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Abstract

This paper examines the relationship between factor complements and the efficiency of providing public inputs in a two-countries model of fiscal competition. The degree of factor complements between capital and public input is characterized by the cross-derivative of the production function. The analysis shows that a stronger degree of complementarity between capital and public inputs leads to higher taxes and higher public inputs than weaker complementarity. In the identical two countries, a stronger degree of complementarity reduces the overprovision of public inputs because one unit of capital increases a smaller unit of marginal productivity of public input. However, if the countries are asymmetric with respect to the degree of complementarity, overprovision of public inputs will occur, at least in the country with a weaker degree of complementarity, if the productivity of public inputs is sufficiently low. Numerical analysis reveals that both countries overprovide public inputs if the productivity of public inputs is sufficiently low.

JEL Classification: F21; H25; H71; H72

Keywords: Tax competition; Public inputs; Capital tax; Factor complements

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1. Introduction

In the global economy, countries compete against each other in their public policy to attract foreign direct investment (FDI). In particular, governments seek to stimulate employment and national income by deriving capital inflow using a good infrastructure and low business costs. Indeed, empirical studies found that public inputs such as infrastructure increase capital inflow (Cheng and Kwan, 2000; Hoffmann, 2003; Kang and Lee, 2007; Duan et al., 2021).[1] Such an environment naturally backgrounds government competition in fiscal policy.

Fiscal competition has been examined theoretically since the seminal works on tax competition presented by Wilson (1986) and Zodrow and Mieszkowski (1986). Their tax competition models showed that such competition leads to inefficiently low tax rates, and the governments cannot provide public goods sufficient to their optimum.[2] In particular, Zodrow and Mieszkowski (1986) argue that such undersupply occurs even if the public inputs are incorporated. However, the empirical evidence seems to support the governments' competition in public inputs.

Numerous Numerous theoretical studies analyze the possibility of overprovision of public inputs (e.g., Noiset, 1995; Bayindir-Upmann, 1998; Matsumoto, 1998, 2000; Dhillon et al., 2007).[3] When the expenditures of public inputs are financed by capital tax, two effects exist: capital outflow by raising the tax rate of the own country and capital inflow effect by increasing public inputs through the factor complements. If the former effect dominates the latter, governments overprovide public input (Noiset, 1995). However, this result of overprovision of public inputs depends on the types of public inputs (production technology) and mobilities of capital and firms (Matsumoto, 1998, 2000).

From the empirical analysis, Bénassy-Quéré et al. (2007) found that an increase in public input by raising the corporate tax rate decreases inward FDI: -1.1 for the tax elasticity of capital inflow and +0.2 for the public input elasticity. This estimated result implies that tax competition may lead to the underprovision of public inputs. On the other hand, they suggest that fiscal competition could lead to more diverse outcomes than a simple "race to the bottom" (low tax and low public inputs) situation. Different countries may experience different outcomes depending on local circumstances.[4]

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[1] This effect is not only limited to a usual concept of infrastructure: Globerman and Shapiro (2002) found that the governance infrastructure, such as the political, institutional, and legal environment, has a positive effect on FDI inflow.
[2] Zodrow (2010) provides an excellent survey of the literature on tax competition.
[3] Bayindir-Upmann (1998) focused on the choice of policy instruments (tax vs expenditure). Dhillon et al. (2007) characterize the provision efficiency depending on the concepts of local vs global concavity.
[4] They also found that a "high equilibrium" (high tax and high public input) can be reached from a "low equilibrium" (low tax and low public input) if households have a sufficiently strong preference for public goods.

This paper reexamines the efficiency of public input provision characterized by the factor complements and asymmetric countries in a two-countries model of fiscal competition. The analysis of this paper is related to those presented by Gugl and Zodrow (2015, 2019) and Matsumoto and Sugahara (2017), who focus on the properties of the production technology: log modularity of production technology and the neutrality of technological changes, respectively.

Instead of their concepts, this paper derives the condition for the overprovision of public inputs characterized by q-complements presented in Hicks (1970) under the specified production function. Our main findings are summarized as follows: First, the degree of q-complements between capital and public inputs characterizes the socially optimal policy of tax and public inputs in the two countries with the asymmetricities of the degree of q-complements. The country with a larger degree of q-complements attracts more capital and sets a higher tax rate to provide higher public inputs than the other with a smaller degree of q-complements.

Second, we derive the necessary and sufficient condition for the overprovision of public inputs under symmetric countries, corresponding to the specified condition shown by Gugl and Zodrow (2019). Under full symmetricity, a larger degree of q-complements decreases the possibility of overprovision of public inputs because the tax elasticity of capital inflow tends to exceed the public input elasticity. Moreover, higher productivity of public inputs leads to underprovision of public inputs regardless of the degree of q-complements.

Third, we find that the country with a smaller degree of q-complements oversupplies public inputs if the productivity of public inputs is sufficiently low. The country with a smaller degree of q-complements is incentivized to raise the tax rate to attract capital. As a result of the other's policy for attracting capital, the country loses national income through capital outflow if the government does not take any counter-action. Furthermore, numerical analysis shows that both countries overprovide public inputs if the productivity of public inputs is sufficiently low.

The remainder of this paper is organized as follows. Section 2 explains the settings of our basic model. Section 3 characterizes the outcomes of decentralized equilibrium in two cases of symmetric and asymmetric two countries, comparing social optimum derived from social planner's optimization problem. Section 4 provides further analysis of public input provision and tax competition. Finally, Section 5 delivers the conclusions of this paper.

2. The model

We consider a two-countries model of tax competition with public input provision based on the model of Zodrow and Mieszkowski (1986). There exists a continuum of firms in each region. The total mass of firms is normalized to unity. The firms in country i ($i = 1,2$) have the production function, $y_i = F_i(k_i, g_i, l_i)$, where y_i is the output of homogenous good, k_i is the capital input, g_i is the public input provision, and l_i, is the labor input, and z_i the land input. To ensure the possibility of overprovision of public inputs, we assume that F_i is a constant-returns-to-scale and increasing in each input (Matsumoto, 1998).

The labor supply of each region is also normalized to unity ($l_i = 1$). The capital is mobile across two counties, while the residents of each region who supply the labor input are stuck to each region. Moreover, the total supply of capital (endowment) is fixed at $\bar{k} > 0$. We assume that the residents of the two countries equally share the capital endowment. Hence, the representative resident of each country owns the capital $\bar{k}/2$.

For given g_i, the firms choose k_i to maximize

$$\pi_i = f_i(k_i, g_i) - (r + t_i)k_i, \tag{1}$$

where $f_i(k_i, g_i) \equiv F_i(k_i, g_i, 1)$, r is the interest rate common with two countries, and t_i is the unit tax on capital. The first-order conditions for a firm's optimization problem are

$$r = f_{ik} - t_i, \tag{2}$$

where $f_{ik} \equiv \partial f_i(k_i, g_i)/\partial k_i$. Eq. (2) derives the demand function of capital as the function of r, t_i, and g_i.

The country i's government provides public input, financing its expenditure by the tax on capital used in the country. The government's budget constraint is

$$g_i = t_i k_i. \tag{3}$$

Note that the public input has no spillover effect.

The residents of each country receive capital income, and the other income is the reward for supplying inputs. Since the residents own $\bar{k}/2$ unit of capital. The representative resident's budget constraint is

$$x_i = \pi_i + \frac{r\bar{k}}{2}, \tag{4}$$

where x_i denotes private consumption.

The resident's preference is assumed to be $u(x_i) = x_i$. Using Eqs. (1), (2), and (4), we have the following utility function:

$$u_i(x_i) = f(k_i, g_i) - r\left(k_i - \frac{\bar{k}}{2}\right). \tag{5}$$

We now consider the equilibrium conditions of factor markets. When a total capital endowment is $\bar{k}$ and the population of each country is normalized to unity, the capital market equilibrium condition becomes

$$k_1 + k_2 = \bar{k}. \tag{6}$$

In the labor market of each country, we have $l_i = 1$ in its equilibrium.

Hereafter, we specify the production function as

$$f_i(k_i, g_i) = \alpha_i k_i + \beta_i g_i - \frac{A_i k_i^2 + B_i g_i^2}{2} + \gamma_i k_i g_i, \tag{7}$$

where $\alpha_i > 0$, $\beta_i > 0$, $A_i > 0$, $B_i > 0$, and $\gamma_i^2 < A_i B_i$.[5] If $\gamma_i > 0$, the capital and public input are q-complements (Sato and Koizumi, 1973).[6] Under this specification, Eq. (1) can be rewritten as

$$r = \alpha_i - A_i k_i + \gamma_i g_i - t_i. \tag{8}$$

For $i = 1,2$ and $i \neq j$, Eqs. (1)–(3) and (5)–(7) derive

$$r = \frac{(\alpha_1 - t_1)(A_2 - \gamma_2 t_2) + (\alpha_2 - t_2)(A_1 - \gamma_1 t_1) - (A_1 - \gamma_1 t_1)(A_2 - \gamma_2 t_2)\bar{k}}{A_1 + A_2 - \gamma_1 t_1 - \gamma_2 t_2}, \tag{9a}$$

$$k_i = \frac{\alpha_i - \alpha_j - t_i + t_j + (A_j - \gamma_j t_j)\bar{k}}{A_1 + A_2 - \gamma_1 t_1 - \gamma_2 t_2}. \tag{9b}$$

In equilibrium, the interest rate and capital employed in country i are functions of t_1 and t_2. By the presence of public input, there exist partial positive effects of the taxes on the interest rate and capital through the productivity effects of public input.

[5] It is necessary to be $F_{ikk}F_{igg} - F_{ikg}^2 = A_i B_i - \gamma_i^2 > 0$ for the concavity of the production function. This type of production function is widely used in the literature on tax competition (e.g., Kempf and Rota-Graziosi, 2010; Kikuchi and Tamai, 2024).

[6] If $\gamma_i < 0$, then these two inputs are q-substitutes. We exclude the q-substitute case from the main part of our analysis, based on empirical evidence. However, we will treat the q-substitute case for a purely theoretical possibility.

3. Equilibrium analysis

3.1. Social optimum

We now consider the centrally planned economy as the criteria for optimality. The social planner seeks to maximize the social welfare subject to Eqs. (6) and (7). The objective function of the planner is given by the Benthamite social welfare function $W = u(x_1) + u(x_2)$; we have

$$W = f(k_1, g_1) + f(k_2, g_2) - g_1 - g_2, \tag{10}$$

where W denotes the social welfare.

The optimality conditions for the social planner's optimization problem are

$$f_{1k} = f_{2k}, \tag{11a}$$

$$f_{ig} = 1, \tag{11b}$$

where $f_{ik} \equiv \partial f_i(k_i, g_i)/\partial g_i$. Eq. (11a) stands for the equalization of marginal products of capital in two countries. Eq. (11b) is the Kaizuka condition for optimal provision of public input.

Using Eqs. (6), (7), (11a), and (11b), we obtain

$$k_i^o = \frac{(\alpha_i - \alpha_j)B_1B_2 + (A_jB_j - \gamma_j^2)B_i\bar{k} + (\beta_i - 1)B_j\gamma_i - (\beta_j - 1)B_i\gamma_j}{(A_1B_1 - \gamma_1^2)B_2 + (A_2B_2 - \gamma_2^2)B_1}, \tag{12a}$$

$$g_i^o = \frac{(\beta_i - 1)A_iB_j + (A_jB_j - \gamma_j^2)(\beta_i + \gamma_i\bar{k} - 1) - (\beta_j - 1)\gamma_1\gamma_2 + (\alpha_i - \alpha_j)B_j\gamma_i}{(A_1B_1 - \gamma_1^2)B_2 + (A_2B_2 - \gamma_2^2)B_1}. \tag{12b}$$

The optimal "tax rate" on capital can be driven from g_i^o/k_i^o. Characterizing the socially optimal outcome with respect to factor complements (i.e., γ_i), we should focus on symmetric countries except for γ_i.

Using Eqs. (12a) and (12b), we obtain the relationship between the equilibrium values of capital and public input and γ_i (see Appendix A for the proof of Proposition 1):

Proposition 1. *Suppose that two countries are symmetric with respect to* α_i, β_i, A_i, *and* B_i $(i = 1,2)$. *Then, the following relationship holds:* $k_1^o \gtreqless k_2^o \Leftrightarrow g_1^o \gtreqless g_2^o \Leftrightarrow \gamma_1 \gtreqless \gamma_2$.

The intuition of Proposition 1 is straightforward: Suppose that $\gamma_1 = \gamma_2$ holds initially. If γ_1 increases (i.e., $\gamma_1 > \gamma_2$), for given initial values of k_i^o and g_i^o, satisfying Eqs. (6), (11a), and (11b), the marginal product of capital in Country 1 exceeds that of Country 2. Then, the capital moves from Country 2 to Country 1. The complementarity between capital and public input must increase public input provision. Therefore, a larger degree of complementarity in Country 1 than in Country 2 leads to larger capital input and public input in Country 1 than in Country 2.

3.2. Decentralized equilibrium

In this part, we characterize the decentralized equilibrium, satisfying Eqs. (1)–(9b) and tax policies derived from each government's optimization problem. Before the full analysis, we impose the following assumption:

Assumption 1. $\frac{t_i}{k_i}\frac{\partial k_i}{\partial t_i} > -1.$

Assumption 1 requires that the elasticity of capital with respect to the capital tax rate is larger than minus unity; An increase in the tax rate increases the government revenue (see Eq. (3)).

The government of country i seeks to maximize the regional welfare Eq. (5) subject to Eqs. (9a) and (9b) for given t_j. The first-order condition of the country-i government's optimization problem is

$$\frac{\partial u_i}{\partial t_i} = t_i\frac{\partial k_i}{\partial t_i} + [\beta - t_i k_i + \gamma_i k_i]\left[k_i + t_i\frac{\partial k_i}{\partial t_i}\right] + \left(\frac{\bar{k}}{2} - k_i\right)\frac{\partial r}{\partial t_i} = 0. \tag{13}$$

Note that the coefficient, $\beta - t_i k_i + \gamma_i k_i$, is the marginal product of public input, which should be positive. Eqs. (3), (6), (9a), (9b), and (13) derive the system of t_i, k_i, and r. The solutions of the system constitute Nash equilibrium if they exist. Hereafter, the superscript "*" denotes the values of Nash equilibrium (e.g., t_i^*, k_i^*, and r^*).

On the right-hand side of Eq. (13), the first term denotes the negative welfare effect of increased tax through a decrease in tax base; the second term stands for the positive welfare effect of increased tax through the productivity effect of public input; the third term represents welfare effect of increased through a change in net capital income (the effect of pecuniary externality). Without public input, an increase in the tax rate decreases capital and the interest rate. However, with public input, these effects are ambiguous, even though we should focus on the negative effects of t_i^* on r^* and k_i^*.

To obtain the explicit analytical outcome, we introduce the following assumption about the symmetries of two countries and normalization:

Assumption 2. $\alpha_i = \alpha,\ \beta_i = \beta,\ A_i = B_i = 1,$ *and* $\bar{k} = 2.$

With full symmetries (i.e., $\gamma_i = \gamma$), we have the following proposition (see Appendix B for the proof of Proposition 2):

Proposition 2. *If* $\gamma_i = \gamma$, *it holds* $t_i^* \gtreqless t_i^o \Leftrightarrow g_i^* \gtreqless g_i^o \Leftrightarrow \beta + \gamma \lesseqgtr 2.$

Note that $k_i^* = 1$ and $k_i^o = 1$ hold in Nash and social optimal equilibria under Assumption 2. Hence, we have $t_i^* = g_i^*$ and $t_i^o = g_i^o$.

Public input is underprovided if $\beta > 2$. However, for $\beta < 2$, the governments overprovide public input when $\gamma < 2 - \beta$. We should note that Proposition 2 is along with the result shown by Gugl and Zodrow (2019).[7] They show that one of the key determinants of public input provision is the elasticity of the marginal product of public input with respect to capital, defined as follows:

$$\varepsilon_{f_g,k} \equiv \frac{k_i f_{igk}}{f_{ig}} = \frac{\gamma}{\beta - t_i + \gamma},$$

where $f_{igk} \equiv \partial^2 f_i / (\partial k_i \partial g_i)$.

If $\varepsilon_{f_g,k} < 1$ ($\varepsilon_{f_g,k} > 1$), it implies underprovision (overprovision) of public input (Gugl and Zodrow, 2019) because one unit of capital increases smaller (larger) unit of marginal productivity of public input than unity.[8] If $\beta + \gamma > 2$, $\varepsilon_{f_g,k} < 1$ because that the capital tax rate t_i^* satisfies $0 < t_i^* < 1$ within Assumption 2 and $\gamma_i = \gamma < 1$.[9] However, when $\beta + \gamma < 2$, $\varepsilon_{f_g,k}$ can be larger than unity, depending on the tax rate. In our model, $\varepsilon_{f_g,k} > 1$ holds for the equilibrium tax rate t_i^* if $\beta + \gamma < 2$. We provide specified necessary and sufficient conditions for the overprovision of public input.

When full symmetricities of two countries do not hold (i.e., $\gamma_1 \neq \gamma_2$), we cannot ensure the uniqueness and existence of Nash equilibrium for all domains of the parameters set. Hence, we consider some specified cases with fixed parameters of β and γ_i, hereafter. Without loss of generality, we set $\gamma_1 = \gamma$ and $\gamma_2 = 0$ and examine the two cases of β: $\beta = 1$ and $\beta = 2$.

The former case implies that there remains a complement effect in the productivity effect of public

[7] Matsumoto and Sugawara (2017) derive the condition, based on the homotheticity of production technology. If the production technology exhibits the Solow-neutrality or the Harrod-neutrality, then there exists expenditure inefficiency unless the elasticity of factor substitution is equal to one. However, with the Hicks-neutrality, there is no inefficiency of public input provision.

[8] This condition is related to the stability condition of Zodrow-Mieszkowski critiqued by Noiset (1995).

[9] With full symmetricity of countries, the first-order condition (13) becomes $-t_i + (\beta - t_i + \gamma)(1 - t_i) = 0$. To ensure positive value of g_i^*, $0 < t_i^* < 1$ is derived from the first-order condition.

input. At the same time, the latter indicates that the underprovision of public input always occurs in two identical countries. Under Assumption 2, the domain of γ must be $0 \leq \gamma < 1$. Figure 1 illustrates two panels of the best response planes of two countries. Each panel of Figure 1 shows the existence of the set of Nash equilibria for $\gamma \in [0,1)$. Especially, Figure 2 indicates the uniqueness of Nash equilibrium for four values of γ. The continuity of t_i in γ will ensure the existence and uniqueness of Nash equilibrium for almost the range of $\gamma \in [0,1)$.

We focus on $\beta = 1$ to consider the effect of asymmetricity of factor complements on the equilibrium outcome of tax competition. Then, we obtain the following proposition (see Appendix C for the proof of Proposition 3):

Proposition 3. *If* $\beta = 1$, $\gamma_1 = \gamma$, *and* $\gamma_2 = 0$, *then* $t_1^* > t_1^o$ *and* $t_2^* > t_2^o = 0$ *holds.*

By the presence of factor complements for Country 1's advantage, Country 1 can raise the tax rate on capital more than Country 2; Capital 1 attracts more capital than Country 2 if the gap of factor complements (i.e., $\gamma = \gamma_1 - \gamma_2$) is larger. Country 2 takes the counteraction to prevent the capital outflow from its region to the other; Country 2 loses national income if not. Therefore, Country 2 as well as Country 1 set the inefficiently higher tax rate than their optimum as a result of fiscal competition.

However, an inefficient high tax rate on capital does not imply overprovision of public inputs. When the countries are asymmetric, the pecuniary externality effect occurs in addition to fiscal externality and the factor complement effects. The pecuniary externality effect increases the cost of tax financing for the capital exporter while it decreases the cost for the capital importer. Attracting capital makes the capital exporter country pay the cost of tax-financing, decreasing the incentive to raise the tax rate.

Propositions 2 and 3 suggest that governments might oversupply public input under certain conditions. However, it is difficult to obtain the clear implication for public input provision because we do not have the explicit form of the equilibrium outcomes due to the strong nonlinearity of the equilibrium system. Alternatively, to solve it explicitly, we rely on numerical analysis. For instance, regarding the relationship between t_i and γ_i, the panels (a)–(d) in Figure 2 show that an increase in γ_i increases the equilibrium tax rates of both countries, as it is predicted above.

Tables 1 and 2 display the equilibrium and social optimal values of the tax rate, capital, and public inputs for $\beta = 1$ and $\beta = 2$, respectively. Table 1 shows that Proposition 3 holds, and that $k_1^* < k_1^o$ and $k_2^* > k_2^o$. Country 2 attracts more capital than its optimum by exceeding the tax rate from the optimal rate; Country 1 loses its capital by fiscal competition. However, Country 1 covers the loss by raising the tax rate to provide more public inputs because of factor complements. Therefore, both countries overprovide public input. The pecuniary externality weakens (strengthens) the capital attracting-effect for its importer (exporter). In Table 1, k_i^* is non-monotonically changed by an increase in γ.

Table 2 demonstrates the case where Proposition 3 does not hold. As shown in Proposition 2, $\beta = 2$ is a sufficient condition for underprovision of public inputs in the symmetric economy. In contrast with $\beta = 1$, both countries undersupply public inputs. Regarding the tax rate on capital, Country 2 chooses a smaller tax rate than its optimum, while Country 1 sets a higher tax rate than its optimum for large γ_1. Tables 1 and 2 show that the equilibrium tax rate of the country with stronger factor complements tends to be large and to exceed its optimum. This result may not be true for the country with weaker factor complements.

3.3. Discussion

In the previous parts, we assumed that capital and public inputs are q-complement (i.e., $\gamma_i > 0$). However, there is a theoretical possibility of q-substitute (i.e., $\gamma_i < 0$). Hence, we should consider how the relaxation of the parameter restriction affects the equilibrium outcome.

A modification of Proposition 1. We can easily verify the magnitude relationship between k_1 and k_2 as well as g_1 and g_2; The result of Proposition 1 can be rewritten as $k_1^o \gtreqless k_2^o \Leftrightarrow g_1^o \lesseqgtr g_2^o \Leftrightarrow \gamma_1 \lesseqgtr \gamma_2$ for $\gamma_i < 0$. Since smaller γ_i leads to higher productivity of capital for $\gamma_i < 0$ in contrast with the case of $\gamma_i > 0$, the country with smaller γ_i obtains more capital than the other with larger γ_i. Smaller γ_i yields lower productivity of public input; the country with smaller γ_i provides smaller public inputs than the other with larger γ_i.

A modification of Propositions 2 and 3. For $\gamma_i < 0$, Proposition 2 still holds. Since $\gamma_i < 0$, the sufficient condition for the overprovision of public inputs is easier to hold than $\gamma_i > 0$. Proposition 3 cannot be obtained because $g_1^o < 0$ holds if $\beta = 1$. $\beta > 1$ is needed to have $g_1^o > 0$. However, based on the results of Propositions 1 and 2, the equilibrium tax rate may be large, and it leads to the overprovision of public inputs.

A choice of policy instruments. Bayindir-Upmann (1998) examined which one of tax and expenditure is better for improving the regional welfare in fiscal competition equilibrium with public inputs using numerical analysis. Hauptmeier et al. (2012) also considered the choice of a business tax rate and public input and estimated a model of strategic interaction in both policy instruments. Based on the estimation, they found that governments use both the business tax rate and public inputs to attract capital: Governments respond to tax cuts by reducing their taxes and boosting public investments. In response to higher public inputs, governments raise their expenditure on public inputs.

These theoretical and empirical findings imply the importance of selecting policy instruments. The non-linearity of the best response functions causes the analytical difficulty. The specified production function provides a simpler structure of the best response function than the more general form of production function. Therefore, our basic and extended analyses are useful in treating this issue.

4. Conclusion

This paper examined the efficiency of public input provision under competition in capital taxes and public input. We conducted qualitative and quantitative analyses using the specified production function that exhibits modularity in capital and public inputs. Especially in the analysis, we focus on the factor complementarity and the asymmetries between the two countries concerning the technological relationship between capital and public input in production.

The main results of this paper are summarized as follows: We show that the country with a larger degree of q-complements attains higher tax and higher public inputs than the other with a smaller degree of q-complements. With full symmetricity, stronger q-complements reduce the overprovision of public inputs because one unit of capital increases a smaller unit of marginal productivity of public input.

When the countries are asymmetric with respect to the degree of q-complements, overprovision of public inputs occurs in the country with a smaller degree of q-complements if the productivity of public inputs is sufficiently low. The country with a smaller degree of q-complements is willing to raise the tax rate to prevent capital outflow. Based on numerical analysis, both countries overprovide public inputs if the productivity of public inputs is sufficiently low.

The analysis developed in this paper can be further extended along with the issues tackled by the previous studies. For instance, it is insightful to consider what taxes are more efficient for providing public services (e.g., Gugl and Zodrow, 2019; Kikuchi and Tamai, 2024). This paper implies that such an extension needs to introduce the asymmetricity of countries. A choice of tax instruments will be affected by the pecuniary externality (i.e., terms of trade effect) as well as the production technology. Asymmetricity casts doubt on simultaneous move game (Kempf and Rota-Graziosi, 2010). Naturally, one more extension is an endogenous leadership under competition in tax and public input. input.

Appendix

A. Proof of Proposition 1

With $\alpha_i = \alpha$, $\beta_i = \beta$, $A_i = A$, and $B_i = B$ $(i = 1,2)$, the difference in k_i^o is

$$k_1^o - k_2^o = \frac{(\gamma_1 - \gamma_2)(\gamma_1 + \gamma_2)\bar{k}}{2AB - \gamma_1^2 - \gamma_2^2}.$$

By a similar way to deriving the equation mentioned above, we have

$$g_1^o - g_2^o = \frac{(\gamma_1 - \gamma_2)\left[AB\bar{k} + \gamma_1\gamma_2\bar{k} + (\gamma_1 + \gamma_2)(\beta - 1)\right]}{(2AB - \gamma_1^2 - \gamma_2^2)B}.$$

B. Proof of Proposition 2

Under Assumption 2, Eqs. (12a) and (12b) yield $k_i^o = 1$ and $g_i^o = \beta + \gamma - 1$. Naturally, we obtain $k_i^* = k_i^o = 1$. Then, the solution of Eq. (13) can be derived as

$$t_i^* = g_i^* = \frac{3 + \beta + \beta\gamma + \gamma^2 - \sqrt{[3 + \beta + \beta\gamma + \gamma^2]^2 - 8(1 + \gamma)(\beta + \gamma)}}{2(1 + \gamma)} > 0.$$

Hence, we obtain

$$g_i^* - g_i^o = \frac{5 - \beta - \beta\gamma - \gamma^2 - \sqrt{[3 + \beta + \beta\gamma + \gamma^2]^2 - 8(1 + \gamma)(\beta + \gamma)}}{2(1 + \gamma)}.$$

Focusing on the sign, we arrive at

$$\begin{aligned} \operatorname{sgn}(g_i^* - g_i^o) &= \operatorname{sgn}\{[5 - \beta - \beta\gamma - \gamma^2]^2 - [3 + \beta + \beta\gamma + \gamma^2]^2 + 8(1 + \gamma)(\beta + \gamma)\} \\ &= \operatorname{sgn}(2 - \beta - \gamma). \end{aligned}$$

C. Proof of Proposition 3

With $\beta = 1$, Eqs. (12a) and (12b) lead to

$$(k_1^o, k_2^o, g_1^o, g_2^o) = \left(\frac{2}{2 - \gamma^2}, \frac{2(1 - \gamma^2)}{2 - \gamma^2}, \frac{2\gamma}{2 - \gamma^2}, 0\right).$$

Hence, the optimal tax rates on capital are $t_1^o = \gamma$ and $t_2^o = 0$, derived from g_1^o/k_1^o and g_2^o/k_2^o, respectively.

Using Eq. (13), we obtain

$$\left.\frac{\partial u_1}{\partial t_1}\right|_{t_1=0} = \frac{t_2^2\gamma + 2(1+\gamma)(4+3t_2)}{8} > 0, \tag{C1}$$

$$\left.\frac{\partial u_2}{\partial t_2}\right|_{t_2=0} = \frac{4 - t_1^2[(2-3\gamma)\gamma] + t_1(3-7\gamma)}{(2-t_1\gamma)^2}. \tag{C2}$$

For small t_1, Eq. (C2) has negative sign (e.g., $t_1 = 0$):

$$\left.\frac{\partial u_2}{\partial t_2}\right|_{t_1=0,t_2=0} = 1.$$

Therefore, each government has an incentive to raise their tax rates at least from zero.

The next point is to show that Country 1 has an incentive to increase the tax rate from $t_1 = \gamma$: We have

$$\left.\frac{\partial u_1}{\partial t_1}\right|_{t_1=\gamma} = \frac{2(1-\gamma)^2(4+3\gamma) + \gamma^3(2+2\gamma-3\gamma^2) + t_2[6 - t_2\gamma - 2\gamma - \gamma^2 + \gamma^3(1-\gamma^2)]}{(2-\gamma^2)^3} \tag{C3}$$

To ensure $k_2 > 0$, it must be $t_2 < 2 + \gamma - 2\gamma^2$ for $t_1 = \gamma$. We have

$$\begin{aligned} t_2[6 - t_2\gamma - 2\gamma - \gamma^2 + \gamma^3(1-\gamma^2)] &> t_2[6 - (2+\gamma-2\gamma^2)\gamma - 2\gamma - \gamma^2 + \gamma^3(1-\gamma^2)] \\ &= t_2[4(1-\gamma) + 2(1-\gamma^2) + \gamma^3(3-\gamma^2)] > 0. \end{aligned}$$

Therefore, Eq. (C3) has a negative sign.

Since the upper bound of t_1 is $2/\gamma$, we have

$$\left.\frac{\partial u_1}{\partial t_1}\right|_{t_1=\frac{2}{\gamma}} = -\frac{[2-(2+t_2)\gamma]^2(4-\gamma^2)}{\gamma^3} \lim_{t_1\to\frac{2}{\gamma}} \frac{1}{(2-t_1\gamma)^3} = -\infty. \tag{C4}$$

Eqs. (C1) and (C4) show that at least one solution exists satisfying Eq. (13) for $i = 1$. Since Eq. (C3) has a negative sign, $t_1^* > \gamma = t_1^o$ holds.

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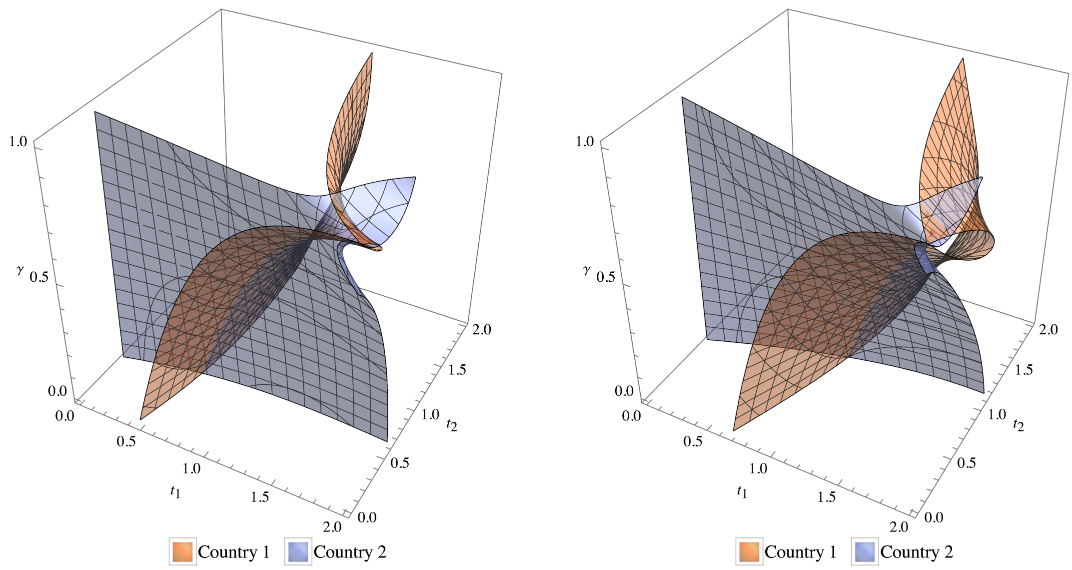
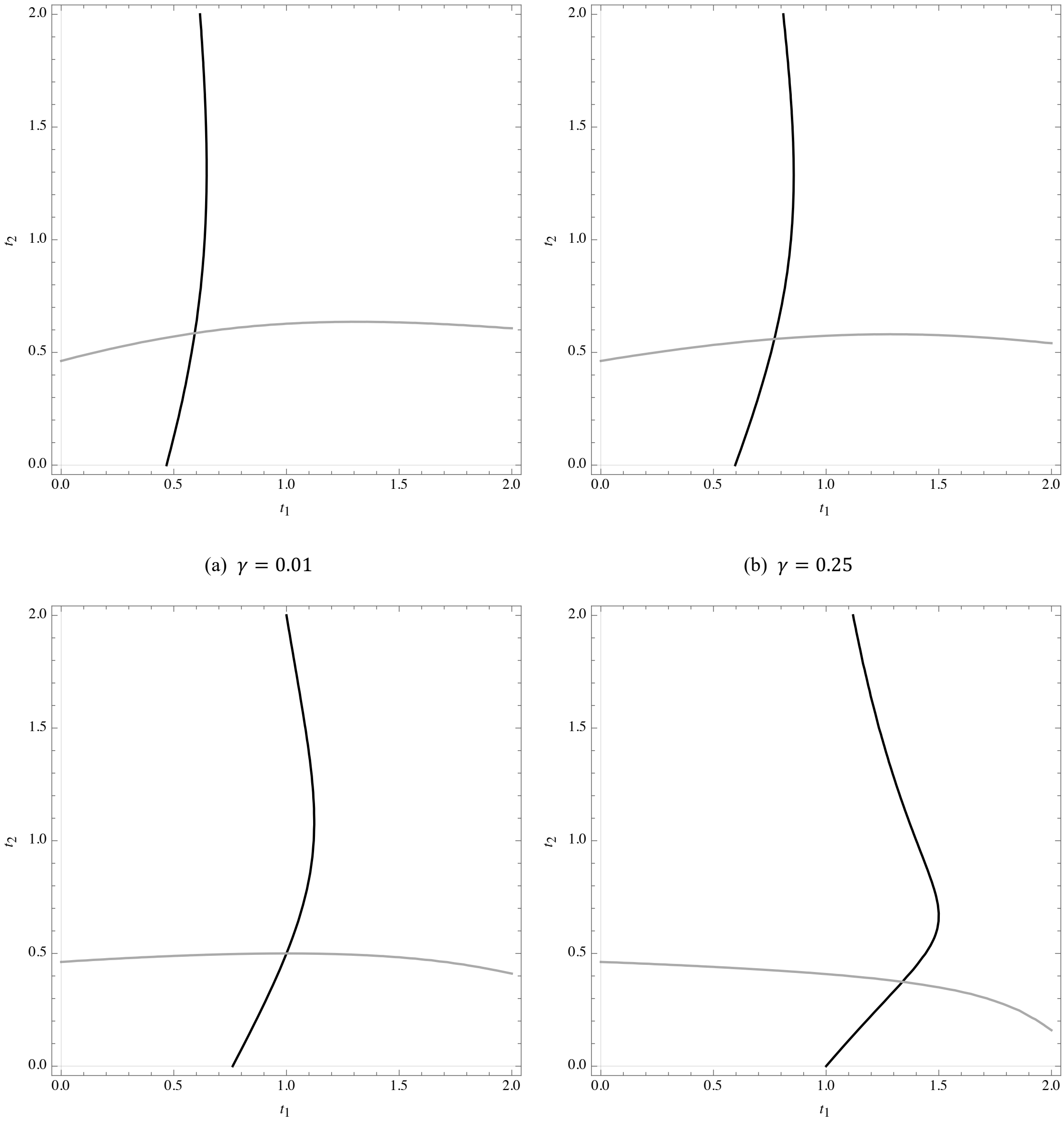
Figures



(a) $\beta = 1$ (b) $\beta = 2$

Figure 1. The best response planes of Countries 1 and 2 ($\gamma_1 = \gamma,\ \gamma_2 = 0$)



(a) $\gamma = 0.01$ (b) $\gamma = 0.25$

(c) $\gamma = 0.5$ (d) $\gamma = 0.75$

Figure 2. The best response curves of Countries 1 and 2 ($\beta = 1,\ \gamma_1 = \gamma,\ \gamma_2 = 0$)

Tables

Table 1. Equilibrium values of capital tax, capital input, and public infrastructure ($\beta = 1$)

	Country 1		Country 2	
	(Decentralized)	(Optimum)	(Decentralized)	(Optimum)
t_i	0.593	0.010	0.585	0.000
	0.770	0.250	0.559	0.000
	1.000	0.500	0.500	0.000
	1.339	0.750	0.374	0.000
k_i	0.999	1.000	1.001	1.000
	0.990	1.032	1.010	0.968
	1.000	1.143	1.000	0.857
	1.039	1.391	0.961	0.609
g_i	0.592	0.010	0.586	0.000
	0.762	0.258	0.565	0.000
	1.000	0.571	0.500	0.000
	1.391	1.044	0.359	0.000

Note: From the top of the line in sequence, $\gamma = 0.01$, $\gamma = 0.25$, $\gamma = 0.5$, and $\gamma = 0.75$ for each variable

Table 2. Equilibrium values of capital tax, capital input, and public infrastructure ($\beta = 2$)

	Country 1		Country 2	
	(Decentralized)	(Optimum)	(Decentralized)	(Optimum)
t_i	1.001	1.005	0.997	1.005
	1.181	1.111	0.892	1.192
	1.401	1.200	0.720	1.750
	1.685	1.273	0.441	11.50
k_i	1.000	1.005	1.000	0.995
	1.004	1.161	0.996	0.839
	1.015	1.429	0.985	0.571
	1.026	1.913	0.974	0.087
g_i	1.007	1.010	0.997	1.000
	1.185	1.290	0.889	1.000
	1.421	1.714	0.709	1.000
	1.730	2.435	0.429	1.000

Note: From the top of the line in sequence, $\gamma = 0.01$, $\gamma = 0.25$, $\gamma = 0.5$, and $\gamma = 0.75$ for each variable