

equilibria with endogenous cycles and economic fluctuations caused by political turnover. Moreover, they contribute to clarifying mechanisms of economic fluctuations and removing inefficiency or inconsistencies between the theoretical findings and the data observed.[10] However, population aging and its economic consequences are beyond their scope.

Kamiguchi and Tamai (2019) examine the growth and welfare effects of population aging through public investment under specialized fiscal rules and majority voting using a continuous-time OLG model developed by Yaari (1965) and Blanchard (1985). They focus on public investment expenditures financed by public bonds. They show that population aging raises the equilibrium tax rate and also increases the ratio of public debt to GDP. However, their analyses do not involve the relationship among population aging, government expenditure composition, and economic growth.

Our work is also related to the literature analyzing the effects of population aging on government expenditure composition through voting under intergenerational conflict. Song et al. (2012), Müller et al. (2016), Arai et al. (2018), Ono and Uchida (2024) study the politics of public expenditure and public debt in an overlapping-generations model of the politico-economy. However, none of these studies focused on productive government spending. Our contribution relative to these papers lies in exploring the politics of productive government spending and linking population aging to growth.

Two notable exceptions are Kuehnel (2011) and Gonzalez-Eiras and Niepelt (2012). These two studies analyze how population aging affects productive government spending and economic growth under voting, which are methodologically similar to the present study. Kuehnel (2011) examines the political effects of aging through voting on both productive and unproductive government spending in a simple two-period overlapping generations model with endogenous growth. The result shows that aging does not politically affect productive public expenditure since intergenerational conflict over productive public spending is abstract from the analysis. Conversely, the present work considers an intergenerational conflict over productive public investment and shows its impacts on growth and welfare.

Gonzalez-Eiras and Niepelt (2012) analyze the difference between the political and economic effects of aging on economic growth through voting on productive public spending and pension in a two-period overlapping-generations model. Instead of focusing on the difference between the economic and political effects of aging, we focus especially on the effects of aging under democracies, and we analytically derive the political effect of aging on not only economic growth but also welfare

[10] Using panel data from US states, Azzimonti (2015) shows that the findings are empirically supported.

through public choices on productive public spending.

3. The model

3.1. Individuals, firms, and government

Consider a closed economy with identical individuals and firms. Time is discrete and denoted by subscript t. Each individual lives for two periods (i.e., young and old), and they face a risk of death from youth to old-age.[11] The population of youth who is belong to the generation born at period t is normalized to unity. Expected utility function for the individual born at time t is formulated as follows:[12]

$$EU_t = \left[\alpha \log c_t^y + (1-\alpha)\log z_t\right] + \rho[\alpha \log c_{t+1}^o + (1-\alpha)\log z_{t+1}], \tag{1}$$

where E is the mathematical operator of expected value, U_t is the generation-t's utility level, c_t^y is the generation-t's youth consumption, z_t is the public goods supply in period-t, α is the preference parameter for private good consumption ($0 < \alpha < 1$), ρ is the survival rate of the old-age ($0 < \rho \leq 1$), and c_{t+1}^o is the generation-t's old-age consumption.

While individuals are young, they work to obtain labor income net of income tax and save money for their old-age consumption. In their old-age, individuals live on their savings and interests. Assuming the perfect annuity market, interests are divided by survived people (Yakita, 2001). Hence, the generation-t's budget equations are

$$c_t^y + s_t = (1-\tau_t)w_t, \tag{2a}$$

$$c_{t+1}^o = \frac{R_{t+1}}{\rho}s_t, \tag{2b}$$

where s_t is the saving, τ_t is the labor income tax rate, w_t is the labor income, R_{t+1} is the interest factor.

Individuals choose their youth and old-age consumption levels to maximize their utility subject to their budget equations. Solving the utility maximization problem yields

[11] This setting is widely used for the analysis on population aging (e.g., Yakita, 2001) in Diamond OLG models and one of analogue in discrete type of Blanchard–Yaari model (i.e., Yaari, 1965; Blanchard, 1985).

[12] Several studies examine the robustness of the findings under the log-utility function. For instance, Lopez-Velasco (2024) successfully derives the closed-form solution of a social security model under the CRRA utility function and shows that the derivation of the equilibrium is significantly more complicated than in the log-utility case. We adopt the log-utility for analytical tractability to avoid additional complexity in our model.

$$c_t^y = \frac{1}{1+\rho}(1-\tau_t)w_t, \tag{3a}$$

$$c_{t+1}^o = \frac{1}{1+\rho}\frac{R_{t+1}}{\rho}(1-\tau_t)w_t, \tag{3b}$$

$$s_t = \frac{\rho}{1+\rho}(1-\tau_t)w_t. \tag{3c}$$

Eqs. (3a) and (3b) are the youth and old-age consumption functions, respectively. Furthermore, Eq. (3c) is the saving function.

A homogenous good is producible using private capital k_t and labor inputs l_t. Each firm has the following production function:

$$y_t = Ak_t^\theta(h_t l_t)^{1-\theta}, \tag{4}$$

where y_t is the output, A is the total factor productivity parameter ($A > 0$), h_t is the labor productivity, and $0 < \theta < 1$. Profit maximization leads to factor prices as follows:

$$R_t = \theta\frac{y_t}{k_t}, \tag{5a}$$

$$w_t = (1-\theta)\frac{y_t}{l_t}. \tag{5b}$$

Following Kalaitzidakis and Kalyvitis (2004) and Yakita (2008), labor productivity positively depends on the average private capital $\bar{k}_t$ and public capital g_t:

$$h_t = \bar{k}_t^\mu g_t^{1-\mu}, \tag{6}$$

where $0 < \mu < 1$.[13] In equilibrium, $\bar{k}_t = k_t$ holds. Hence, Eqs. (4) and (6) provide

$$y_t = Ak_t^{1-\eta}g_t^\eta, \tag{7}$$

where $\eta = (1-\theta)(1-\mu)$. Let be $x_t \equiv g_t/k_t$. Using Eqs. (5) and (7), we obtain

$$R_t = \theta Ak_t^{-\eta}g_t^\eta = \theta Ax_t^\eta, \tag{8a}$$

$$w_t = (1-\theta)Ak_t^{1-\eta}g_t^\eta = (1-\theta)Ax_t^\eta k_t. \tag{8b}$$

Government taxes labor income to finance public investment and public goods provision. Hence, the government's budget equation becomes

$$g_{t+1} + z_t = \tau_t w_t = (1-\theta)\tau_t Ak_t^{1-\eta}g_t^\eta. \tag{9}$$

Based on Eq. (9), the allocation rates of public investment and public service expenditures are defined as

[13] Eq. (5) is a combination of the externalities presented by Romer (1986), Barro (1990), and Futagami et al. (1993). For $\mu \to 1$, the production function becomes that developed by Romer (1986). Furthermore, it is closed to that of Barro (1990) and Futagami et al. (1993) when $\mu \to 0$.

$$\pi_g \equiv \frac{g_{t+1}}{\tau_t w_t}, \pi_z \equiv \frac{z_t}{\tau_t w_t},$$

satisfying $\pi_g + \pi_z = 1$.

The government decides the policy variables to maximize the following objective function:

$$W \equiv \sigma V^o + V^y,$$

where $\sigma > 0$ denotes the degree of elderly citizens' political power (i.e., bargaining power of the elderly's party in parliament). This formulation is based on the bargaining model presented by Marsiliani and Renström (2007).[14] In addition to this formulation, the most critical assumption in this paper is that the elderly's political power positively depends on the relative share of elderly citizens: $\sigma = \sigma(\rho)$, where $\sigma'(\rho) > 0$.

With a larger pool of elderly voters, most parties have pandered to the elderly's preferences as compared to those of the young (Parijs, 1998; Berry, 2008). The political parties tend to target the vote of elderly citizens rather than those of younger ones because an individual's turnout increases with age (Binstock, 2012; Davidson, 2012). Therefore, more numerical superiority of the senior citizens in the vote gives larger weight to their preferences in parliament. This evidence supports that population aging nonlinearly increases the bargaining power of the elderly party in parliament.[15]

Using Eqs. (3c) and (8b) and capital market equilibrium condition, the law of motion of private capital is

$$k_{t+1} = s_t = \frac{\rho}{1+\rho}(1-\tau_t)w_t = \frac{\rho}{1+\rho}(1-\tau_t)(1-\theta)Ak_t^{1-\eta}g_t^{\eta}. \tag{10}$$

Eq. (9) represents the law of motion of public capital. Given that the sequences of τ_t and z_t are obtained, Eqs. (9) and (10) gives the full motions of state variables.

In our model, the government policy is determined as the solution of the government optimization problem, satisfying the individuals' and firm's optimization conditions, their budget equations and production technologies, and the government's budget equation. Using Eqs. (1), (3a), (3b), (3c), (8a), and (8b), and adopting recursive notation with primes denoting the *next* period variables (e.g., $x' = x_{t+1}$ when the current period is at t), we have the following indirect utility functions of the young and the old:

[14] This is similar to the populational welfare function (see Hori, 1997; Aoki and Nishimura, 2017). Moreover, the government's objective function can be interpreted as an analog formulation of probabilistic voting model (Lindbeck and Weibull, 1987; Grossman and Helpman, 1998).

[15] Several studies reported that the political participation gap between people aged 25 or 30 and under and those in their 50s and 60s reaches 25 or 30 percentage points (e.g., Grasso, 2014; Holbein and Hillygus, 2016; Achen and Wang, 2019). Moreover, Sevi (2021) found that voters are more likely to vote for candidates who are close in age to themselves. See Stockemer and Sundström (2023) for the excellent survey of the literature on politics referred to here.

$$V^o(\tau, k, g, g') \simeq \rho\{\alpha[(1-\eta)\log k + \eta \log g] + (1-\alpha)\log z\}, \tag{11a}$$

$$V^y(\tau, k, g, \tau', k', g') \simeq \alpha\{\log(1-\tau) + (1-\eta)\log k + \eta \log g\} + (1-\alpha)\log z$$
$$+\rho\{\alpha[(1-\eta)\log k' + \eta \log g'] + (1-\alpha)\log z'\}, \tag{11b}$$

where

$$z = (1-\theta)\tau A k^{1-\eta} g^{\eta} - g'.$$

Hence, the period-t government chooses τ and g' for the optimization problem with anticipated τ'. The optimization problem can be formulated as

$$W(k, g) = \max_{\tau, g'}[\sigma V^o(\tau, k, g, g') + V^y(\tau, k, g, \tau', k', g')]. \tag{12}$$

Before we implement the formal definition of the politico-economic equilibrium, we briefly mention Markov perfect equilibrium as the solution of politico-economic model described in the previous section. Under the concept of Markov perfect equilibrium, the policy variables today are functions of the current payoff-relevant state variables. In the present framework, the payoff-relevant state variables are the physical capital, k, and the public capital, g. Then, the equilibrium policies determined in period-t must be $k' = K(k, g)$, $g' = G(k, g)$, $\tau = T(k, g)$, and $z = Z(k, g)$. Furthermore, we highlight that taxes in this equilibrium will be constant owing to our special parametric assumptions. The conjecture gives

$$\tau = \tau', \tag{13}$$

and the other policy variables are the linear function of y.

3.2. Politico-economic equilibrium

This subsection analyzes the nature of equilibrium policy including government expenditure composition. First, we examine the existence and uniqueness of the politico-economic equilibrium. Subsequently, comparative statics analysis provides the theoretical relationship between political power and the expenditure allocation rates.

We briefly discussed the concept of equilibrium policy in the previous section. Here, the definition of politico-economic equilibrium is given as follows:

Definition 1. *Politico-economic equilibrium is a Markov perfect equilibrium that is a set of a sequences of Markov perfect strategy* $\{K(k_t, g_t), G(k_t, g_t), T(k_t, g_t), Z(k_t, g_t)\}$*, satisfying (3a), (3b),* *(3c), (5a), (5b), (7), (9), (11a), (11b), and (12).*

We now turn to the optimality conditions of the government's optimization problem with Eqs. (9), (10), and (13). First-order conditions for maximizing W are

$$(1-\alpha)\rho\sigma\frac{g'}{z}\frac{\partial z}{\partial g'}+\left\{(1-\alpha)\frac{g'}{z}\frac{\partial z}{\partial g'}+\rho\left[\alpha\eta+(1-\alpha)\frac{g'}{z'}\frac{\partial z'}{\partial g'}\right]\right\}=0, \tag{14a}$$

$$(1-\alpha)\rho\sigma\frac{\tau}{z}\frac{\partial z}{\partial \tau}+\left\{(1-\alpha)\frac{\tau}{z}\frac{\partial z}{\partial \tau}+(1-\alpha)\rho\frac{\tau}{z'}\frac{\partial z'}{\partial \tau}-[1+(1-\eta)\rho]\alpha\frac{\tau}{1-\tau}\right\}=0. \tag{14b}$$

Eq. (14a) implies that the benefit of public investment (i.e., increasing future income) equals to the cost of public investment (i.e., decreasing the current public goods consumption). Eq. (14b) indicates that the benefit of increased income tax (i.e., increasing the current public goods consumption and future income) balances the cost of increased tax (i.e., decreasing the private goods consumption).

As mentioned above, the policy variables except for the tax rate is assumed to be the linear function of y. Our guess is

$$g'=\beta y=\beta Ak^{1-\eta}g^{\eta}, \tag{15a}$$

$$z=\zeta y=\zeta Ak^{1-\eta}g^{\eta}, \tag{15b}$$

where β is a positive constant. Let be $\zeta\equiv z/y$. Then, Eqs. (13) and (15b) lead to $\zeta=(1-\theta)\tau-\beta$. Eqs. (10) and (15b) give k' as the function of y.

We now verify the existence and uniqueness of the politico-economic equilibrium. Using Eqs. (13), (15a), and (15b), Eqs. (14a) and (14b) become

$$\beta=\frac{(1-\theta)\rho\eta}{(1-\alpha)(1+\rho\sigma)+\eta\rho}\tau, \tag{16a}$$

$$\beta=\frac{\{1+[1-\eta+(1-\alpha)(1+\sigma)]\rho\}(1-\theta)}{\alpha+(1-\eta)\rho}\tau-\frac{[1+(1+\sigma)\rho](1-\alpha)(1-\theta)}{\alpha+(1-\eta)\rho}. \tag{16b}$$

Figure 2 illustrates two curves derived from Eqs. (16a) and (16b). Eq. (16a) provides the trajectory of β and τ satisfying equilibrium public investment condition (hereafter, *public investment curve*). Eq. (16b) provides the trajectory of β and τ satisfying equilibrium condition for the income tax (hereafter, *income tax curve*).

Ratio of public investment to labor income tax rate takes the value less than unity because of the government budget constraint. Hence, the gradient of public investment curve is less than unity. The gradient of income tax curve is larger than that of public investment curve. Furthermore, the intercept of income tax curve is below the origin. Figure 2 illustrates that public investment and income tax curves have only one intersection point. Therefore, we have the following result:

Proposition 1. *A unique politico-economic equilibrium exists such that*

$$\tau^* = \frac{[1+(1+\sigma)\rho][(1-\alpha)(1+\rho\sigma)+\eta\rho]}{\{1+[1-\eta+(1-\alpha)(1+\sigma)]\rho\}(1+\rho\sigma)+[1+(1+\sigma)\rho]\eta\rho},$$

$$\beta^* = \frac{(1-\theta)[1+(1+\sigma)\rho]\eta\rho}{\{1+[1-\eta+(1-\alpha)(1+\sigma)]\rho\}(1+\rho\sigma)+[1+(1+\sigma)\rho]\eta\rho},$$

$$\zeta^* = \frac{(1-\alpha)(1-\theta)(1+\rho\sigma)[1+(1+\sigma)\rho]}{\{1+[1-\eta+(1-\alpha)(1+\sigma)]\rho\}(1+\rho\sigma)+[1+(1+\sigma)\rho]\eta\rho}.$$

Proposition 1 implies that the policy variables are all constant over time. We can verify that the equilibrium policy variables (i.e., the ratio of the expenditure to GDP and tax rates) are interior solutions.[16] Inserting the equilibrium policy variables in Proposition 1 into Eqs. (10) and using Eqs. (15) yield

$$\gamma^* = \frac{\rho}{1+\rho}(1-\tau^*)(1-\theta)A(x^*)^\eta, \tag{17}$$

$$x^* = \frac{(1+\rho)\beta^*}{(1-\theta)(1-\tau^*)\rho} = \frac{(1+\rho)(1-\theta)[1+(1+\sigma)\rho]\eta}{(1-\theta)[\alpha+(1-\eta)\rho](1+\rho\sigma)}. \tag{18}$$

Eqs. (17) and (18) denote the equilibrium growth factor and the ratio of public to private capital, respectively. Eq. (17) implies that the effect of an increase in σ affects the equilibrium growth factor through two channels—equilibrium tax rate and ratio of public to private capital. Eq. (17) also show that the effect of an increase in ρ on the equilibrium growth factor has three channels: saving rate, equilibrium tax rate, and ratio of public to private capital.

Using the definitions of π_g and π_z, Proposition 1 leads to the following result:

Corollary 1. *Equilibrium government expenditure allocation satisfies*

$$\pi_g^* = \frac{\beta^*}{(1-\theta)\tau^*} = \frac{\eta\rho}{(1-\alpha)(1+\rho\sigma)+\eta\rho} < 1,$$

$$\pi_z^* = \frac{\zeta^*}{(1-\theta)\tau^*} = \frac{(1-\alpha)(1+\rho\sigma)}{(1-\alpha)(1+\rho\sigma)+\eta\rho} < 1,$$

$$\frac{\pi_g^*}{\pi_z^*} = \frac{\eta\rho}{(1-\alpha)(1+\rho\sigma)} \gtreqless 1 \Leftrightarrow \frac{\eta}{1-\alpha} \gtreqless \frac{1+\rho\sigma}{\rho}.$$

[16] Eq. (16a) shows that $\beta^* < \tau^*$ if there exists a politico-equilibrium pair of (β^*, τ^*). The government's budget equation implies $0 < \zeta^* = (1-\theta)\tau^* - \beta^* < 1$. Finally, we have

$$1-\tau^* = \frac{[\alpha+(1-\eta)\rho](1+\rho\sigma)}{\{1+[1-\eta+(1-\alpha)(1+\sigma)]\rho\}(1+\rho\sigma)+[1+(1+\sigma)\rho]\eta\rho} > 0.$$

Corollary 1 provides government expenditure allocation between public investment and current expenditure for public service under balanced budget. Allocation rates of two expenditures depend on the parameters of preference, production technology, political power, and survival rate. On whether π_g^* or π_z^* (i.e., β^* and ζ^*) is larger depends on the ratio of the output elasticity of public capital to the taste for public goods. If this ratio is sufficiently large (small), $\pi_g^* > \pi_z^*$ ($\pi_g^* < \pi_z^*$) holds. Related conditions imply that higher (lower) value of σ tends to being $\pi_g^* < \pi_z^*$ ($\pi_g^* > \pi_z^*$), whereas larger value of ρ leads to $\pi_g^* > \pi_z^*$ ($\pi_g^* < \pi_z^*$).

4. Macroeconomic effects of population aging

In this section, we examine the relationship among equilibrium policy variables, equilibrium growth rate, and population aging. Population aging affects the equilibrium government policy through two effects: direct effect by an increase in the survival rate and indirect effect through an increase in elderly citizens' political power. Degree of political power of the elderly is denoted by σ because it reflects the population share of elderly citizens (e.g., Parijs, 1998; Berry, 2008; Binstock, 2012; Davidson, 2012; Stockemer and Sundström, 2023). Hence, population aging by an increase in ρ leads to an increase in σ. Moreover, these effects on government expenditure allocation were already examined in previous sections. However, considering the effects of population aging on public investment, public service expenditure, and economic growth, we need to analyze the direct effects of a rise in ρ on economic variables because it directly affects individuals' consumption and saving.

4.1. Effect of a rise in elderly citizens' political power

We begin our comparative statics analysis by investigating the effects of a rise in σ. Larger σ yields larger cost of public investment owing to substitution between public investment and public service expenditure (Eq. (14a)). Hence, smaller β will be chosen for given τ. Figure 3 indicates that an increase in σ moves public investment curve downward (Eq. (16a)). On the other hand, a rise in σ has a positive effect on the benefit of increased tax (Eq. (14b)); higher tax rate will be preferred for given β. Figure 3 indicates a downward movement of the income tax curve (Eq. (16b)). Therefore, the equilibrium tax rate is increased, whereas the equilibrium ratio of public investment to GDP is decreased.

Based on the geometric analysis developed above, we obtain the following result (See Appendix B for the proof of Lemma 1):

Lemma 1. *An increase in* σ *shows*

$$\frac{\partial \pi_g^*}{\partial \sigma} = -\frac{\partial \pi_z^*}{\partial \sigma} < 0 \; with \; \frac{\partial \tau^*}{\partial \sigma} > 0, \frac{\partial \beta^*}{\partial \sigma} < 0, \frac{\partial \zeta^*}{\partial \sigma} > 0.$$

A rise in σ increases the income tax rate and the current government expenditure to GDP ratio, while it reduces the public investment to GDP ratio. The result of Lemma 1 is consistent with the empirical findings such that population aging increases welfare and health expenditures (Sanz and Velazquez, 2007) and decreases public investment (Jäger and Schmidt, 2016) by the elderly demands. Hence, an increase in elderly citizens' political power decreases the allocation rate of public investment and increases the allocation rate of current public service expenditure. Allocation rate of public service expenditure is more increased than the rate before an increase in σ. Therefore, $\pi_g^* < \pi_z^*$ (i.e., $\beta^* < \zeta^*$) will be plausible.

We now turn to the effect of a rise in σ on equilibrium growth rate. Partial derivatives of Eqs. (17) and (18) yield

$$\frac{\sigma}{\gamma^*}\frac{\partial \gamma^*}{\partial \sigma} = -\left(\frac{\tau^*}{1-\tau^*}\right)\underbrace{\frac{\sigma}{\tau^*}\frac{\partial \tau^*}{\partial \sigma}}_{(+)} + \eta \underbrace{\frac{\sigma}{x^*}\frac{\partial x^*}{\partial \sigma}}_{(-)} < 0, \tag{19}$$

where

$$\frac{\sigma}{x^*}\frac{\partial x^*}{\partial \sigma} = \underbrace{\frac{\sigma}{\beta^*}\frac{\partial \beta^*}{\partial \sigma}}_{(-)} + \left(\frac{\tau^*}{1-\tau^*}\right)\underbrace{\frac{\sigma}{\tau^*}\frac{\partial \tau^*}{\partial \sigma}}_{(+)} = \frac{\sigma\rho}{1+(1+\sigma)\rho} - \frac{\rho\sigma}{1+\rho\sigma} < 0. \tag{20}$$

Therefore, we arrive at the following result:

Proposition 2. *A rise in* σ *decreases the equilibrium growth rate.*

Increasing the elderly political power leads to a budget cut in public investment and increased tax for budget increase in public goods expenditure. Decreased public investment induces less public capital accumulation. Increased tax slows private capital accumulation. These two effects oppositely affect the ratio of public to private capital. The former effect dominates the latter effect; a rise in σ reduces x^* as shown by Eq. (20). Then, an increase in σ affects equilibrium growth rate through two channels: private and public capital accumulation. A rise in σ reduces disposable income through an increased tax. A decrease in disposable income decreases saving (investment), leading to less private capital accumulation. Furthermore, a rise in σ impedes public capital accumulation. As shown in Eq. (19), these two effects are negatively associated with the equilibrium growth rate.

4.2. Effect of population aging

Elderly citizens' political power negatively affects equilibrium growth rate because a rise in its degree increases the income tax rate to finance an increase in public service expenditure but decreases the public investment. Their political power originates from elderly citizens' population share (e.g., Parijs, 1998; Berry, 2008; Binstock, 2012; Davidson, 2012; Stockemer and Sundström, 2023). Hence, strengthening the political power coincides with population aging. Since population aging means extension of longevity, it also affects individual's consumption and saving behaviors. Therefore, the total effect of population aging on the policy choices in the politico-economic equilibrium include its *direct* and *indirect* effects; the former represents the effects of a rise in ρ on equilibrium policy functions, and the latter stands for those of a rise in σ on equilibrium policy functions.

First, we investigate the *direct* effects of population aging on equilibrium policy variables in the politico-economic equilibrium. A rise in ρ increases both the benefit and cost of public investment (Eq. (14a)). The effect of an increase in ρ on public investment benefit dominates that on cost. For a given τ, the democratic government is willing to choose larger β. Figure 4 indicates that an increase in β moves a public investment curve upward (Eq. (16a)). An increase in ρ raises both benefit and cost of increased tax (Eq. (14b)). The former effect outweighs the latter effect. Hence, larger values of tax rate will be preferred for a given β. Figure 4 indicates a downward movement in the income tax curve (Eq. (16b)). As a result, both β^* and τ^* are increased by the extension of longevity.

By the analysis developed above, the following result holds (see Appendix C for the proof of Lemma 2):

Lemma 2. *Suppose that* σ *is fixed. Then,*

$$\frac{\partial \pi_g^*}{\partial \rho} = -\frac{\partial \pi_z^*}{\partial \rho} > 0 \; with \; \frac{\partial \tau^*}{\partial \rho} > 0, \frac{\partial \beta^*}{\partial \rho} > 0, \frac{\partial \zeta^*}{\partial \rho} > 0.$$

A rise in ρ increases the income tax rate τ^*, the current government expenditure to GDP ratio ζ^*, and the public investment to GDP ratio β^*. Lemma 2 implies that an increase in public investment is larger than an increase in income tax revenue, whereas an increase in public service expenditure is smaller than an increase in income tax revenue. Hence, an increase in ρ raises the allocation rate of public investment to tax revenue and reduces that of public service expenditure.

Similarly to derive the growth effect of a rise in σ, the partial derivative of Eq. (17) with respect to ρ gives

$$\frac{\rho}{\gamma^*}\frac{\partial \gamma^*}{\partial \rho} = \frac{1}{1+\rho} - \left(\frac{\tau^*}{1-\tau^*}\right)\underbrace{\frac{\rho}{\tau^*}\frac{\partial \tau^*}{\partial \rho}}_{(+)} + \eta \underbrace{\frac{\rho}{x^*}\frac{\partial x^*}{\partial \rho}}_{(+/-)}, \tag{21}$$

where

$$\frac{\rho}{x^*}\frac{\partial x^*}{\partial \rho} = -\frac{1}{1+\rho} + \underbrace{\frac{\rho}{\beta^*}\frac{\partial \beta^*}{\partial \rho}}_{(+)} + \left(\frac{\tau^*}{1-\tau^*}\right)\underbrace{\frac{\rho}{\tau^*}\frac{\partial \tau^*}{\partial \rho}}_{(-)}. \tag{22}$$

The first and second terms on the RHS of Eq. (21), respectively, represent a positive growth effect by increased saving and a negative growth effect through decreased savings by increased tax. Furthermore, the third term indicates that an increase in ρ causes one more growth effect of a change in x, which is denoted by Eq. (22). However, the sign of Eq. (22) is ambiguous because larger ρ leads to lower x by increased saving, whereas it simultaneously yields the opposite effect by increased public investment. If the productivity effect of public capital is sufficiently large, the sign of Eq. (22) becomes positive.[17]

Even if the third term on the right-hand side of Eq. (21) is positive, determining the sign of Eq. (21) is difficuelt because the second term of Eq. (21) is negative. Indeed, we have

[17] To clarify the sign of Eq. (22), we need to impose the parameter restrictions. After some calculations, we obtain

$$\frac{\alpha}{1-\eta} \geq \frac{\left[\frac{1}{1+\rho} - \frac{\rho}{[1+(1+\sigma)\rho](1+\rho\sigma)}\right]}{\left[\frac{1}{1+\rho} + \frac{1}{[1+(1+\sigma)\rho](1+\rho\sigma)}\right]}$$

$$\Rightarrow \frac{\rho}{x^*}\frac{\partial x^*}{\partial \rho} = \alpha\left[\frac{\rho}{1+\rho} + \frac{\rho}{[1+(1+\sigma)\rho](1+\rho\sigma)}\right] - (1-\eta)\left[\frac{\rho}{1+\rho} - \frac{\rho^2}{[1+(1+\sigma)\rho](1+\rho\sigma)}\right] \geq 0.$$

$$\frac{\rho}{\gamma^*}\frac{\partial \gamma^*}{\partial \rho} = \frac{1}{1+\rho} + (1-\eta)\left[\frac{(1-\eta)\rho}{\alpha + (1-\eta)\rho} + \frac{\sigma\rho}{1+\sigma\rho}\right] + \eta\left[\frac{\rho}{1+\rho} + \frac{(1+\sigma)\rho}{1+(1+\sigma)\rho}\right] - \rho\Phi \gtreqless 0,$$

where

$$\Phi = \frac{\sigma + [1+(1-\alpha)(1+\sigma)](1+2\rho\sigma) + 2\eta\rho}{\{1+[1+(1-\alpha)(1+\sigma)]\rho\}(1+\rho\sigma) + \eta\rho^2} > 0.$$

Because of the term of $\rho\Phi$, the growth effect of a rise in ρ is ambiguous without any additional assumptions.

Based on the empirical studies, the plausible values of α and η are $0.5 < \alpha < 1$ and $0 < \eta < 0.5$.[18] Hereafter, we impose the following assumption:

Assumption 1. $\eta < 0.5 < \alpha$.

Considering ρ is approximately equal to zero, the effect of a marginal increase in ρ on γ^* becomes

$$\left.\frac{\rho}{\gamma^*}\frac{\partial \gamma^*}{\partial \rho}\right|_{\rho=0} = 1 > 0.$$

For extremely small ρ ($\rho \to 0$), the effects of a rise in ρ on economic growth is positive.

In contrast, if $\rho \to 1$, the growth effect of an increase in ρ can be evaluated as

$$\left.\frac{\rho}{\gamma^*}\frac{\partial \gamma^*}{\partial \rho}\right|_{\rho=1} = \frac{1}{2} + \left[\frac{1-\eta}{1-\eta+\alpha} + \frac{\sigma}{1+\sigma}\right](1-\eta) + \left[\frac{1}{2} + \frac{1+\sigma}{2+\sigma}\right]\eta - \Phi|_{\rho=1},$$

where

$$\Phi|_{\rho=1} = \frac{\sigma + [1+(1-\alpha)(1+\sigma)](1+2\sigma) + 2\eta}{[2+(1-\alpha)(1+\sigma)](1+\sigma) + \eta} > 0.$$

Depending on the values of α, η, and σ, a rise in ρ has either positive or negative effects on economic growth. Therefore, we have the following result (see Appendix D for the proof of Proposition 3):

Proposition 3. *Suppose that σ is independent of a change in ρ. If $\rho \approx 0$, a rise in ρ increases equilibrium growth rate. Otherwise, an increase in ρ might reduce or raise equilibrium growth rate depending on the values of α, ρ, η, and σ; a rise in ρ increases equilibrium growth rate, focusing on the case of $\sigma = 1$.*

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[18] In reality, public service expenditure is smaller than private consumption expenditure. This implies $\alpha \geq 0.5$. According to Bom and Ligthart (2014), $\eta = 0.2$ is plausible based on their Meta analysis.

Kamiguchi and Tamai (2019) show that population aging enhances economic growth through an increase in debt-financed public investment using Yaari–Blanchard model. In their model, population aging is characterized as extending the longevity (or decreasing the probability of death). Hence, longer expected lifetime leads to increased benefits from higher growth through public investment for all existing generations, similar to our model. However, considering income tax financing and democratic determination of tax policy, longer lifetime has an adverse effect on economic growth through increased tax. Under democracies in the aged societies, the indirect effect of politically determined public policy cannot be ignored.

Uniting the results of Lemmas 1 and 2 and Propositions 2 and 3, we can derive total effect of population aging on equilibrium policy and growth rate:

$$\frac{\rho}{\beta^*}\frac{d\beta^*}{d\rho} = \frac{\rho}{\beta^*}\frac{\partial\beta^*}{\partial\rho} + \left(\frac{\sigma}{\beta^*}\frac{\partial\beta^*}{\partial\sigma}\right)\varepsilon_\sigma \gtreqless 0 \Leftrightarrow \varepsilon_\sigma \lesseqgtr -\frac{\frac{\rho}{\beta^*}\frac{\partial\beta^*}{\partial\rho}}{\frac{\sigma}{\beta^*}\frac{\partial\beta^*}{\partial\sigma}}, \tag{23a}$$

$$\frac{\rho}{\tau^*}\frac{d\tau^*}{d\rho} = \frac{\rho}{\tau^*}\frac{\partial\tau^*}{\partial\rho} + \left(\frac{\sigma}{\tau^*}\frac{\partial\tau^*}{\partial\sigma}\right)\varepsilon_\sigma > 0, \tag{23b}$$

$$\frac{\rho}{\zeta^*}\frac{d\zeta^*}{d\rho} = \frac{\rho}{\zeta^*}\frac{\partial\zeta^*}{\partial\rho} + \left(\frac{\sigma}{\zeta^*}\frac{\partial\zeta^*}{\partial\sigma}\right)\varepsilon_\sigma > 0, \tag{23c}$$

where

$$\varepsilon_\sigma \equiv \frac{\rho}{\sigma}\frac{d\sigma}{d\rho} > 0.$$

Population aging has positive effects on equilibrium income tax rate and ratio of public service expenditure to GDP, demonstrated in Eqs. (23b) and (23c), because the direct and indirect effects of population aging on the values are both positive. In contrast, as shown in Eq. (23a), the effect of population aging on the public investment ratio depends on the size of elasticity of σ with respect to ρ relative to the direct and indirect effects of population aging. The key determinant of the effect of population aging on public investment is how the old-age citizens' political power is strengthened by population aging.

Finally, we arrive at the following formulas of the total effects of population aging on government expenditure composition and economic growth:

$$\frac{\rho}{\pi_g^*}\frac{d\pi_g^*}{d\rho} = \frac{\rho}{\pi_g^*}\frac{\partial\pi_g^*}{\partial\rho} + \left(\frac{\sigma}{\pi_g^*}\frac{\partial\pi_g^*}{\partial\sigma}\right)\varepsilon_\sigma \gtreqless 0 \Leftrightarrow \varepsilon_\sigma \gtreqless -\frac{\frac{\rho}{\pi_g^*}\frac{\partial\pi_g^*}{\partial\rho}}{\frac{\sigma}{\pi_g^*}\frac{\partial\pi_g^*}{\partial\sigma}}, \tag{24a}$$

$$\frac{\rho}{\gamma^*}\frac{d\gamma^*}{d\rho} = \frac{\rho}{\gamma^*}\frac{\partial\gamma^*}{\partial\rho} + \left(\frac{\sigma}{\gamma^*}\frac{\partial\gamma^*}{\partial\sigma}\right)\left(\frac{\rho}{\sigma}\frac{d\sigma}{d\rho}\right) \gtreqless 0 \Leftrightarrow \varepsilon_\sigma \lesseqgtr -\frac{\frac{\rho}{\gamma^*}\frac{\partial\gamma^*}{\partial\rho}}{\frac{\sigma}{\gamma^*}\frac{\partial\gamma^*}{\partial\sigma}}. \tag{24b}$$

Eq. (24a) suggests that a rise in survival rate affects the equilibrium allocation rates through two ways: direct effect of a rise in ρ on the allocation rates and indirect effect through a change in σ. The latter effect is explained above. Larger ρ yields larger allocation rate of public investment and smaller allocation rate of public service expenditure. Therefore, the direct and indirect effects oppose each other.

Eq. (24b) indicates that the first term on the RHS represents the direct growth effect of population aging, and the second term denotes the indirect growth effect of population aging through the change in the old-age people political power. The direct growth effect may be positive or negative; however, the indirect growth effect is negative. Larger elasticity of σ with respect to ρ leads to a larger indirect growth effect. Therefore, if population aging largely increases elderly citizens' political power, the indirect growth effect of population aging dominates its direct growth effect.

Using Lemmas 1 and 2 and Propositions 2 and 3, the results developed above are summarized as the following proposition:

Proposition 4. *If* ε_σ *is sufficiently large, population aging reduces the public investment to GDP ratio and equilibrium growth rate, whereas it raises the income tax and the current government expenditure to GDP ratio. Government revenue is less allocated to public investment. In contrast, if* ε_σ *is sufficiently small, population aging increases the public investment to GDP ratio, the income tax, and the current government expenditure to GDP ratio, and might also raise equilibrium growth rate. Then, government revenue is more allocated to public investment.*

Based on Jäger and Schmidt (2016), population aging has a negative effect on public investment. Furthermore, Katsimi and Sarantides (2012) found that election increases current expenditure at the cost of public investment. These empirical findings imply that the elasticity of old-age citizens' political power with respect to longevity is sufficiently large. On the other hand, some countries exhibit a positive relationship between population aging and public investment for small values of old-age dependency ratio. Several countries have smaller old-age dependency ratio than that of Japan. In sum, a non-monotonic relationship between public investment and population ageing—economic growth and population ageing—might exist. This case could be true if ε_σ is increasing in ρ. We examine this possibility in the next section.

5. Further analyses

This section develops welfare analysis of population aging through its direct and indirect (political) effects. Furthermore, the numerical analysis provides quantitative examples for indicating theoretical findings and covers cases wherein the sufficient conditions are not satisfied.

5.1. Welfare analysis

Several criteria exist for welfare analysis. In this study, we follow the conventional approach using the following social welfare function:

$$W = EU_{-1} + E\sum_{t=0}^{\infty}\left(\frac{1}{1+\delta}\right)^t U_t,$$

where $EU_{-1} \equiv \rho[\alpha \log c_0^o + (1-\alpha)\log z_0]$. Evidently, the politico-economic equilibrium outcomes differ from the social optimum solutions. Hence, we consider the welfare effects of population aging rather than the comparison between different equilibria.

The social welfare function W in the politico-economic equilibrium becomes

$$W = V_0^o + \sum_{t=0}^{\infty}\left(\frac{1}{1+\delta}\right)^t V_t^y, \tag{25}$$

where

$$V_0^o = \rho\Omega,$$
$$V_t^y = \alpha\log(1-\tau^*) + (1+\rho)\Omega + (1+t)\rho\log\gamma^*,$$
$$\Omega \equiv \log k_0 + \eta\log x^* + (1-\alpha)\log\zeta^* + \log A.$$

V_0^o and V_t^y, respectively, denote the indirect utility functions for elderly (survivors) individuals at period 0 and for the young at period t. We introduce the following assumption to ensure $V_0^o \geq 0$ and $V_t^y > 0$ for $\rho \geq 0$.[19]

[19] Under Assumption 2, $V_0^o = \rho\Omega \geq 0$ and $V_t^y \approx -\alpha\tau^* + (1+\rho)\alpha + (1+t)\rho\log\gamma^* = (1-\tau^*+\rho)\alpha + (1+t)\rho\log\gamma^* > 0$ for $\rho \geq 0$.

Assumption 2. $\Omega \geq \alpha$.

We now consider the welfare effect of a rise in σ. Partial derivatives of the indirect utility functions are

$$\frac{\partial V_0^o}{\partial \sigma} = \rho \frac{\partial \Omega}{\partial \sigma}, \tag{26a}$$

$$\frac{\partial V_t^y}{\partial \sigma} = (1+\rho)\frac{\partial \Omega}{\partial \sigma} - \frac{\alpha}{1-\tau^*}\frac{\partial \tau^*}{\partial \sigma} + (1+t)\rho \frac{1}{\gamma^*}\frac{\partial \gamma^*}{\partial \sigma}, \tag{26b}$$

with

$$\begin{aligned}\frac{\partial \Omega}{\partial \sigma} &= \eta \frac{1}{x^*}\frac{\partial x^*}{\partial \sigma} + (1-\alpha)\frac{1}{\zeta^*}\frac{\partial \zeta^*}{\partial \sigma} \\ &= (1-\alpha+\eta)\frac{\rho}{1+(1+\sigma)\rho} + (1-\alpha-\eta)\frac{\rho}{1+\rho\sigma} - (1-\alpha)\Lambda,\end{aligned}$$

where

$$\Lambda \equiv \frac{\{1+[1-\eta+(1-\alpha)(1+\sigma)]\rho\}\rho + (1-\alpha)(1+\rho\sigma)\rho + \eta\rho^2}{\{1+[1-\eta+(1-\alpha)(1+\sigma)]\rho\}(1+\rho\sigma) + [1+(1+\sigma)\rho]\eta\rho}.$$

Eq. (26a) represents the marginal effect of strengthening elderly citizens' political power on the old-age's utility through private and public consumptions. This term should be positive based on real observations. Given that the intensity of public goods preference is larger than the output elasticity of public capital (i.e., $\alpha + \eta \leq 1$), Eq. (26a) takes a positive value. Hence, benefits from public goods consumption outweigh the cost of public goods consumption by decreasing public investment.

Eq. (26b) comprises three terms: the first-term on the RHS is marginal effect of strengthening elderly citizens' political power on the youth and elderly utility through the public goods consumption; the second-term represents distortionary tax effect; and the last term denotes growth effects. The former measures positive welfare effect, whereas the latter two measure negative welfare effects. Without additional assumptions on parameters, the sign of Eq. (26b) cannot be determined.

Using Eq. (26a) and (26b), the partial derivative of Eq. (25) with respect to σ yields

$$\begin{aligned}\frac{\partial W}{\partial \sigma} &= \frac{\partial V_0^o}{\partial \sigma} + \sum_{t=0}^{\infty}\left(\frac{1}{1+\delta}\right)^t \frac{\partial V_t^y}{\partial \sigma} \\ &= \rho\frac{\partial \Omega}{\partial \sigma} + \left(\frac{1+\delta}{\delta}\right)\left\{(1+\rho)\frac{\partial \Omega}{\partial \sigma} - \frac{\alpha}{1-\tau^*}\frac{\partial \tau^*}{\partial \sigma} + \rho\left(\frac{1+\delta}{\delta}\right)\frac{1}{\gamma^*}\frac{\partial \gamma^*}{\partial \sigma}\right\}.\end{aligned} \tag{27}$$

Eq. (27) involves three effects: The terms related to Ω represent the welfare effects of a change in σ

through public service expenditure; The terms related to τ^* denotes the welfare effects of a change in σ through income tax; The terms related to γ^* stands for the welfare effects of a change in σ through economic growth.

The key factor for determining the sign of Eq. (27) is the social discount rate. $\delta \to 0$ denotes that negative growth effects are extremely large. Its negative welfare effects dominate all other effects. Hence, a rise in σ decreases the social welfare when $\delta \to 0$. On the other hand, if $\delta \to \infty$, all terms except for the first period will vanish because all the generations after initial periods are disregarded. This implies that the welfare effect of a rise in σ is evaluated by

$$\lim_{\delta\to\infty} \frac{\partial W}{\partial \sigma} = (1+2\rho)\frac{\partial \Omega}{\partial \sigma} - \frac{\alpha}{1-\tau^*}\frac{\partial \tau^*}{\partial \sigma} + \rho\left(\frac{1+\delta}{\delta}\right)\frac{1}{\gamma^*}\frac{\partial \gamma^*}{\partial \sigma}.$$

Therefore, if $\eta + \alpha \leq 1$, a rise in σ might increase the social welfare. However, if not, a rise in σ might not have positive welfare effect. These results are summarized as follows (see Appendix E for the proof of Proposition 5):

Proposition 5. *(i) If the social discount rate is sufficiently small, then a rise in σ decreases the social welfare. (ii) Given that the social discount rate is sufficiently large, a rise in σ might improve (worsens) the social welfare if $\sigma < 1$ and η is sufficiently small (large). However, if $\sigma \geq 1$, a rise in σ reduces the social welfare.*

As an increase in σ impedes economic growth (Proposition 2), the negative growth effect of a rise in σ lingers throughout whole periods if the society has the strong social preference for intergenerational equity. Even if the society is myopic and does not care for future generations, the negative growth effect of a rise in σ on existing young generations is counted. Hence, an increase in σ cannot improve social welfare, at least without a positive welfare effect of increasing public service expenditure by a rise in σ; this requires $\eta + \alpha \leq 1$. When $\eta + \alpha \geq 1$, the productivity effect of public capital is larger than the welfare effect of public service expenditure. Then, a rise in σ cannot effectively increase social welfare even though elderly citizens have a hope of improving their welfare.

Next, we analyze the direct welfare effect of a rise in ρ. Partial differentiation of the indirect utility functions with respect to ρ engenders

$$\frac{\partial V^o}{\partial \rho} = \Omega + \rho\frac{\partial \Omega}{\partial \rho}, \tag{28a}$$

$$\frac{\partial V^y}{\partial \rho} = \Omega + (1+\rho)\frac{\partial \Omega}{\partial \rho} - \frac{\alpha}{1-\tau^*}\frac{\partial \tau^*}{\partial \rho} + (1+t)\rho \log \gamma^* + (1+t)\rho \frac{1}{\gamma^*}\frac{\partial \gamma^*}{\partial \rho}, \tag{28b}$$

with

$$\frac{\partial \Omega}{\partial \rho} = \eta \frac{1}{x^*}\frac{\partial x^*}{\partial \rho} + (1-\alpha)\frac{1}{\zeta^*}\frac{\partial \zeta^*}{\partial \rho}$$
$$= (1-\alpha-\eta)\left\{\frac{1+\sigma}{1+(1+\sigma)\rho} + \frac{\sigma}{1+\rho\sigma} - \frac{1}{(1+\rho)[\alpha+(1-\eta)\rho]}\right\} - (1-\alpha)\Phi,$$

where

$$\Phi \equiv \frac{\sigma + [1+(1-\alpha)(1+\sigma)](1+2\rho\sigma) + 2\eta\rho}{\{1+[1+(1-\alpha)(1+\sigma)]\rho\}(1+\rho\sigma) + \eta\rho^2}.$$

The intuitions of Eqs. (28a) and (28b) are similar to those of Eqs. (26a) and (26b). In addition to the explained effects of Eqs. (26a) and (26b), Eqs. (28a) and (28b) have the *direct benefits* of extending the longevity: The first terms on RHS in (28a) and (28b) (public service expenditure) and the third term on RHS in (28b) (economic growth). These effects indicate that extending longevity increases the possibility of survival and therefore people enjoy consuming more public services and more private goods through their old-age income.

Using Eqs. (28a) and (28b), the direct effect of population aging on social welfare is calculated as

$$\frac{\partial W}{\partial \rho} = \frac{\partial V_0^o}{\partial \sigma} + \sum_{t=0}^{\infty}\left(\frac{1}{1+\delta}\right)^t \frac{\partial V_t^y}{\partial \sigma}$$
$$= \Omega + \rho\frac{\partial \Omega}{\partial \rho} + \left(\frac{1+\delta}{\delta}\right)\left\{\Omega + (1+\rho)\frac{\partial \Omega}{\partial \rho} - \frac{\alpha}{1-\tau^*}\frac{\partial \tau^*}{\partial \rho} + \rho\left(\frac{1+\delta}{\delta}\right)\left[\log \gamma^* + \frac{1}{\gamma^*}\frac{\partial \gamma^*}{\partial \rho}\right]\right\}. \tag{29}$$

If $\delta \to 0$, the welfare effect of population aging through its growth effect outweighs the other welfare effects. The results of Proposition 3 indicate that an increase in ρ raises economic growth rate for a small value of ρ. In this case, population aging improves social welfare.

When $\delta \to \infty$, Eq. (29) can be reduced to

$$\lim_{\delta \to \infty}\frac{\partial W}{\partial \rho} = 2\Omega + (2+\rho)\frac{\partial \Omega}{\partial \rho} - \frac{\alpha}{1-\tau^*}\frac{\partial \tau^*}{\partial \rho} + \rho\left[\log \gamma^* + \frac{1}{\gamma^*}\frac{\partial \gamma^*}{\partial \rho}\right].$$

Unlike the welfare effect of a rise in σ, we find the direct benefits of population aging through extending the longevity. Hence, the direct welfare effect of population aging tends to be positive if such direct benefits are sufficiently large. This requires $\alpha + \eta \ll 1$.

Formally, we have the following proposition (see Appendix F for the proof of Proposition 6).

Proposition 6. *(i) If the social discount rate is sufficiently small, then a rise in* ρ *increases (might* *decrease) the social welfare for initially low (high) ρ. (ii) Suppose that the social discount rate is sufficiently large. For small value of ρ, a rise in ρ increases the social welfare if $\alpha + \eta \ll 1$ and $(1-\alpha)\alpha^{-1} < \sigma \leq (4-\alpha)\alpha^{-1}$.*

An increase in ρ might enhance economic growth (Proposition 3). For societies with strong social preference for intergenerational equity, higher economic growth rate generates larger welfare. Moreover, this might be true for a large ρ and not only a small ρ. However, this is not certain without any additional conditions (Proposition 3). When the society is myopic and cares only for existing generations, the direct benefits of extending the longevity exists even if a rise in ρ has a negative growth effect. Hence, if the direct benefits are large enough to cover the negative welfare effects of increased tax and the others, population aging could improve social welfare. This requires at least $\eta + \alpha \leq 1$ and not too strong the elderly political power.

Finally, we consider the total effect of population aging on social welfare. Using Eqs. (27) and (29), we arrive at

$$\frac{\rho}{W}\frac{dW}{d\rho} = \frac{\rho}{W}\frac{\partial W}{\partial \rho} + \frac{\sigma}{W}\frac{\partial W}{\partial \sigma}\varepsilon_\sigma. \tag{30}$$

Eq. (30) implies that the elasticity of elderly citizens' political power with respect to the longevity ε_σ is a key determinant of the total welfare effect of population aging. Incorporating this into the results of Propositions 5 and 6, we obtain the following proposition:

Proposition 7. *(i) Suppose that the social discount rate is sufficiently small. If ε_σ is sufficiently small, then a rise in the longevity might increase social welfare for a short longevity, whereas it might decrease social welfare for long life expectancy. However, if ε_σ is sufficiently large, a rise in the longevity decreases social welfare. (ii) Suppose that the social discount rate is sufficiently large. If ε_σ is sufficiently small, a rise in the longevity increases the social welfare for small values of σ and η. In contrast, when ε_σ is sufficiently large, a rise in the longevity reduces (might increase) the social welfare for large value of σ (small values of σ and η).*

Population aging could improve social welfare through the direct benefits of extending the longevity if their effects remain sufficiently large to cover the negative welfare effects of the other factors (Proposition 6). However, if the indirect negative welfare effects under democratic determination of public policy increase, population aging prejudices social welfare (Proposition 5). In particular, the larger productivity effect of public capital and weaker preference for public service expenditures generate smaller positive welfare effects of increasing current welfare expenditure.

Furthermore, the sensitivity of elderly citizens' political power concerning population aging is crucial to derive the total welfare effect of population aging because it identifies which of the direct and indirect effects of population aging is larger. Hence, Proposition 7 also implies that an inverted U-shaped-shaped relationship between social welfare and population aging might exist if σ is increasing in ρ. Kamiguchi and Tamai (2019) suggest that the possibility of non-monotonic relationship between welfare and population aging differs from this conclusion. We verify this claim in the next section.

5.2. Quantitative analysis

This subsection develops a numerical analysis as an example of analytical results in the previous sections. The parameters are set as follows: $\alpha = 0.7$, $\eta = 0.2$, and $\theta = 0.3$. Since the value of α is considered to lie between 0.6–0.8, we adopt the intermediate case (e.g., Tamai, 2022). Output elasticity η is estimated between 0.1–0.4. According to Bom and Ligthart (2014), $\eta = 0.2$ is plausible for the infrastructure.[20] Output elasticity of private capital θ is reported between 0.2–0.5. We adopt $\theta = 0.3$ used in the numerous studies.

First, we consider the relationship between equilibrium values of key variables, ρ and σ. Figure 5 illustrates the curved surfaces of (a) the income tax rate τ^*, (b) the ratio of public investment to tax revenue π_g^*, (c) economic growth rate γ^*, and (d) social welfare W with respect to ρ $(0 \leq \rho \leq 1)$ and σ $(0 \leq \sigma \leq 3)$. Brighter (darker) of colors mean higher (lower) value of each variable. Panel (a) shows that larger ρ or σ leads to larger equilibrium tax rate. In contrast, Panel (b) demonstrates that an increase in σ reduces the ratio of public investment to tax revenue, whereas a rise in ρ increases it. These observations are consistent with Lemmas 1 and 2. In Panels (c) and (d), the bottom right (top left) is brighter (darker) as similar to Panel (b). An increase in ρ raises growth rate and social welfare. However, an increase in σ reduces them. These results correspond to Propositions 2 and 5 and support the analytical results of Propositions 3 and 6. Within realistic values of key parameters, the effects of a change in ρ and σ are monotonic.

[20] They reported that the long-run output elasticities of public capital are 0.193 for regional level and 0.170 for national level, which are nearly equal to 0.2.

We now analyze the total effects of population aging which depend on ε_σ. According to OECD (2016), the average ratio of the voter turnout rate in OECD for age 18–24 to age 25–50 is 0.835. Within G5 countries, the ratios of the voter turnout rate for age 16–35 to age over 55 are 0.761, 0.787, 0.535, and 0.720 in Canada, France, Germany, Italy, the UK, and the US, respectively. A survey report on 2017 House of Representatives general election in Japan indicates that the ratio of age 20–55 to age over 55 in Japan is 0.730.[21] The average ratio of the voter turnout rate in G5 is 0.707. This fact means that the voter turnout rate of the seniors is about 1.4 times that of the young. Even though the data coverage does not perfectly match our model setting, the elderly citizens' political power might be approximately 1.4 times the younger citizens' political power.[22]

Population aging could rapidly strengthen their power. As an example, we specify $\sigma(\rho) = 0.2e^{4\rho}$, leading to $\varepsilon_\sigma = 4\rho$. Survival to age 65 (male, % of cohort) indicates, 86, 86, 90, 88, and 80, in France, Germany, Japan, the UK, and the US, respectively (World Development Indicators, World Bank). For OECD members, its average value is 83%. If $\rho = 0.85$, we obtain $\sigma = 5.993$ and $\varepsilon_\sigma = 3.4$. Hence, we consider the situation that elderly citizens have strong political power, and its marginal increase in response to population ageing is drastic for large value of ρ.

Figure 6 indicates that the ratio of public investment to GDP has an inverted U-shaped relationship with respect to the longevity. Conversely, each of the income tax rate and ratio of public service expenditure to GDP is a monotonically increasing in the longevity. As $\varepsilon_\sigma = 4\rho$ is small for a small value of ρ, the negative indirect effect of population aging on β^* (Lemma 1) is weak and outweighed by the positive direct effect of population aging on β^* (Lemma 2). Larger ρ leads to a larger ε_σ. The indirect effect overweighs the direct one at a certain value of ρ. Hence, a non-monotonic relationship between ρ and β^* exists.

In contrast, an increase in ρ monotonically increases each of τ^* and ζ^* because both direct and indirect effects are positive (Lemmas 1 and 2), and a higher ρ boosts up the latter. Consequently, the share of public investment to tax revenue also has an inverted U-shaped relationship with respect to ρ (Figure 7). This implies that the share of public service expenditure to tax revenue exhibits a U-shaped relationship concerning population aging. These results provide specified cases of Proposition 4.

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[21] OECD (2016) does not report the value for Japan. Alternatively, we use the data from a survey report on 2017 House of Representatives general election for Japan.

[22] Furthermore, the OECD (2019) reports the percentage of people reporting to be uninterested in politics in OECD countries. Based on the report, the ratios of those aged 15–29 to total are 1.36, 1.19, 1.36, 2.06, 1.45, and 1.79 for the OECD average, France, Germany, Japan, the UK, and the US, respectively.

Figure 8 illustrates the relationship among economic growth, social welfare, and population aging. It shows that an inverted U-shaped relationship between population aging and economic growth exists. An increase in ρ has two opposite growth effects: direct and indirect effects under democracies (Propositions 3 and 4). Since elderly citizens' political power is acceleratingly increasing in the longevity, the indirect negative effect of population aging is gradually strengthened relative to the direct positive growth effect of population aging, and the former finally dominates the latter.

Regarding social welfare, Figure 9 indicates the inverted U-shaped curve in the plane of W and ρ. A rise in ρ reduces social welfare through its negative growth effect through a rise in σ (Proposition 5). On the other hand, increase in longevity has a positive welfare effect through itself and cumulative income effect by high economic growth (Proposition 6). When ε_{σ} is rapidly increasing in ρ, the positive direct effect of population aging on welfare is outweighed by the negative indirect welfare effect for a large value of ρ. Conversely, the latter is dominated by the former for the small value of ρ. Since population ageing holds the direct benefits, the value of ρ for maximizing the social welfare is larger than that for maximizing economic growth rate.

6. Conclusion

This paper examines the relationship among government expenditure composition, economic growth, and population aging in democracies as key determinants of long-run welfare. Incorporating probabilistic death from young to old and analogue of bargaining in parliament, we developed an OLG model to illustrate that young and old generations face intergenerational conflicts concerning the determination of public policy. In the model, government revenue based on labor income tax is allocated between public investment and public service expenditure. Public investment promotes the economy-wide productivity in future, and public service expenditure includes current welfare expenditures as a public consumption good. Hence, the retired citizens favor increased tax to finance their increased current welfare expenditures, whereas younger citizens paying income tax favor more public investment to obtain future returns.

Equilibrium policy consists of income tax rate, public investment, and public service expenditure, which are democratically determined as a result of political power balance between young and old generations. Hence, population aging leading to power imbalance affects government revenue, expenditure composition, economic growth, and social welfare. Macroeconomic effects of population aging can be decomposed into direct and indirect effects. The former represents the effects of extending the longevity, whereas the latter represents those of strengthening elderly citizens' political power relative to the young. In particular, indirect effects capture the political imbalance effects in the aged societies. These outcomes are consistent with real observations found by the various empirical studies.

The indirect effects indicate that population aging increases the income tax rate and public service expenditure, and that decreases public investment and therefore economic growth rate and social welfare. In contrast, the direct effects indicate that population aging raises not only all of the income tax rate, public investment, and public service expenditure but also economic growth rate and social welfare. This is because extending longevity requires more savings to cover old-age consumption and provides time for enjoying larger returns on public investment, leading to greater capital accumulation and higher utility. Total effects of population aging depend on which of direct and indirect effects are larger than the other. For almost key variables, direct and indirect effects work in opposite directions with each other. Therefore, an inverted U-shaped relationship exists between population ageing and each of public investment share to tax revenue, economic growth rate, and social welfare if elderly citizens' political power is increased with population aging.

Finally, we should consider future research direction. First, financial sources of government expenditure could vary in reality. For example, corporate income, capital income, and consumption taxes account for large percentages of tax revenue. Debt-financing has been widely observed in various developed and developing countries. Baiardi at al. (2019) empirically found that the composition of taxes significantly affects economic growth—negative and significant correlation between shifting from income taxes to recurrent taxes on immovable property and economic growth. Furthermore, Beqiraj et al. (2018) show that a reduction in the tax burden by fiscal deficits increases the economic growth rate using the panel data of 19 OECD and 12 European countries. Considering the financial sources provides further insights for the analysis of fiscal policy under population aging.

Political institutions and households' belief or preferences are key in the macroeconomic effects of public investment and welfare expenditure. Using data of 80 countries over the 1970–2010, Morozumi and Veiga (2016) show that public capital spending under an accountable government promotes economic growth for various financing sources, including expenditure composition change, increased revenue, and budget deficits. Tamai (2022, 2023) theoretically demonstrates that altruistic

OLG models generate future bias, and that optimal government spending policy is not neutral for intertemporal resource allocation under future bias. In the present study, we use an OLG model without complicated political institutions or intergenerational altruism. We believe these recent theoretical and empirical findings could be incorporated into future research. Finally, our study provides the analytical basis for these future extensions.

Appendix

A. Derivation of Eqs. (16a) and (16b)

Using Eqs. (13) and (15), we have

$$\frac{g'}{z}\frac{\partial z}{\partial g'} = -\frac{\beta}{(1-\theta)\tau-\beta} < 0, \frac{\tau}{z}\frac{\partial z}{\partial \tau} = \frac{(1-\theta)\tau}{(1-\theta)\tau-\beta} > 0, \tag{A1}$$

$$\frac{g'}{z'}\frac{\partial z'}{\partial g'} = \eta > 0, \frac{\tau}{z'}\frac{\partial z'}{\partial \tau} = \frac{(1-\theta)\tau}{(1-\theta)\tau-\beta} - \frac{(1-\eta)\tau}{1-\tau}. \tag{A2}$$

Inserting Eqs. (A1) and (A2) into Eqs. (14a) and (14b) yields

$$-\frac{(1+\sigma\rho)(1-\alpha)\beta}{(1-\theta)\tau-\beta} + \rho\eta = 0 \text{ and } \frac{[1+(1+\sigma)\rho](1-\alpha)(1-\theta)}{(1-\theta)\tau-\beta} - \frac{\alpha+(1-\eta)\rho}{1-\tau} = 0.$$

Solving each equation with respect to β, we arrive at Eqs. (16a) and (16b).

B. Proof of Lemma 1

Total differentiation of Eqs. (16a) and (16b) give

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} d\beta^* \\ d\tau^* \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}\begin{pmatrix} d\sigma \\ d\rho \end{pmatrix}, \tag{A3}$$

where

$$A_{11} = A_{21} = 1, A_{12} = -\frac{(1-\theta)\rho\eta}{1-\alpha+[\eta+(1-\alpha)\sigma]\rho} = -\frac{\beta^*}{\tau^*} > -1,$$

$$A_{22} = -\frac{\{1+[1-\eta+(1-\alpha)(1+\sigma)]\rho\}(1-\theta)}{\alpha+(1-\eta)\rho} < A_{12} < 0,$$

$$B_{11} = -\frac{(1-\alpha)(1-\theta)\eta\rho^2}{\{1-\alpha+[\eta+(1-\alpha)\sigma]\rho\}^2}\tau^* = -\frac{(1-\alpha)\rho\beta^*}{1-\alpha+[\eta+(1-\alpha)\sigma]\rho} < 0,$$

$$B_{12} = \frac{(1-\alpha)(1-\theta)\eta\tau^*}{\{(1-\alpha)(1+\rho\sigma)+\eta\rho\}^2} > 0,$$

$$B_{21} = -\frac{(1-\alpha)(1-\theta)(1-\tau^*)\rho}{\alpha+(1-\eta)\rho} = -\frac{(1-\alpha)[1+\sigma\rho]\beta^*}{[1+(1+\sigma)\rho]\eta} < 0,$$

$$B_{22} = \frac{[1-(1+\sigma)\alpha-\eta](1-\alpha)(1-\tau)(1-\theta)}{[\alpha+(1-\eta)\rho]^2} < 0.$$

The determinant of the coefficient matrix of Eq. (A3) is

$$D \equiv \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} = A_{11}A_{22} - A_{12}A_{21} = A_{22} - A_{12} < 0.$$

Applying the Cramer's formula to Eq. (A3), we obtain

$$\frac{\partial \beta^*}{\partial \sigma} = \frac{B_{11}A_{22} - A_{12}B_{21}}{D} \text{ and } \frac{\partial \tau^*}{\partial \sigma} = \frac{B_{21} - B_{11}}{D}. \tag{A4}$$

We now verify the sign of (A4). Some calculations show

$$B_{21} - B_{11} = -\frac{(1-\alpha)\rho^2\sigma^2 + 2(1-\alpha)\rho\sigma + (1-\alpha) - \eta\rho^2}{\{1-\alpha+[\eta+(1-\alpha)\sigma]\rho\}[1+(1+\sigma)\rho]\eta}$$

$$= -\frac{(1-\alpha)(1+\rho\sigma)^2 - \eta\rho^2}{\{1-\alpha+[\eta+(1-\beta)\sigma]\rho\}[1+(1+\sigma)\rho]\eta}.$$

By Assumption 1, we have $\sigma \geq \eta \Rightarrow B_{21} - B_{11} < 0 \Rightarrow B_{21} < B_{11} < 0$. Therefore, Eq. (A4) shows

$$\frac{\partial \tau^*}{\partial \sigma} = \frac{\overbrace{B_{21} - B_{11}}^{(-)}}{\underbrace{D}_{(-)}} > 0. \tag{A5}$$

We have the following equations:

$$B_{11}A_{22} = \frac{(1-\alpha)\rho\beta^*}{1-\alpha+[\eta+(1-\alpha)\sigma]\rho}\frac{\{1+[1-\eta+(1-\alpha)(1+\sigma)]\rho\}(1-\theta)}{\alpha+(1-\eta)\rho},$$

$$A_{12}B_{21} = \frac{(1-\theta)\rho\eta}{1-\alpha+[\eta+(1-\alpha)\sigma]\rho}\frac{(1-\alpha)(1-\theta)(1-\tau^*)\rho}{\alpha+(1-\eta)\rho}.$$

Using these equations yields

$$B_{11}A_{22} - A_{12}B_{21} = \frac{(1-\alpha)(1-\theta)\rho\{[1+[1-\eta+(1-\alpha)(1+\sigma)]\rho]\beta^* - (1-\theta)(1-\tau^*)\rho\eta\}}{\{1-\alpha+[\eta+(1-\alpha)\sigma]\rho\}[\alpha+(1-\eta)\rho]}$$

$$= \frac{(1-\alpha)(1-\theta)^2\rho^2\eta\left\{\frac{1+[1-\eta+(1-\alpha)(1+\sigma)]\rho}{1-\alpha+[\eta+(1-\alpha)\sigma]\rho} - \frac{1-\tau^*}{\tau^*}\right\}}{\{1-\alpha+[\eta+(1-\alpha)\sigma]\rho\}[\alpha+(1-\eta)\rho]}.$$

The equilibrium tax rate shown in Proposition 1 leads to

$$\frac{1-\tau^*}{\tau^*} = \frac{[\alpha+(1-\eta)\rho](1+\rho\sigma)}{\{1-\alpha+[\eta+(1-\alpha)\sigma]\rho\}[1+(1+\sigma)\rho]}. \tag{A6}$$

Using Eq. (A6), we obtain

$$\frac{1+[1-\eta+(1-\alpha)(1+\sigma)]\rho}{1-\alpha+[\eta+(1-\alpha)\sigma]\rho} - \frac{1-\tau^*}{\tau^*}$$

$$= \frac{(1-\alpha)(1+\rho\sigma)^2 + [2(1-\alpha)\rho\sigma + 2 - \alpha + (2-\alpha-\eta)\rho]\rho}{\{1-\alpha+[\eta+(1-\alpha)\sigma]\rho\}[1+(1+\sigma)\rho]} > 0.$$

The above equation and (A4) derive

$$\frac{\partial \beta^*}{\partial \sigma} = \frac{\overbrace{B_1 A_{22} - A_{12} B_2}^{(+)}}{\underbrace{D}_{(-)}} < 0. \tag{A7}$$

Regarding the effect of a change in σ on ζ^*, using Eqs. (A5) and (A7), the partial derivative of ζ^* with respect to σ becomes

$$\frac{\partial \zeta^*}{\partial \sigma} = (1-\theta)\underbrace{\frac{\partial \tau^*}{\partial \sigma}}_{(+)} - \underbrace{\frac{\partial \beta^*}{\partial \sigma}}_{(-)} > 0.$$

C. Proof of Lemma 2

By the similar way to derive (A5) and (A7), we obtain

$$\frac{\partial \tau^*}{\partial \rho} = \frac{\overbrace{B_{22} - B_{12}}^{(-)}}{\underbrace{D}_{(-)}} > 0, \tag{A8}$$

$$\frac{\partial \beta^*}{\partial \rho} = \frac{\overbrace{B_{12} A_{22} - A_{12} B_{22}}^{(-)}}{\underbrace{D}_{(-)}} > 0. \tag{A9}$$

Using (A8) and (A9), the partial derivative of ζ^* with respect to ρ yields

$$\frac{\partial \zeta^*}{\partial \rho} = (1-\theta)\frac{\partial \tau^*}{\partial \rho} - \frac{\partial \beta^*}{\partial \rho} = \frac{\overbrace{(1-\theta+A_{12})}^{(+)}\overbrace{B_{22}}^{(-)} - \overbrace{(1-\theta-A_{22})}^{(+)}\overbrace{B_{12}}^{(+)}}{\underbrace{D}_{(-)}} > 0,$$

where

$$1-\theta+A_{12} = 1-\theta-\frac{\beta^*}{\tau^*} = (1-\theta)\left[1-\frac{\eta\rho}{(1-\alpha)(1+\rho\sigma)+\eta\rho}\right] > 0.$$

D. Proof of Proposition 3

Evaluating the formula of the direct growth effect of a change in ρ at $\rho = 0$, we obtain

$$\left.\frac{\rho}{\gamma^*}\frac{\partial \gamma^*}{\partial \rho}\right|_{\rho=0} = 1 > 0. \tag{A10}$$

Furthermore, when $\rho = 1$ and $\sigma = 1$,

$$\left.\frac{\rho}{\gamma^*}\frac{\partial \gamma^*}{\partial \rho}\right|_{\rho=1,\sigma=1} = \frac{1}{2} + \left[\frac{1-\eta}{1-\eta+\alpha} + \frac{\sigma}{1+\sigma}\right](1-\eta) + \left[\frac{1}{2} + \frac{1+\sigma}{2+\sigma}\right]\eta - \Phi|_{\rho=1,\sigma=1}$$

$$= \frac{1}{2} + \left[\frac{1}{2} + \frac{1-\eta}{1+\alpha-\eta}\right](1-\eta) + \frac{7\eta}{6} - \frac{2[3(2-\alpha)-1+\eta]}{4(2-\alpha)+\eta} > 0. \tag{A11}$$

We now have

$$\frac{\partial}{\partial\rho}\left(\frac{\rho}{\gamma^*}\frac{\partial\gamma^*}{\partial\rho}\bigg|_{\sigma=1}\right) = -\frac{1}{\rho^2} - \frac{4\eta}{(1+2\rho)^2} - \frac{(1-\eta)^3}{(\alpha+\rho-\eta\rho)^2}$$

$$+ \frac{4(1-\alpha)^2-4\eta}{\{1+\rho[4+(3+\eta)\rho-2\alpha(1+\rho)]\}^2} + \frac{2(3+\eta)-4\alpha}{1+\rho[4+(3+\eta)\rho-2\alpha(1+\rho)]} < 0. \tag{A12}$$

Equations (A10)–(A12) shows that γ^* is monotonically increasing in ρ if $\sigma = 1$:

$$\frac{\rho}{\gamma^*}\frac{\partial\gamma^*}{\partial\rho}\bigg|_{\sigma=1} > 0 \text{ for } 0 \leq \rho \leq 1.$$

E. Proof of Proposition 5

Differentiating Ω with respect to σ yields

$$\frac{\partial\Omega}{\partial\sigma} = (1-\alpha+\eta)\frac{\rho}{1+(1+\sigma)\rho} + (1-\alpha-\eta)\frac{\rho}{1+\rho\sigma} - (1-\alpha)\Lambda. \tag{A13}$$

At $\eta = 1-\alpha$, (A13) becomes

$$\frac{\partial\Omega}{\partial\sigma}\bigg|_{\eta=1-\alpha} = \rho\eta\left\{\frac{2}{1+(1+\sigma)\rho} - \frac{[1+(1+\eta\sigma)\rho]+\eta[1+\rho(1+\sigma)]}{\{1+[1+\eta\sigma]\rho\}(1+\rho\sigma)+[1+(1+\sigma)\rho]\eta\rho}\right\} > 0.$$

With $\eta = 0$, (A13) is

$$\frac{\partial\Omega}{\partial\sigma}\bigg|_{\eta=0} = (1-\alpha)\left[\frac{\rho}{1+(1+\sigma)\rho} + \frac{\rho}{1+\rho\sigma} - \Lambda|_{\eta=0}\right]$$

$$= (1-\alpha)\rho\left[\frac{\alpha+\rho}{[1+(1+\sigma)\rho]\{1+[1+(1-\alpha)(1+\sigma)]\rho\}}\right] > 0,$$

and

$$\frac{\partial^2\Omega}{\partial\eta\partial\sigma} = \frac{\rho}{1+(1+\sigma)\rho} - \frac{\rho}{1+\rho\sigma} - (1-\alpha)\frac{\partial\Lambda}{\partial\eta} = \left\{\frac{\zeta^*\Lambda-(1-\theta)}{(1-\theta)(1+\rho\sigma)[1+(1+\sigma)\rho]}\right\}\rho^2 \geq 0.$$

Therefore, we arrive at

$$\frac{\partial\Omega}{\partial\sigma} > 0 \text{ for } 0 < \eta \leq 1-\alpha.$$

Finally, we consider the sign of (27). If $\delta \to 0$, we obtain

$$\text{sgn}\left(\lim_{\delta\to 0}\frac{\partial W}{\partial\sigma}\right) = \text{sgn}\left(\frac{1}{\gamma^*}\frac{\partial\gamma^*}{\partial\sigma}\right) < 0.$$

If $\delta \to \infty$, Eq. (27) becomes

$$\lim_{\delta\to\infty}\frac{\partial W}{\partial\sigma} = (1+2\rho)\frac{\partial\Omega}{\partial\sigma} + \rho\frac{1}{\gamma^*}\frac{\partial\gamma^*}{\partial\sigma} + \alpha\frac{1}{1-\tau^*}\frac{\partial(1-\tau^*)}{\partial\sigma}$$

$$= (1+2\rho)\left[(1-\alpha+\eta)\frac{\rho}{1+(1+\sigma)\rho} + (1-\alpha-\eta)\frac{\rho}{1+\rho\sigma} - (1-\alpha)\Lambda\right]$$

$$+\rho\left[(1-\eta)\frac{\rho}{1+\rho\sigma} + \eta\frac{\rho}{1+(1+\sigma)\rho} - \Lambda\right] + \alpha\left[\frac{\rho}{1+\rho\sigma} - \Lambda\right]. \quad \text{(A14)}$$

When $\eta = 0$, Eq. (A14) can be written as

$$\lim_{\delta\to\infty}\frac{\partial W}{\partial\sigma}\bigg|_{\eta=0} = (1+2\rho)\frac{\partial\Omega}{\partial\sigma} + \rho\frac{1}{\gamma^*}\frac{\partial\gamma^*}{\partial\sigma} + \alpha\frac{1}{1-\tau^*}\frac{\partial(1-\tau^*)}{\partial\sigma}$$

$$= \frac{(1-\alpha)(\alpha+\rho)(1-\sigma)\rho^2}{[1+(1+\sigma)\rho]\{1+\rho[2-\alpha+(1-\alpha)\sigma]\}} \gtreqless 0 \Leftrightarrow \sigma \lesseqgtr 1.$$

At the same time, we have

$$\frac{\partial}{\partial\eta}\left(\lim_{\delta\to\infty}\frac{\partial W}{\partial\sigma}\right) = -\frac{\rho^2(1+3\rho)}{(1+\rho\sigma)[1+(1+\sigma)\rho]}$$

$$-\frac{\rho^3[1+(3-2\alpha)\rho][(2-\alpha)(1+\rho)+2(1-\alpha)\rho\sigma]}{\{1+\rho[2+\eta\rho-\alpha(1+\sigma)(1+\rho\sigma)+\sigma(2+\rho(2+\sigma))]\}^2} < 0.$$

These properties show

$$\sigma \geq 1 \Rightarrow \lim_{\delta\to\infty}\frac{\partial W}{\partial\sigma}\bigg|_{\eta=0} \leq 0 \Rightarrow \lim_{\delta\to\infty}\frac{\partial W}{\partial\sigma} \leq 0 \text{ for } \eta \geq 0,$$

$$\sigma < 1 \Rightarrow \lim_{\delta\to\infty}\frac{\partial W}{\partial\sigma}\bigg|_{\eta=0} > 0 \Rightarrow \lim_{\delta\to\infty}\frac{\partial W}{\partial\sigma} > 0 \ (<) \text{ for } \eta < \hat{\eta} \ (>) \text{ where } \lim_{\delta\to\infty}\frac{\partial W}{\partial\sigma}\bigg|_{\eta=\hat{\eta}} = 0.$$

F. Proof of Proposition 6

Partial differentiation of Ω with respect to ρ gives

$$\rho\frac{\partial\Omega}{\partial\rho} = \eta\frac{\rho}{x^*}\frac{\partial x^*}{\partial\rho} + (1-\alpha)\frac{\rho}{\zeta^*}\frac{\partial\zeta^*}{\partial\rho}$$

$$= (1-\alpha-\eta)\left\{\frac{(1+\sigma)\rho}{1+(1+\sigma)\rho} + \frac{\rho\sigma}{1+\rho\sigma} - \frac{\rho}{(1+\rho)[\alpha+(1-\eta)\rho]}\right\} - (1-\alpha)\rho\Phi. \quad \text{(A15)}$$

Evaluating (A15) at $\eta = 1-\alpha$ and $\eta = 0$

$$\frac{\partial\Omega}{\partial\rho}\bigg|_{\eta=1-\alpha} = -(1-\alpha)\rho\Phi|_{\eta=1-\alpha} < 0,$$

$$\left.\frac{\partial\Omega}{\partial\rho}\right|_{\eta=0} = (1-\alpha)\left[\frac{\rho\sigma}{1+\rho\sigma}+\frac{(1+\sigma)\rho}{1+(1+\sigma)\rho}-\frac{\rho\sigma+\rho[1+(1-\alpha)(1+\sigma)](1+2\rho\sigma)}{\{1+[1+(1-\alpha)(1+\sigma)]\rho\}(1+\rho\sigma)}\right]$$

$$= \frac{(1-\alpha)[\alpha(1+\sigma)-1]\rho}{[1+(1+\sigma)\rho]\{1+[1+(1-\alpha)(1+\sigma)]\rho\}} \gtreqless 0 \Leftrightarrow \sigma \gtreqless \frac{1-\alpha}{\alpha},$$

with

$$\frac{\partial^2\Omega}{\partial\eta\partial\rho} = -\frac{\sigma}{1+\rho\sigma}-\frac{\{\alpha^2+[2(1-\eta)-1]\alpha+(1-\eta)^2\rho^2\}(1+\sigma)-\alpha}{[1+(1+\sigma)\rho][\alpha+(1-\eta)\rho]^2}-(1-\alpha)\frac{\partial\Phi}{\partial\eta}<0,$$

where

$$\frac{\partial\Phi}{\partial\eta} = \frac{2+\rho\sigma+[1+(1-\alpha)(1+\sigma)]\rho}{\left\{\{1+[1+(1-\alpha)(1+\sigma)]\rho\}(1+\rho\sigma)+\eta\rho^2\right\}^2}\rho > 0.$$

These properties of Ω show that Ω is increasing in ρ if σ is sufficiently large and η is sufficiently small. Therefore, positive direct effect of population aging on Ω requires

$$\sigma > \frac{1-\alpha}{\alpha} \text{ and } \alpha+\eta \ll 1.$$

We now consider the sign of (29). When $\delta \to 0$, we obtain

$$\operatorname{sgn}\left(\lim_{\delta\to 0}\frac{\partial W}{\partial\rho}\right) = \operatorname{sgn}\left(\frac{\partial\gamma^*}{\partial\rho}\right) > 0 \text{ as } \rho \to 0.$$

If $\delta \to \infty$, Eq. (29) leads to

$$\lim_{\delta\to\infty}\frac{\partial W}{\partial\rho} = 2\Omega+(2+\rho)\frac{\partial\Omega}{\partial\rho}-\frac{\alpha}{1-\tau^*}\frac{\partial\tau^*}{\partial\rho}+\rho\left[\log\gamma^*+\frac{1}{\gamma^*}\frac{\partial\gamma^*}{\partial\rho}\right]. \qquad \text{(A16)}$$

When $\sigma > (1-\alpha)\alpha^{-1}$ and $\alpha+\eta \ll 1$, Eq. (A16) is

$$\lim_{\delta\to\infty}\frac{\partial W}{\partial\rho} = 2\Omega-\frac{\alpha}{1-\tau^*}\frac{\partial\tau^*}{\partial\rho}+\underbrace{(2+\rho)\frac{\partial\Omega}{\partial\rho}+\rho\left[\log\gamma^*+\frac{1}{\gamma^*}\frac{\partial\gamma^*}{\partial\rho}\right]}_{(+)}.$$

Using $\Omega \geq \alpha$ (Assumption 2),

$$2\Omega-\frac{\alpha}{1-\tau^*}\frac{\partial\tau^*}{\partial\rho} > 0 \text{ if } \Phi < 2.$$

After some calculations, we have

$$2-\Phi$$
$$= \frac{(2\rho+(1-\alpha)\rho-1)\sigma+(2-\alpha)\rho+2[1-(1-\rho)\eta\rho]+[1+(1-\alpha)(1+\sigma)](1+2\rho\sigma)}{\{1+[1+(1-\alpha)(1+\sigma)]\rho\}(1+\rho\sigma)+\eta\rho^2}.$$

Evaluating the equation mentioned above at $\rho = 0$,

$$2-\Phi|_{\rho=0} = 4-(1+\sigma)\alpha \gtreqless 0 \Leftrightarrow \sigma \lesseqgtr \frac{4-\alpha}{\alpha}.$$

Therefore, if $\alpha + \eta \ll 1$ and $(1-\alpha)\alpha^{-1} < \sigma \leq (4-\alpha)\alpha^{-1}$, we obtain

$$\lim_{\delta \to \infty} \frac{\partial W}{\partial \rho} > 0 \text{ for } \rho \approx 0.$$

References

Achen, C. and T.Y. Wang (2019), Declining voter turnout in Taiwan: a generational effect? *Electoral Studies*, 58 (1), 113–124.

Afonso, A. and J. Alegre (2011), Economic growth and budgetary components: a panel assessment for the EU, *Empirical Economics*, 41 (3), 703–723.

An, Z., A. Kangur, and C. Papageorgiou (2019), On the substitution of private and public capital in production, *European Economic Review*, 118, 296–311.

Angelopoulos, K., G. Economides, and P. Kammas (2007), Tax-spending policies and economic growth: theoretical predictions and evidence from the OECD, *European Journal of Political Economy*, 23 (4), 885–902.

Aoki, T. and K. Nishimura (2017), Global convergence in an overlapping generations model with two-sided altruism, *Journal of Evolutionary Economics*, 27 (5), 1205–1220.

Arai, R., K. Naito, and T. Ono (2018), Intergenerational policies, public debt, and economic growth: a politico-economic analysis, *Journal of Public Economics*, 166, 39–52.

Arin, K.P., E. Braunfels, and G. Doppelhofer (2019), Revisiting the growth effects of fiscal policy: a Bayesian model averaging approach, *Journal of Macroeconomics*, 62, 103158.

Arrow, K.J. and M. Kurz (1970), *Public Investment, the Rate of Return, and Optimal Fiscal Policy*, Johns Hopkins University Press, Baltimore, US.

Aschauer, D.A. (1989), Is public expenditure productive?, *Journal of Monetary Economics*, 23 (2), 177–200.

Azzimonti, M. (2015), The dynamics of public investment under persistent electoral advantage, *Review of Economic Dynamics*, 18 (3), 653–678.

Baiardi, D., P. Profeta, R. Puglisi, and S. Scabrosetti (2019), Tax policy and economic growth: does it really matter?, *International Tax and Public Finance*, 26 (2), 282–316.

Barro, R.J. (1990), Government spending in a simple model of endogenous growth, *Journal of Political Economy*, 98 (5), part2, S103–S125.

Berry, C. (2008), Labour's lost youth: young people and the Labour Party's youth section, *Political Quarterly*, 79 (2), 366–376.

Beqiraj, E., S. Fedeli, and F. Forte (2018), Public budgetary rules and GDP growth: an empirical study on OECD and twelve European countries, *Southern Economic Journal*, 85 (1), 170–188.

Binstock, R.H. (2012), Older voters and the 2010 U.S. election: implications for 2012 and beyond? *The Gerontologist*, 52 (3), 408–417.

Blanchard, O.J. (1985), Debt, deficits, and finite horizons, *Journal of Political Economics*, 93 (2), 223–247.

Bleaney, M., N. Gemmell, and R. Kneller (2001), Testing the endogenous growth model: public expenditure, taxation, and growth over the long-run, *Canadian Journal of Economics*, 34 (1), 36–57.

Bom, P.R.D. and J.E. Ligthart (2014), What have we learned from three decades of research on the productivity of public capital?, *Journal of Economic Surveys*, 28 (5), 889–916.

Brunner, E.J. and E.B. Johnson (2016), Intergenerational conflict and the political economy of higher education funding, *Journal of Urban Economics*, 91, 73–87.

Chen, B.-L. (2006), Economic growth with an optimal public spending composition, *Oxford Economic Papers*, 58 (1), 123–136.

Chu, T.T., J. Hölscher, and D. McCarthy (2020), The impact of productive and non-productive government expenditure on economic growth: an empirical analysis in high-income versus low- to middle-income economies, *Empirical Economics*, 58 (5), 2403–2430.

Conde-Ruiz, J.I. and V. Galasso (2005), Positive arithmetic of the Welfare State, *Journal of Public Economics*, 89, 933–955.

Davidson, S. (2012), *Going Grey: The Mediation of Politics in an Ageing Society*, Routledge, London, UK.

Devarajan, S., V. Swaroop, and H. Zou (1996), The composition of public expenditure and economic growth, *Journal of Monetary Economics*, 37 (2), 313–344.

Diamond, P.A. (1965), National debt in a neoclassical growth model, *American Economic Review*, 55 (5), 1126–1150.

Economides, G., H. Park, and A. Philippopoulos (2011), How should the government allocate its tax revenues between productivity-enhancing and utility-enhancing public goods? *Macroeconomic Dynamics*, 15 (3), 336–364.

Felice, G. (2016), Size and composition of public investment, sectoral composition and growth, *European Journal of Political Economy*, 44, 136–158.

Fisher, W.H. and S.J. Turnovsky (1998), Public investment, congestion and private capital accumulation, *Economic Journal*, 108 (447), 399–413.

Futagami, K., Y. Morita, and A. Shibata (1993), Dynamic analysis of an endogenous growth model with public capital, *Scandinavian Journal of Economics*, 95 (4), 607–625.

Galasso, V. and P. Profeta (2002), The political economy of social security: a survey, *European Journal of Political Economy*, 18, 1–29.

Galasso, V., P. Profeta, J. Haskel and J. Ventura (2004), Lessons for an aging society: the political sustainability of social security systems, *Economic Policy*, 19 (38), 63–115.

Glomm, G. and B. Ravikumar (1997), Productive government expenditures and long-run growth, *Journal of Economic Dynamics and Control*, 21 (1), 183–204.

Ghosh, S. and U. Roy (2004), Fiscal policy, long-run growth, and welfare in a stock-flow model of public goods, *Canadian Journal of Economics*, 37 (3), 742–756.

Grasso, M.T. (2014), Age, period and cohort analysis in a comparative context: political generations and political participation repertoires in Western Europe, *Electoral Studies*, 33, 63–76.

Grossman, G.M. and E. Helpman (1998), Intergenerational redistribution with short-lived governments, *Economic Journal*, 108 (450), 1299–1329.

Holbein, J.B. and D.S. Hillygus (2016), Making young voters: the impact of preregistration on youth turnout, *American Journal of Political Science*, 60 (2), 364–382.

Hori, H. (1997), Dynamic allocation in an altruistic overlapping generations economy, *Journal of Economic Theory*, 73 (2), 292–315.

Irmen, A., and J. Kuehnel (2009), Productive government expenditure and economic growth, *Journal of Economic Surveys*, 23 (4), 692–733.

Jäger, P. and T. Schmidt (2016), The political economy of public investment when population is aging: a panel cointegration analysis, *European Journal of Political Economy*, 43, 145–158.

Kaas, L. (2003), Productive government spending, growth, and sequential voting, *European Journal of Political Economy*, 19 (2), 227–246.

Kalaitzidakis, P. and S. Kalyvitis (2004), On the macroeconomic implications of maintenance in public capital, *Journal of Public Economics*, 88 (3–4), 695–712.

Kamiguchi, A. and T. Tamai (2019), Public investment, public debt, and population aging under the golden rule of public finance, *Journal of Macroeconomics*, 60, 110–122.

Katsimi, M. and V. Sarantides (2012), Do elections affect the composition of fiscal policy in developed, established democracies?, *Public Choice*, 151 (1–2), 325–362.

Kneller, R., M.F. Bleaney, and N. Gemmell (1999), Fiscal policy and growth: evidence from OECD countries, *Journal of Public Economics*, 74 (2), 171–190.

Kuehnel, J. (2011), Population aging, the composition of government spending, and endogenous economic growth in politico-economic equilibrium, Working Papers 0510, University of Heidelberg, Department of Economics.

Lee, J. (1992), Optimal size and composition of government spending, *Journal of the Japanese and International Economies*, 6 (4), 423–439.

Ligthart, J.E. and R.M.M. Suárez (2011), The productivity of public capital: a Meta-analysis, in W. Manshanden and W. Jonkhoff (eds.), *Infrastructure Productivity Evaluation*, Springer, New York, 5–32.

Lindbeck, A. and J.W. Weibull (1987), Balanced-budget redistribution as the outcome of political competition, *Public Choice*, 52 (3), 273–297.

Lopez-Velasco, A.R. (2024), Markov equilibrium of social security: an analytic solution under CRRA utility and the future of social security, *Economic Modelling*, 132, 106655.

Marsiliani, L. and T.I. Renström (2007), Political institutions and economic growth, *Economics of Governance*, 8 (3), 233–261.

Morozumi, A. and F. Veiga (2016), Public spending and growth: the role of government accountability, *European Economic Review*, 89, 148–171.

Müller, A., K. Storesletten, and F. Zilibotti (2016), The political color of fiscal responsibility, *Journal of the European Economic Association*, 14, 252–302.

Niepelt, D. and M. Gonzalez-Eiras (2012), Ageing, government budgets, retirement, and growth, *European Economic Review*, 56, 97–115.

OECD (2016), *Society at a Glance 2016: OECD Social Indicators,* OECD Publishing, Paris.

OECD (2019), *Society at a Glance 2019: OECD Social Indicators,* OECD Publishing, Paris.

OECD (2022), Elderly population (indicator), Infrastructure investment (indicator). doi: 10.1787/8d805ea1-en; Old-age dependency ratio (indicator). doi: 10.1787/e0255c98-en. (Accessed on 13 June 2022)

Parijs, P.V. (1998), The disfranchisement of the tlderly, and other attempts to secure intergenerational justice, *Philosophy and Public Affairs*, 27 (4), 292–333.

Pereira, A.M. and J.M. Andraz (2013), On the economic effects of public infrastructure investment: a survey of the international evidence, *Journal of Economic Development*, 38 (4), 1–37.

Razin, A., E. Sadka, and P. Swagel (2002), The aging population and the size of the Welfare State, *Journal of Political Economy*, 110 (4), 900–918.

Romero-Ávila, D. and R. Strauch (2008), Public finances and long-term growth in Europe: evidence from a panel data analysis, *European Journal of Political Economy*, 24 (1), 172–191.

Sanz, I. and F.J. Velázquez (2007), The role of ageing in the growth of government and social welfare spending in the OECD, *European Journal of Political Economy*, 23 (4), 917–931.

Sevi, S., (2021), Do young voters vote for young leaders? *Electoral Studies*, 69, 102200.

Song, Z., K. Storesletten, and F. Zilibotti (2012), Rotten parents and disciplined children: a politico-economic theory of public expenditure and debt, *Econometrica*, 80, 2785–2803.

Stockemer, D. and A. Sundström (2023), Age inequalities in political representation, a review article, *Government and Opposition*, 1–18.

Tamai, T. (2022), Economic growth, equilibrium welfare, and public goods provision with intergenerational altruism, *European Journal of Political Economy*, 71, 102068.

Tamai, T. (2023), The rate of discount on public investments with future bias in an altruistic overlapping generations model, *European Journal of Political Economy*, 79, 102416.

Turnovsky, S.J. (1996), Optimal tax, debt, and expenditure policies in a growing economy, *Journal of Public Economics*, 60 (1), 21–44.

Uchida, Y. and T. Ono (2024), Generational distribution of fiscal burdens: a positive analysis, *International Economic Review*, 65, 393–430.

Välilä, T (2020), Infrastructure and growth: a survey of macro-econometric research, *Structural Change and Economic Dynamics*, 53 (C), 39–49.

Yaari, M.E. (1965), Uncertain lifetime, life insurance, and the theory of the consumer, *Review of Economic Studies*, 32 (2), 137–150.

Yakita, A. (2001), Uncertain lifetime, fertility and social security, *Journal of Population Economics*, 14 (4), 635–640.

Yakita, A. (2008), Ageing and public capital accumulation, *International Tax and Public Finance*, 15 (5), 582–598.

Zhang, L., Y. Ru, and J. Li (2016), Optimal tax structure and public expenditure composition in a simple model of endogenous growth, *Economic Modelling*, 59, 352–360.

Figures

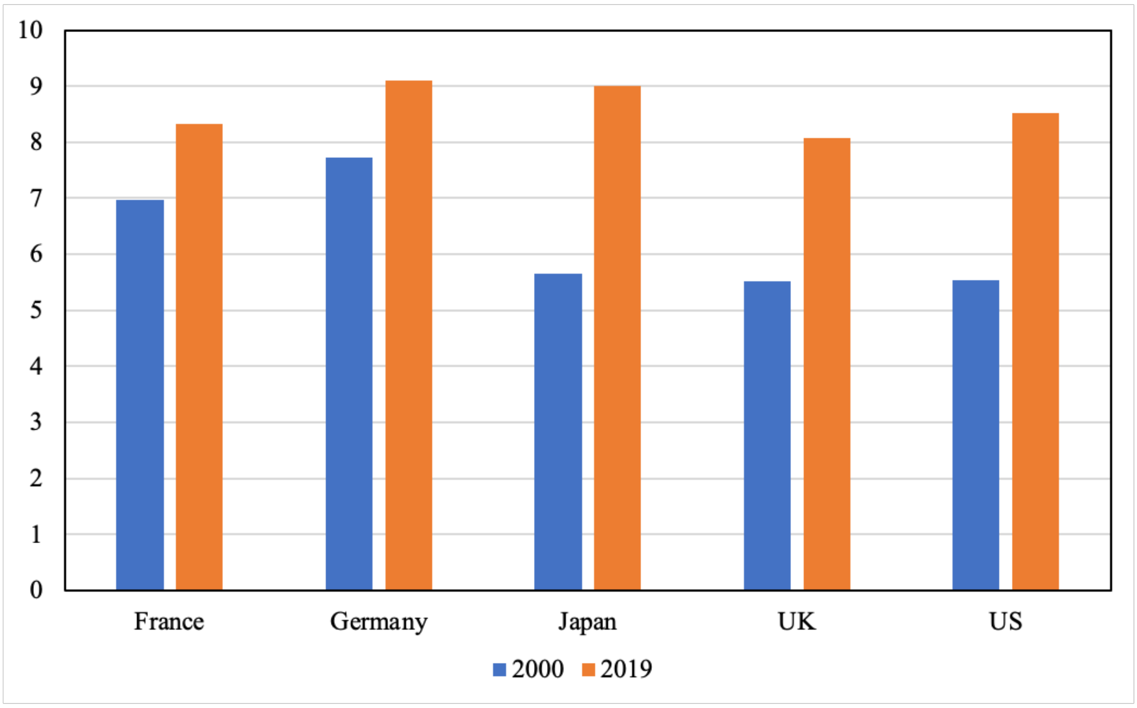


Figure 1. General government health expenditure (% of GDP) in G5 countries

Data: World Development Indicators (World Bank)

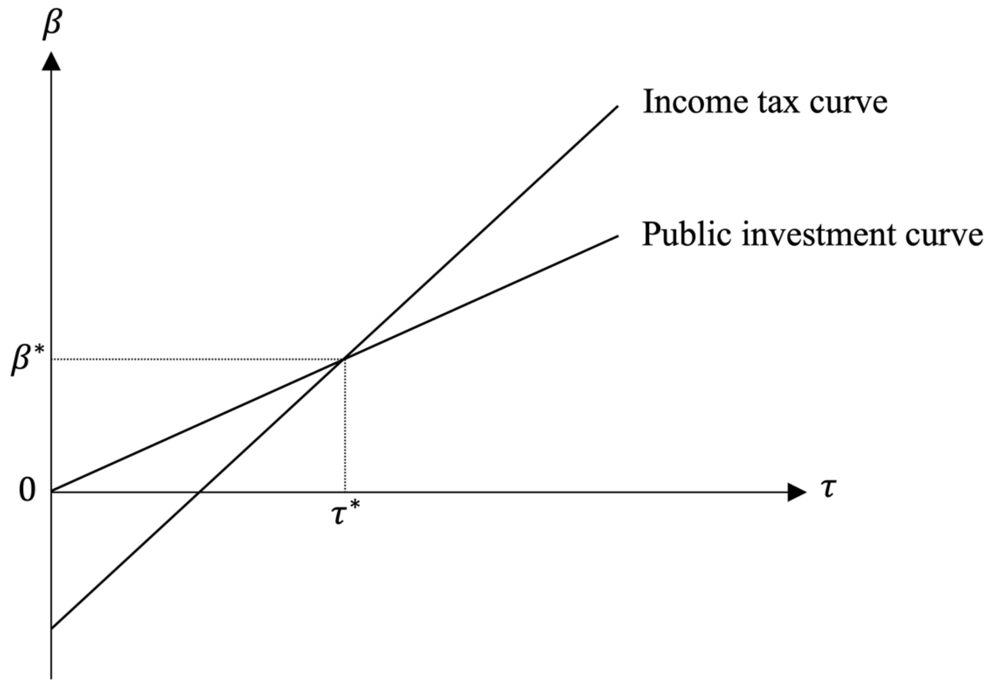


Figure 2. The existence and uniqueness of politico-economic equilibrium

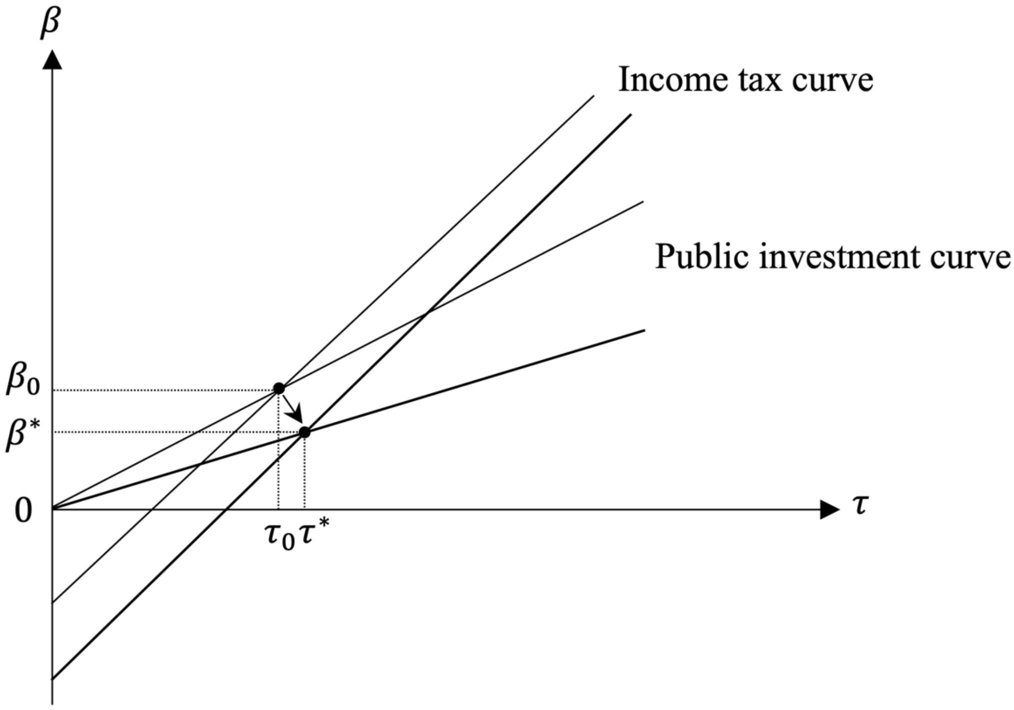


Figure 3. Effects of a rise in σ on equilibrium policies

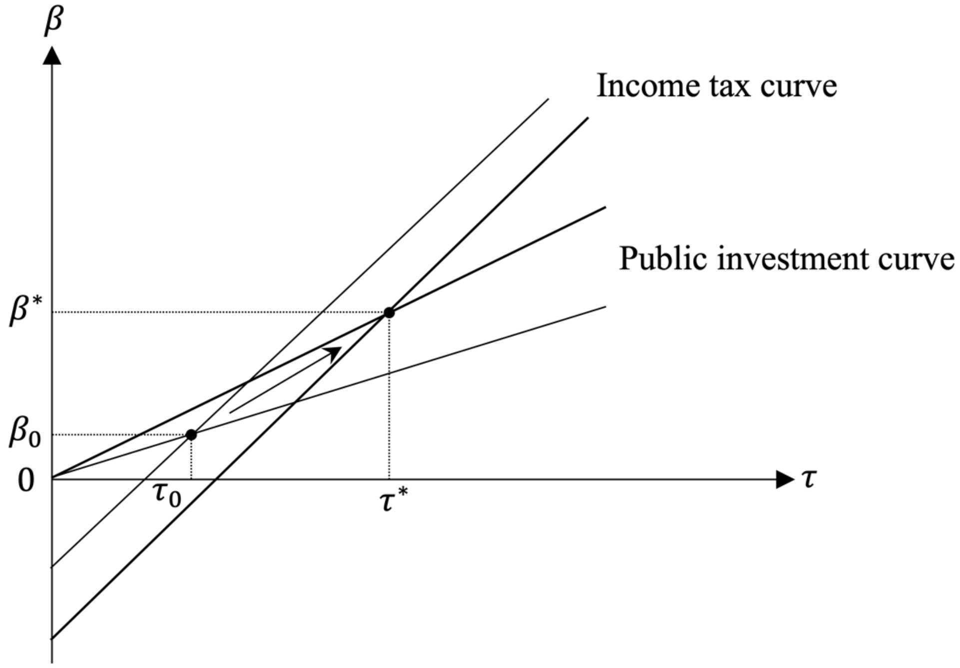
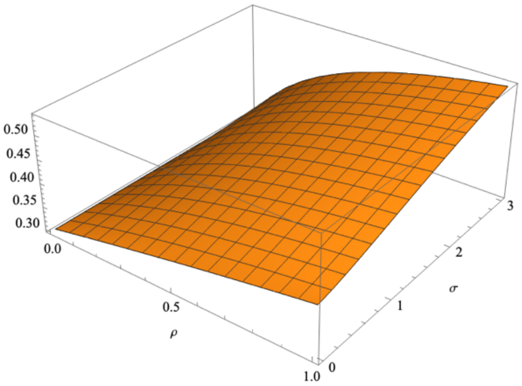
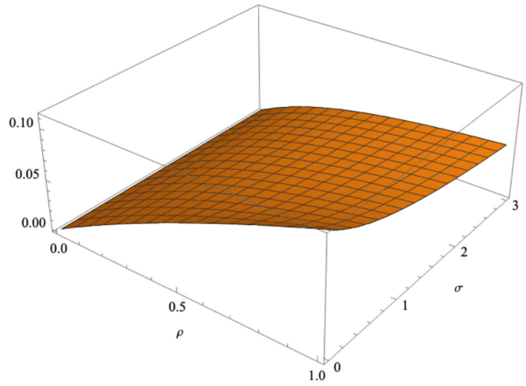


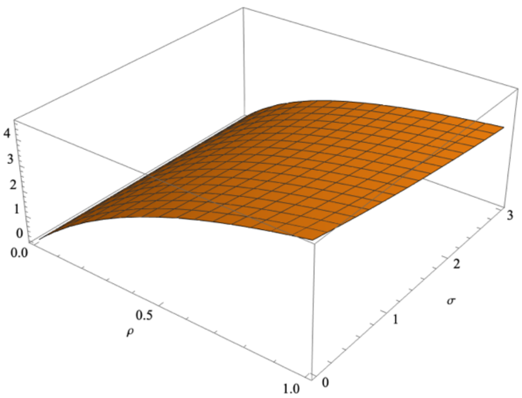
Figure 4. Effects of a rise in ρ on equilibrium policies



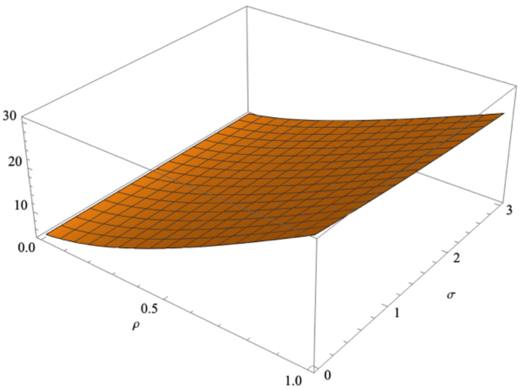
(a) Tax rate



(b) Ratio of public investment to tax revenue



(c) Growth rate



(d) Social welfare

Figure 5. Equilibrium values of some key variables with different values of $\boldsymbol{\rho}$ and $\boldsymbol{\sigma}$

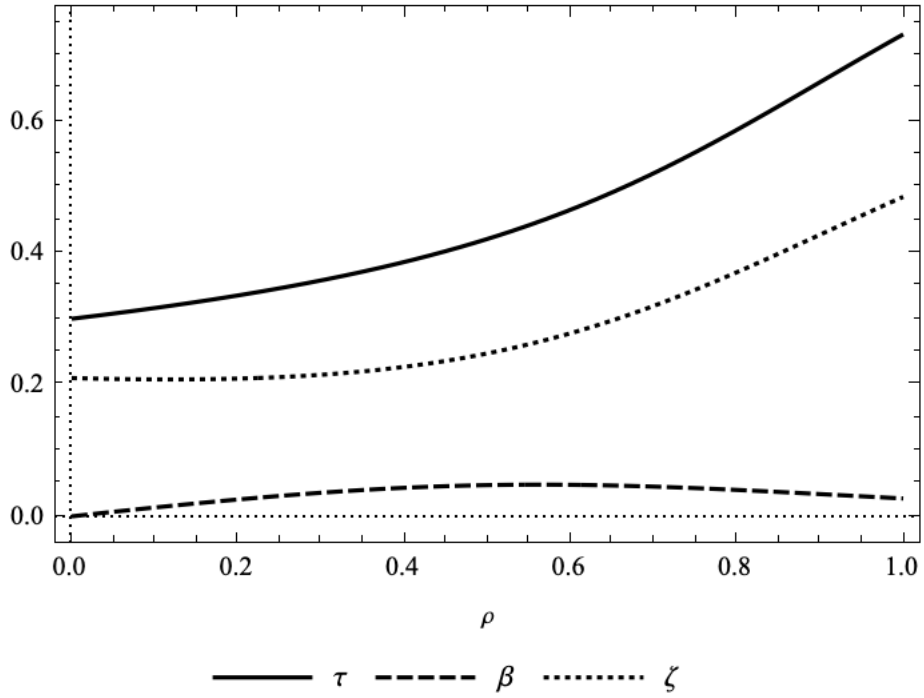


Figure 6. Income tax rate, Expenditure to GDP ratios, and population ageing

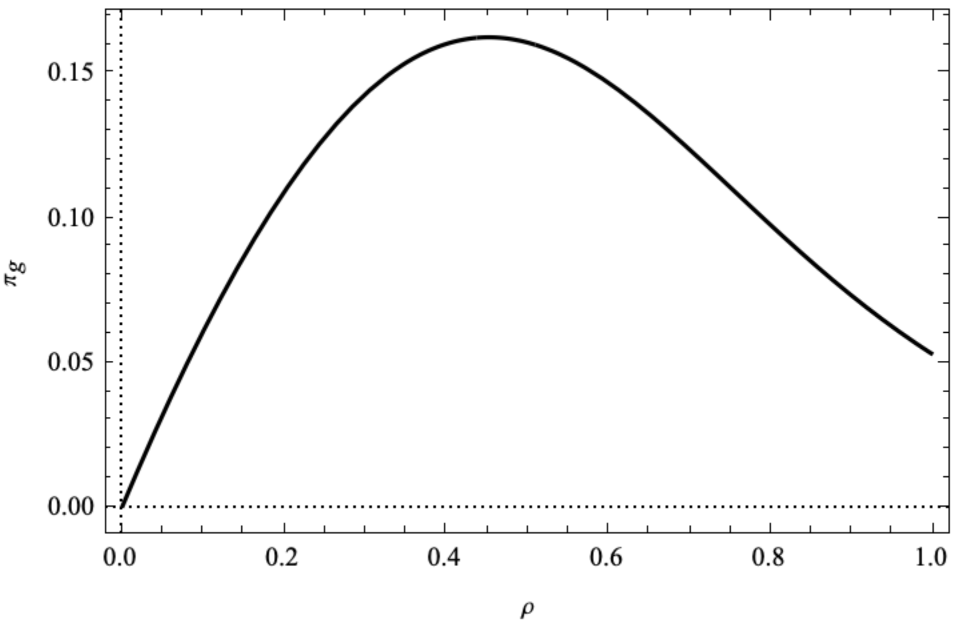


Figure 7. Ratio of public investment to tax revenue and population ageing

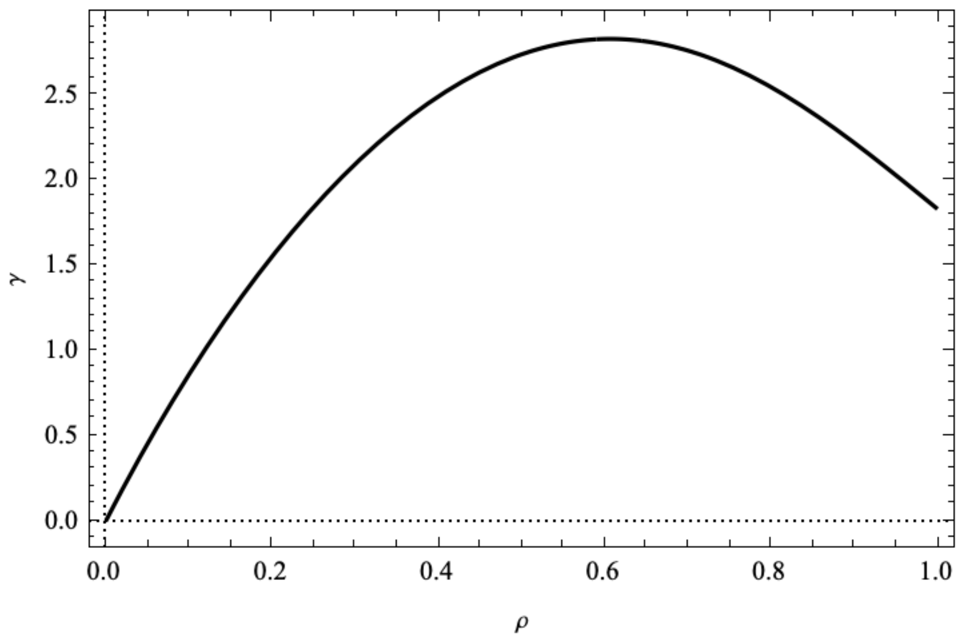


Figure 8. Economic growth and population ageing

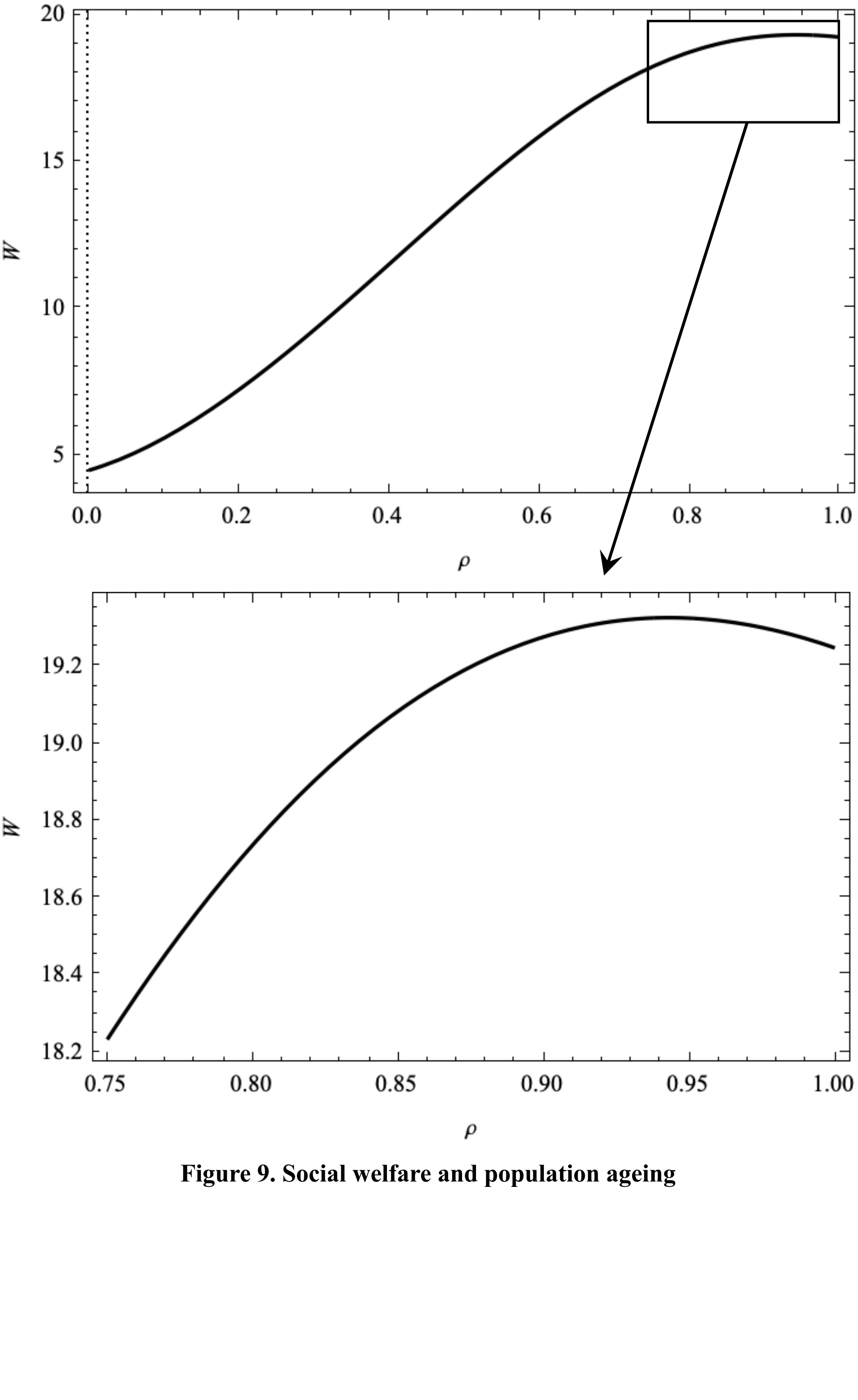


Figure 9. Social welfare and population ageing