Nash Equilibria in Models of Fiscal Competition with Unemployment

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Abstract

This paper examines two different fiscal competition games under labor market imperfection. Given that capital moves across regions and affects regional employment, governments must choose expenditure levels and tax rates on such mobile capital by accounting for the effects of fiscal variables on both capital and labor. Then, governments may play these games with either tax rates on mobile capital or with public expenditures. The presence/absence of absentee ownership of capital and employment externalities are important factors for characterizing two distinct Nash equilibria, one that occurs with tax competition and the other with expenditure competition. In particular, and contrary to the existing literature, tax rates under tax competition are likely to be *more competitive* than under expenditure competition because of employment externalities. Furthermore, in some cases, governments prefer to choose government expenditure as their strategic variable rather than tax rates. The presence of employment externalities motivates governments to use government expenditure as the strategic variable through which it might encourage strategic effects.

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1. Introduction

Fiscal competition has been widely discussed since Zodrow and Mieszkowski (1986) and Wilson (1986) developed theoretical frameworks.[1] One of the important issues in the literature is the existence of unemployment. Some theoretical studies have attempted to explain the relationship between fisacal competition and unemployment (e.g., Ogawa et al. 2006; Aronsson and Wehke 2008; Sato 2009; Eichner and Upmann 2012; Exbrayat *et al.* 2012; Kikuchi and Tamai 2019). Ogawa et al. (2006) and Kikuchi and Tamai (2019) are based on fixed wages model, whereas Sato (2009) introduced job search and recruiting friction to fiscal competition model. Aronsson and Wehke (2008), Eichner and Upmann (2012) and Exbrayat et al. (2012) applied bargaining between unions and firms.

In particular, Ogawa *et al.* (2006) theoretically examined the issue of fiscal competition and employment. It is demonstrated in their study that the efficiency of providing public goods financed by capital taxes depends on the degree of complementarity/substitutability between capital and labor. They also showed that the mechanism behind the result is based on fiscal and employment externalities. Most recently, Kikuchi and Tamai (2019) incorporated fiscal transfers into their model and clarified how externalities affect the motives for choosing tax rates. Their findings imply that the presence of employment externalities is essential for implementing optimal policies.[2] However, their results depend on the assumption that governments use only tax rate as policy variable. In practice, tax competition may no longer exist, and countries are facing intergovernmental/interregional competition for using other policy variable.[3]

Wildasin (1988, 1991) developed theoretical framework to answer the question what instruments governments should implement under the environment of competing for capital with other regions. Wildasin (1988) considered two possible Nash equilibria, in which governments play games either with tax rates on capital or with public expenditures[4]. The comparative analysis of the two equilibria showed that the equilibrium with taxes as the control variable is not identical to the equilibrium with expenditures. The tax rate under competition in public expenditure is *more competitive* than that in tax competition. Wildasin (1991) examined which of the two—tax rates or expenditures—is the best strategic variable and proved that the tax rate is the better strategic variable.

Subsequent theoretical studies have attempted to clarify the strategic choice on policy instruments (e.g., Hoyt 1993; Bayindir-Upmann 1998; Köethenbüerger 2011a).[5] Hoyt (1993) compared the equilibria of tax competition with the equilibria that of expenditure competition in case of Tibout model where residents are mobile among jurisdictions in a metropolitan area. He reversed Wildasin (1988)'s conclusion when the government maximize land rent and the demand for housing is elastic. Bayindir-Upmann (1998) examined two different games of fiscal competition in which local governments provide public inputs. Contrary to the case of public goods analyzed by Wildasin (1988, 1991), this study shows that competition in productive public expenditure does not necessarily lead to more competitive tax rates than in tax competition. In relation to the fiscal competition model, Köethenbüerger (2011a) incorporated fiscal transfers and taxes on labor into the model and examined the issues related to taxes versus expenditures. His model excludes tax-based mobility and analyzes only inter-regional dependencies through fiscal transfer. Köethenbüerger (2011a) showed that the policy variables chosen by governments vary depending on the structure of fiscal transfers.

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[1] Recently, Zodrow (2010) and Keen and Konrad (2013) surveyed the theoretical literature on fiscal competition. On the other hand, Devereux and Loretz (2013) and Revelli (2015) surveyed the recent empirical literature.

[2] Some empirical studies have found evidence to support the effect of taxes on employment. Numerous studies have found significant effects of taxes on employment (*e.g.*, Bettendorf *et al.* 2009; Feld and Kirchgassner 2002; Feldmann 2011; Felix 2009; Harden and Hoyt 2003; Zirgulis and Šarapovas 2017). Although the literature found a negative relationship between taxes and employment, Feldmann (2011) showed that higher corporate taxes might lower the unemployment rate.

[3] Some empirical studies investigated the impact of taxes and public infrastructures on the allocation of private capital. Bénassy-Quéré *et al.* (2007) showed that both the corporate taxes and the stock of public capital are significant using a data set of US FDI in 18 EU countries from 1994 to 2003. Hauptmeier *et al.* (2012) suggested that local governments use both the business taxes and public inputs to compete for mobile capital using a data set of the 1100 German municipalities in the state of Baden-Wuerttemberg from 1998 to 2004.

[4] The existence and characteristics of the tax competition equilibrium have been examined by numerous studies (e.g., Laussel and Le Breton; 1998, Bayindir-Upmann and Ziad, 2005; Petchey and Shapiro, 2009; Taugourdeau and Zaid, 2011; Rota-Graziosi, 2019).

[5] Several studies developed comparisons between taxes and transfers and between unit taxes and ad valorem taxes (Hindriks, 1999; Lockwood, 2004; Akai *et al.*, 2011; Köethenbüerger, 2011b).

The purpose of the present study is to examine two different under-employment equilibria. One is the equilibrium wherein governments compete in terms of tax rates and the other is under competition for government expenditures. It is shown in this study that there exists a unique Nash equilibrium regardless of whether the tax rate or expenditure level is the control variable. Comparative analyses of two equilibria demonstrate that the presence/absence of absentee ownership of capital is essential to characterizing the equilibrium properties and outcomes. If all capital is owned by absentee owners, the tax rates under expenditure competition are *more competitive* (*i.e.*, lower) than that under tax competition. In contrast, the tax rates under expenditure competition are *less competitive* (*i.e.*, higher) than under tax competition, depending on the size of fiscal and employment externalities, when residents equally share the economy-wide capital stock.

Furthermore, welfare analyses are also conducted in this paper. The analyses show that the welfare superiority of the possible equilibria depends on the presence/absence of absentee ownership of capital and the size of the employment externality. With the absentee ownership of capital, government expenditure is chosen as a strategic variable if the employment externality is sufficiently strong and negative. However, it will be unlikely that government expenditure is a strategic variable when residents equally share the economy-wide capital stock.

Wildasin (1988) compares the equilibrium of tax competition with that of expenditure competition in an economy without unemployment. We find that with the absentee ownership of capital, government expenditure is chosen as a strategic variable. Our findings differ from Wildasin (1988), which finds that tax is a strategic variable. Bayindir-Upmann (1998) investigated two equilibria similarly to Wildasin (1988) in a situation where the government supplies the public inputs. Unlike governments supplying public goods, the possibility of negative capital externalities exists when governments supply public inputs. Thus, similar to Bayindir-Upmann (1998), our results suggest that public expenditure is a desirable strategic variable due to employment externalities.[6]

The remainder of this paper is organized as follows. Section 2 describes the basic setup of our analytical framework. Section 3 characterizes the two different equilibria: the tax competition equilibrium and the expenditure competition equilibrium. Comparative analysis of two equilibria is also developed in the section. Finally, Section 4 provides the conclusions of this study.

2. The Model

This section describes our theoretical framework and provides the preliminary results for later analyses. The basic setup is based on Wildasin (1988) and Ogawa *et al.* (2006). The economy consists of N regions ($N \geq 2$). Each region has identical residents and firms. The population in each region is normalized to unity and the residents are immobile. There is a single homogenous private good produced using capital, labor, and land. The production technology is given by a constant-returns-to-scale production function $Y_i = F(H_i, K_i, L_i)$, where Y_i is the output of the private good, H_i is the land input, K_i is the capital input, and L_i is the labor input employed in region i ($i = 1, 2, \cdots, N$). It is assumed that each factor reward equals the value of its marginal product. Treating the amount of land input as fixed, $Y_i = F(\overline{H}_i, K_i, L_i) \equiv f(K_i, L_i)$. We also assume that $f(K_i, L_i)$ is strictly concave, twice continuously differentiable, and increasing in K_i and L_i.

Capital moves across the region while labor and land are fixed in the locality. Summing up the factor demand for capital in all regions yields

$$\sum_{i=1}^{N} K_i = \overline{K},\tag{1}$$

where $\overline{K}$ stands for economy-wide capital stock. The resident as worker and the landlord supplies one unit of labor and land, respectively. In this paper, some of the labor might be unemployed while the land is fully employed. Hence, the factor demands for labor and land satisfy $L_i \leq 1$ and $H_i = 1$.

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[6] Hoyt (1993) showed similar result in some cases using Tiebout model. Köethenbüerger (2011a) showed that the governments choose public expenditure under a specific fiscal transfer.

As already explained, the rewards for the production factors equal their marginal product. Since each regional government imposes a per unit capital tax, the cost of using capital is the sum of the capital price and the unit tax. Taking this into account, the relationship between factor demands and prices are given by

$$\rho = f_K(K_i, L_i) - t_i, \tag{2}$$
$$\overline{w}_i = f_L(K_i, L_i), \tag{3}$$

where ρ is the common factor price of capital, t_i is the unit tax rate on capital in region i, $\overline{w}_i$ is an exogenously fixed wage rate in region i. To ensure the presence of unemployment, we assume that $\overline{w}_i$ is higher than the wage rate determined in a competitive labor market.[7] Equations (2) and (3) determine the factor demands for capital and labor as functions of their factor prices and capital tax rate, respectively.

Each regional government provides a public good and finances its supply cost by a per unit capital tax. The marginal rate of transformation between private and public goods is unity. Local public expenditure G_i stands for the amount of public goods. Therefore, the budget equation for the regional government equals

$$t_i K_i = G_i. \tag{4}$$

Note that $0 \le t \le \overline{t}$ and $\overline{t} \equiv f_K$ must hold from equations (2) and (4).

Since capital moves across regions, the factor price of capital (net return to capital), ρ, is determined from equation (1) as a function of the vector of tax rates $\tau = (t_1, \cdots, t_n)$. We now verify that fact. Note that the following equation is derived from equations (2) and (3):

$$\frac{dK_i}{d(\rho + t_i)} = \frac{f_{LL}(K_i, L_i)}{|J_i|} < 0,$$

where $|J_i| \equiv f_{KK}(K_i, L_i)f_{LL}(K_i, L_i) - [f_{KL}(K_i, L_i)]^2 > 0$. Using this equation and equation (1), we obtain[8]

$$\frac{\partial \rho}{\partial t_i} = - \frac{\dfrac{\varepsilon_i K_i}{\rho + t_i}}{\sum_j \dfrac{\varepsilon_j K_j}{\rho + t_j}} < 0, \tag{5}$$

where

$$\varepsilon_i \equiv \frac{d \log K_i}{d \log(\rho + t_i)} < 0.$$

Equations (2) and (5) yield $K_i = K_i(\rho(\tau) + t_i) = K_i(\tau)$. For a fixed wage level, equation (3) describes the relationship between capital and labor as

$$\mu_i \equiv \frac{dL_i}{dK_i} = - \frac{f_{KL}(K_i, L_i)}{f_{LL}(K_i, L_i)} \gtreqless 0 \Leftrightarrow f_{KL}(K_i, L_i) \gtreqless 0.$$

This equation and $K_i = K_i(\tau)$ provide $L_i = L_i(K_i(\tau)) = L_i(\tau)$. Partial differentiation of $K_i(\tau)$, $L_i(\tau)$, $K_j(\tau)$, and $L_j(\tau)$, with respect to t_i leads to

$$\frac{\partial K_i}{\partial t_i} = \frac{\varepsilon_i K_i}{\rho + t_i} \frac{\sum_{j \neq i} \dfrac{\varepsilon_j K_j}{\rho + t_j}}{\sum_j \dfrac{\varepsilon_j K_j}{\rho + t_j}} < 0, \tag{6}$$

$$\frac{\partial L_i}{\partial t_i} = \frac{\varepsilon_i \mu_i K_i}{\rho + t_i} \frac{\sum_{j \neq i} \dfrac{\varepsilon_j K_j}{\rho + t_j}}{\sum_j \dfrac{\varepsilon_j K_j}{\rho + t_j}} \lesseqgtr 0 \Leftrightarrow \mu_i \gtreqless 0, \tag{7}$$

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[7] We assume the wage determination by a simple model of efficiency wages (*e.g.,* Yellen 1984).
[8] See the Appendix for the derivation of equation (5).

$$\frac{\partial K_j}{\partial t_i} = -\frac{\dfrac{\varepsilon_i K_i}{\rho + t_i}\dfrac{\varepsilon_j K_j}{\rho + t_j}}{\sum_s \dfrac{\varepsilon_s K_s}{\rho + t_s}} > 0, \tag{8}$$

$$\frac{\partial L_j}{\partial t_i} = -\frac{\dfrac{\varepsilon_i \mu_i K_i}{\rho + t_i}\dfrac{\varepsilon_j K_j}{\rho + t_j}}{\sum_s \dfrac{\varepsilon_s K_s}{\rho + t_s}} \gtreqless 0 \Leftrightarrow \mu_j \gtreqless 0. \tag{9}$$

Equations (6) and (8) are essentially the same results derived by Wildasin (1988). Equations (7) and (9) are generalized versions of Ogawa *et al.* (2006). If the regions are symmetrical in all respects, equations (6) through (9) then become

$$\frac{\partial K_i}{\partial t_i} = \frac{(1-n)\varepsilon K}{f_K(K,L)}, \frac{\partial K_j}{\partial t_i} = -\frac{n\varepsilon K}{f_K(K,L)}, \frac{\partial L_i}{\partial t_i} = \frac{(1-n)\varepsilon \mu K}{f_K(K,L)}, \frac{\partial L_j}{\partial t_i} = -\frac{n\varepsilon \mu K}{f_K(K,L)}.$$

Finally, we consider the ownership of capital and social welfare in each region. Total capital income in the economy is represented by $\rho \overline{K}$. Let θ_i be the share of the capital stock owned by the residents in region i. Then, we have $\sum_{i=1}^{N} \theta_i \leq 1$. If $\sum_{i=1}^{N} \theta_i < 1$, there are absentee owners of capital. On the contrary, capital is owned by residents only when $\sum_{i=1}^{N} \theta_i = 1$. With the presence of unemployment, there are two possible states that might characterize the residents: employed and unemployed. To keep the model simple, the quasi-linear preference is assumed. Following Ogawa et al. (2006) and Kikuchi and Tamai (2019), the Benthamite social welfare function is

$$U_i(X_i, G_i) = X_i + v(G_i),$$

where $v'(G_i) > 0$, $v''(G_i) < 0$, $v'(0) = \infty$, $v'(\infty) = 0$, and

$$X_i = f(K_i, L_i) - K_i f_K(K_i, L_i) + \rho \theta_i \overline{K}. \tag{10}$$

The regional government chooses the tax rate or the expenditure level to maximize the social welfare function, subject to some constraints. The next section provides a detailed analysis.

3. Tax Competition versus Expenditure Competition

This section characterizes two different Nash equilibria for states produced by fiscal competition. One is a state in which the local government chooses the tax rate to maximize its objective function. The other is a state in which the regional authority determines the expenditure level in order to maximize its objective function. In the presence of unemployment, the ownership of capital and the number of regions significantly influence the equilibrium outcome of each of these two equilibria.

3.1. Tax Competition Equilibrium

First, we define the Nash equilibrium wherein regional governments compete with each other with regard to their tax rates (*e.g.,* Wilson 1986; Zodrow and Mieszkowski 1986). Following Wildasin (1988), the definition is described as follows:

Definition 1. *T-equilibrium is a vector τ^* that t_i^* is the solution to*

$$\max_{t_i} U_i(X_i, G_i)$$

subject to (4), (10), $\rho = \rho(\tau)$, $K_i = K_i(\tau)$, $L_i = L_i(\tau)$, and $t_j = t_j^*$ $(j \neq i)$.

Solving the optimization problem defined in Definition 1, we have

$$\frac{\partial U_i}{\partial t_i} = \overline{w}\frac{\partial L_i}{\partial t_i} - K_i f_{KL}(K_i, L_i)\frac{\partial L_i}{\partial t_i} - K_i f_{KK}(K_i, L_i)\frac{\partial K_i}{\partial t_i} + \theta_i \overline{K}\frac{\partial \rho}{\partial t_i} + v'(G_i)\left[K_i + t_i\frac{\partial K_i}{\partial t_i}\right] = 0.$$

The first and second terms are originated in the presence of unemployment. In particular, the first term

captures *the employment externality* shown by Ogawa *et al.* (2006).[9] Here, we introduce a standard assumption commonly found in the literature on tax competition:

Assumption 1.

$$1 + \frac{\partial \log K_i}{\partial \log t_i} > 0.$$

Assumption 1 ensures that the economy is located on the left-hand side of the Laffer curve.

We focus on the symmetrical regions in all respects. Furthermore, two cases of capital ownership are considered at this point: (a) when absentee owners of capital have full ownership of capital ($\theta_i = 0$) and (b) when residents equally share the economy-wide capital stock ($\theta_i = n$).

(a) Absentee owners of capital have full ownership of capital ($\theta_i = 0$). Then, the first-order condition for the regional government's optimization problem can be rewritten as

$$v'(G_i) = \frac{1 - n - \frac{\overline{w}}{K_i}\frac{\partial L_i}{\partial t_i}}{1 + \frac{\partial \log K_i}{\partial \log t_i}}, \tag{11}$$

where $n \equiv N^{-1}$. Note that $0 < n < 1$ and $n \to 0$ as $N \to \infty$.

(b) Residents equally share the economy-wide capital stock ($\theta_i = n$). The first-order condition for the regional government's optimization problem then becomes

$$v'(G_i) = \frac{1 - \frac{\overline{w}}{K_i}\frac{\partial L_i}{\partial t_i}}{1 + \frac{\partial \log K_i}{\partial \log t_i}}. \tag{12}$$

Equations (11) and (12) have similar forms except for the term n in the numerator. If the number of regions is infinitely large ($N \to \infty$), these two equations can be equalized. Absentee owners of capital are crucial to determining the marginal cost of public funds (MCPF).

3.2. Expenditure Competition Equilibrium

Another state of fiscal competition is expenditure competition. From equation (4) and $K_i = K_i(\tau)$, we have $G_i = G_i(\tau) = t_i K_i(\tau)$. Let $g = (G_1, \cdots, G_n)$ be the vector of public expenditure. If the system composed of equations (1)-(4) has its inverse matrix, we obtain $\tau = \tau(g)$. Then, we can analyze the expenditure competition equilibrium formally defined as follows:

Definition 2. *G-equilibrium is a vector $g^\star$ that $G_i^\star$ is the solution to*
$$\max_{t_i} U_i(X_i, G_i)$$
subject to (4), (10), $\rho = \rho(\tau(g))$, $K_i = K_i(\tau(g))$, $L_i = L_i(\tau(g))$, and $G_j = G_j^\star$ ($j \neq i$).

The first-order condition for the regional government's optimization problem is
$$\frac{\partial U_i}{\partial G_i} = \left[\overline{w}\frac{\partial L_i}{\partial t_i} - K_i f_{KK}(K_i, L_i)\frac{\partial K_i}{\partial t_i} - K_i f_{KL}(K_i, L_i)\frac{\partial L_i}{\partial t_i} + \theta_i \overline{K}\frac{\partial \rho}{\partial t_i}\right]\frac{\partial t_i}{\partial G_i}$$
$$+ \sum_{j \neq i}\left[\overline{w}\frac{\partial L_i}{\partial t_j} - K_i f_{KK}(K_i, L_i)\frac{\partial K_i}{\partial t_j} - K_i f_{KL}(K_i, L_i)\frac{\partial L_i}{\partial t_j} + \theta_i \overline{K}\frac{\partial \rho}{\partial t_j}\right]\frac{\partial t_j}{\partial G_i} + v'(G_i) = 0$$

The first-bracket term represents the effects of a change in government expenditure in region i through a change in the capital tax rate in region i. The second-bracket term captures the strategic

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[9] In Ogawa *et al.* (2006), it is referred as *an unemployment-exporting externality*. A reduction in the capital tax rate in region i attracts capital to the region. If $f_{KL} > 0$, the capital inflow to the region decreases employment in the other region while it simultaneously increases employment in its own region.

effects of a change in government expenditure in region i through changes in the tax rates in other regions.

Focusing on symmetrical regions, we have[10]

$$\frac{\partial t_i}{\partial G_i} = D^{-1}\left[K_j(\tau) + t_j\frac{\partial K_j(\tau)}{\partial t_j} + \left(\frac{1-2n}{n}\right)t_j\frac{\partial K_j(\tau)}{\partial t_i}\right]$$

$$\frac{\partial t_j}{\partial G_i} = -D^{-1}t_j\frac{\partial K_j(\tau)}{\partial t_i},$$

where

$$D \equiv \left[K_i(\tau) + t_i\frac{\partial K_i(\tau)}{\partial t_i}\right]\left[K_j(\tau) + t_j\frac{\partial K_j(\tau)}{\partial t_j} + \left(\frac{1-2n}{n}\right)t_j\frac{\partial K_j(\tau)}{\partial t_i}\right] - \left(\frac{1-n}{n}\right)t_it_j\frac{\partial K_j(\tau)}{\partial t_i}\frac{\partial K_i(\tau)}{\partial t_j}$$
$$> 0.$$

(a) Absentee owners of capital have full ownership of capital $(\theta_i = 0)$. The first-order condition can be reduced to

$$v'(G_i) = \left[(1-n)K_i - \overline{w}\frac{\partial L_i}{\partial t_i}\right]\frac{\partial t_i}{\partial G_i} - \sum_{j\neq i}\left[nK_i + \overline{w}\frac{\partial L_i}{\partial t_j}\right]\frac{\partial t_j}{\partial G_i}. \tag{13}$$

(b) Residents equally share the economy-wide capital stock $(\theta_i = n)$. The first-order condition then becomes

$$v'(G_i) = \left[K_i - \overline{w}\frac{\partial L_i}{\partial t_i}\right]\frac{\partial t_i}{\partial G_i} - \overline{w}\sum_{j\neq i}\frac{\partial L_i}{\partial t_j}\frac{\partial t_j}{\partial G_i}. \tag{14}$$

In equations (13) and (14), the effects of a change in region i's public expenditure on tax rates in its own and other regions are derived (see the Appendix) as

$$\frac{\partial t_i}{\partial G_i} = D^{-1}K\left[1 + \frac{n\varepsilon t}{f_K(K,L)}\right] > 0, \tag{15}$$

$$\frac{\partial t_j}{\partial G_i} = D^{-1}K\frac{n\varepsilon t}{f_K(K,L)} < 0, \tag{16}$$

where

$$D = \left[1 + \frac{\varepsilon t}{f_K(K,L)}\right]K^2 > 0.$$

Equation (16) captures the strategic effects of a change in government expenditure.

3.3. Comparative Analysis of the Tax and Expenditure Equilibria

We now characterize two different equilibria of fiscal competition through a comparison between the MCPF (*i.e.,* (11) and (13); (12) and (14)). To conduct the comparative analysis, equations (11) through (14) should be transformed to these comparable forms.

(a) Absentee owners of capital have full ownership of capital $(\theta_i = 0)$. Applying the symmetrical conditions into equations (11) and (13) gives

$$\text{T-equilibrium: } v'(G_i) = \frac{\left[1 - \overline{w}\frac{\varepsilon\mu}{f_K(K,L)}\right](1-n)}{1 + \frac{(1-n)\varepsilon t}{f_K(K,L)}} \equiv T\text{-MCPF}, \tag{17}$$

$$\text{G-equilibrium: } v'(G_i) = \frac{\left[1 - \overline{w}\frac{\varepsilon\mu}{f_K(K,L)}\right](1-n)}{1 + \frac{\varepsilon t}{f_K(K,L)}} \equiv G\text{-MCPF}. \tag{18}$$

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[10] See the Appendix for the derivation of these two equations.

Under symmetricity in all aspects, $\varepsilon, \mu, w, K, L,$ and f_K are all positive constants.[11] Hence, we can directly compare equations (17) and (18), evaluated at the given level of the same tax rate:

$$T\text{-MCPF} - G\text{-MCPF} = \frac{\dfrac{n\varepsilon t}{f_K(K,L)}\left[1 - \overline{w}\,\dfrac{\varepsilon\mu}{f_K(K,L)}\right](1-n)}{\left[1 + \dfrac{(1-n)\varepsilon t}{f_K(K,L)}\right]\left[1 + \dfrac{\varepsilon t}{f_K(K,L)}\right]} < 0.$$

This inequality indicates that the marginal cost of supplying a public good in G-equilibrium is larger than the marginal cost in T-equilibrium. Figure 1 illustrates the relationship between the marginal benefits of the public good, the marginal cost of public funds at T-equilibrium (T-MCPF), and the marginal cost of public funds at G-equilibrium (G-MCPF). The intersection points A and B, respectively, correspond to the T-equilibrium and the G-equilibrium. Figure 1 illustrates that the public good supply (capital tax rate) in G-equilibrium is smaller than the supply (capital tax rate) in T-equilibrium.

Using Nash conjecture, in the state of tax competition equilibrium, the government of region i perceives that a rise in its tax rate increases the supply of public goods in other regions due to the resultant capital flight from its own region to other regions. In contrast, the government presumes that governments in other regions aggressively reduce their tax rates in response to region i's increase of its tax rate. Even though the resultant capital flight induces increasing employment or unemployment, the employment effect has a common impact on the MCPFs; See the numerator of equations (17) and (18). Hence, the key determinant of the relative size of the MCPFs is a strategic effect in that governments in other regions decrease their tax rates in order to maintain a balanced budget.

Formally, we have the following proposition (see the Appendix for the proof of Proposition 1):

Proposition 1. *Suppose that absentee owners possess the capital in the economy. There exists a unique symmetrical T-equilibrium and G-equilibrium, respectively. Then, the equilibrium tax rate and the expenditure level satisfy $t^* > t^\star$ and $G^* > G^\star$.*

Proposition 1 is the extended version of Wildasin (1988), who demonstrated the same result.[12] On the other hand, in our model, the T-equilibrium and the G-equilibrium are not necessary to cause the under-provision of public goods because of an employment externality. We should investigate whether the tax rate and the expenditure level are less than optimal. With a lump-sum tax or common tax rates on capital, the first-order condition for the optimal provision of a public good is $v'(G_i) = 1$. This condition yields (G^o, t^o).[13] The optimal conditions and Proposition 1 reveal the following results (see the Appendix for the proof of Proposition 2):

Proposition 2. *Suppose that absentee owners possess the capital in the economy. For $t \in [0, \bar{t})$, the relationships between the capital tax rate and welfare are*

(i) $\quad n + \dfrac{(\overline{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)} \le 0 \Rightarrow t^o \ge t^* > t^\star, U^o \ge U^* > U^\star,$

(ii) $\quad 0 < n + \dfrac{(\overline{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)} < -\dfrac{n\varepsilon t}{f_K(K,L)} \Rightarrow t^* > t^o > t^\star, U^o > \max(U^*, U^\star),$

(iii) $\quad 0 \le -\dfrac{n\varepsilon t}{f_K(K,L)} < n + \dfrac{(\overline{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)} \Rightarrow t^* > t^\star \ge t^o, U^o \ge U^* > U^\star.$

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[11] In the symmetric economy, $K = n\overline{K} \equiv \overline{k}$ holds. Equation (3) leads to $\overline{w} = f_L(\overline{k}, L) \Rightarrow \overline{L} = L(\overline{k}, \overline{w})$. Capital and labor are both constant in equilibrium. Thus, all derivatives of the production function are also constant, as is μ, and

$$\varepsilon = \frac{d\log K}{d\log(\rho + t)} = \frac{f_K(K,L)f_{LL}(K,L)}{[f_{KK}(K,L)f_{LL}(K,L) - f_{KL}^2(K,L)]K}.$$

[12] When $\mu = 0$, the result reverts to the one derived by Wildasin (1988).

[13] We have $G^o = v'^{-1}(1)$. Using equation (4) and $K_i = n\overline{K}$, $t^o = G^o/(n\overline{K})$.

Result (i) of Proposition 2 is parallel to the results obtained by of Wildasin (1988). On the other hand, results (ii) and (iii) differ from the first result. The interpretation of Proposition 3 is as follows. From equations (17) and (18), we have:

$$T\text{-MCPF} \lesseqgtr 1 \Leftrightarrow n + \frac{(\overline{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)} \gtreqless 0,$$

$$G\text{-MCPF} \lesseqgtr 1 \Leftrightarrow n + \frac{(\overline{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)} + \frac{n\varepsilon t}{f_K(K,L)} \gtreqless 0.$$

The presence of the strategic effect characterizes the difference between T-MCPF and G-MCPF. In addition to this result, the employment externality influences the MCPFs, depending on the value of μ.

If $\mu > 0$, there is a positive employment externality. Combining the positive employment externality with the fiscal externality, the MCPFs tend to be larger than unity. Therefore, public goods in the T- and G-equilibria will be under-provided. Furthermore, the supply of public goods in the G-equilibrium is smaller than that in the T-equilibrium due to the existence of the strategic effect. When $\mu < 0$, it means that there is a negative employment externality. The relative size of the strategic effect, the fiscal externality, and the employment externality are all important to maintaining efficiency in the provision of public goods. If the negative employment externality impacts the economy enough to dominate the fiscal externality and the strategic effect, the MCPFs are possibly larger than unity; resulting in the over-provision of public goods. However, G-MCPF is smaller than T-MCPF due to the presence of the strategic effect. The public good may be optimally supplied at a certain value of μ. Therefore, the results of Proposition 2 imply that expenditure competition may be preferable to tax competition.

(b) Residents equally share the economy-wide capital stock ($\theta_i = n$). Using the symmetrical conditions, equations (12) and (14) can be reduced to

$$\text{T-equilibrium: } v'(G_i) = \frac{1 - \overline{w}\dfrac{(1-n)\varepsilon\mu}{f_K(K,L)}}{1 + \dfrac{(1-n)\varepsilon t}{f_K(K,L)}}, \tag{19}$$

$$\text{G-equilibrium: } v'(G_i) = \frac{1 - \overline{w}\dfrac{(1-n)\varepsilon\mu}{f_K(K,L)} + \dfrac{n\varepsilon t}{f_K(K,L)}}{1 + \dfrac{\varepsilon t}{f_K(K,L)}}. \tag{20}$$

The assumption of symmetrical regions leads to a set of constant values for $\varepsilon, \mu, w, K, L,$ and f_K. Similar to how we demonstrated Proposition 1, we now have

$$T\text{-MCPF} - G\text{-MCPF} = -\frac{(\overline{w}\mu + t)\dfrac{(1-n)n\varepsilon^2 t}{[f_K(K,L)]^2}}{\left[1 + \dfrac{(1-n)\varepsilon t}{f_K(K,L)}\right]\left[1 + \dfrac{\varepsilon t}{f_K(K,L)}\right]} \lesseqgtr 0 \Leftrightarrow \overline{w}\mu + t \gtreqless 0.$$

In contrast to case (a), the strategic effect has no impact on the order of the MCPFs. The strategic effect appears as the third term of the numerator in equation (20). The strategic effect is offset by the effect of fiscal externality, which corresponds to the second term of the denominator in equation (19). Thus, only the sign of $(\overline{w}\mu + t)$ is important for determining the inequality between the two MCPFs.

In addition to Assumption 1, we impose the following assumption:

Assumption 2.

$$v''\overline{k} < \frac{(1-n)(\overline{w}\mu + t)\left(\dfrac{\overline{\varepsilon}}{\overline{f}_K}\right)^2}{\left(1 + \dfrac{\overline{\varepsilon}t}{\overline{f}_K}\right)^2}.$$

This condition displays the *infimum* of μ, which represents the employment externality. Under Assumptions 1 and 2, the analysis developed above leads to the following proposition (see the Appendix for the proof of Proposition 3):

Proposition 3. *Suppose that the ownership of economy-wide capital is equally distributed among residents. There exists a unique symmetrical T-equilibrium and a unique symmetrical G-equilibrium, respectively. These equilibria are characterized as* $t^* \gtreqqless t^\star \Leftrightarrow G^* \gtreqqless G^\star \Leftrightarrow \bar{w}\mu + t \gtreqqless 0$ *for* $t \in [0, \bar{t})$.

Comparing the results of Propositions 1 and 3, the ownership of capital is crucial to determine the equilibrium tax rate and the supply of the public good. In particular, it affects not only the first-order condition of each equilibrium state, but also the strategic effect of the *G*-equilibrium. When the residents equally own the capital, the strategic effect is negligible when comparing the tax rates at both *T*-equilibrium and *G*-equilibrium.

According to Proposition 3, a positive employment externality, $\mu > 0$, leads to $\bar{w}\mu + t > 0$. Therefore, a positive external effect is generated by fiscal and employment externalities, which are both positive. A similar result shown by Wildasin (1988) holds in such a case. A negative employment externality, $\mu < 0$, yields a non-trivial outcome. Especially, if $\bar{w}\mu + t < 0$, we have the opposite result in comparison to the case where $\bar{w}\mu + t > 0$; $t^* < t^\star$ and $G^* < G^\star$. The critical level of the tax rate, $t = -\bar{w}\mu$, corresponds to *the optimal tax rate* derived by Ogawa *et al.* (2006) when the lump-sum tax is available.[14] Whether the equilibrium tax rate is larger or smaller than the critical level is an essential factor in determining whether public goods are over-provided or not. Indeed, we obtain the following results (see the Appendix for the proof of Proposition 4):

Proposition 4. *Suppose that the ownership of economy-wide capital is equally distributed among residents. For* $t \in [0, \bar{t})$*, the relationships between the capital tax rate and welfare are*
  (i) $\bar{w}\mu + t > 0 \Rightarrow t^o > t^* > t^\star, U^o > U^* > U^\star$,
  (ii) $\bar{w}\mu + t < 0 \Rightarrow t^o < t^* < t^\star, U^o > U^* > U^\star$.

The equilibrium tax rates of case (i) are illustrated in Figure 1 and on the right side of $t = -\bar{w}\mu$ in Figure 2. In contrast, the equilibrium tax levels of case (ii) are placed on the left side of $t = -\bar{w}\mu$ in Figure 2. Equations (19) and (20), respectively, lead to

$$T\text{-MCPF} - 1 = -\frac{\dfrac{(\bar{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)}}{1 + \dfrac{(1-n)\varepsilon t}{f_K(K,L)}} \gtreqqless 0 \Leftrightarrow \bar{w}\mu + t \gtreqqless 0,$$

$$G\text{-MCPF} - 1 = -\frac{\dfrac{(\bar{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)}}{1 + \dfrac{\varepsilon t}{f_K(K,L)}} \gtreqqless 0 \Leftrightarrow \bar{w}\mu + t \gtreqqless 0.$$

These equations show that the relationship between degree of the employment externality and the equilibrium tax rate is important for maintaining efficiency in providing public goods.

If the employment externality is positive, or not too negative, the MCPFs tend to be larger than unity. Hence, an under-provision of public goods occurs, and the tax rate in *G*-equilibrium corresponds to the lowest one. When the employment externality is negative and has a sufficiently powerful impact, the MCPFs will be less than unity. This means that an over-provision of public goods occurs. Then, the highest tax rate is that at *G*-equilibrium. In contrast to case (a), *T*-equilibrium may be welfare-superior to *G*-equilibrium (recall Proposition 2).

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[14] In our model, we need $t > 0$ because the capital tax is the only financial source for supplying the public good. Hence, $\mu < 0$ is a required condition. The results shown by Kikuchi and Tamai (2019) imply that the optimal provision of public goods is attainable regardless of the presence of the intergovernmental transfer if $t = -\bar{w}\mu$ holds in equilibrium.

4. Conclusion

This paper examined two different fiscal competition games operating under labor market imperfections. We demonstrated that a unique Nash equilibrium exists for both tax competition and expenditure competition. Some conditions are required to ensure the existence and uniqueness of the Nash equilibria, even though they are reasonable, they are not strong. Comparing these two equilibria revealed that the presence/absence of absentee ownership of capital and employment externalities are significant factors to be considered in order to characterize the equilibria properties.

Focusing on the most interesting implications, in some cases, this paper showed that tax rates under expenditure competition are *less competitive* (*i.e.*, higher) than those under tax competition, which is contrary to the dominant view in the existing literature. This is only possible when residents equally share the capital. If absentee owners own the capital, such a situation does not occur. Furthermore, welfare analyses demonstrated that governments choose government expenditure as their strategic variable if the negative employment externality is sufficiently strong and absentee owners possess the capital. In contrast, the tax rate is likely chosen as the strategic variable if the capital is evenly distributed among the residents.

Finally, the future direction of this research should be mentioned. In the basic model, a fixed wage is assumed for analytical simplicity and tractability. This assumption does not vitiate our main findings under the presence of employment externalities. However, different settings for labor markets might alter these results. Recent insights about labor economics should be incorporated in order to examine the robustness of our results. Numerical analyses under specific utility and production functions would also be helpful to conduct such research and provide a clear relationship between the equilibria and demonstrate the best response curve. The present study will be the analytical basis for such extensions of this research.

Appendix

A. Derivations of Equations (5) through (9)

The total differentiation of equation (2) leads to
$$d(\rho + t_i) = f_{KK}(K_i, L_i)dK_i + f_{KL}(K_i, L_i)dL_i$$
$$= \frac{f_{KK}(K_i, L_i)f_{LL}(K_i, L_i) - [f_{KL}(K_i, L_i)]^2}{f_{LL}(K_i, L_i)}dK_i = \frac{|J_i|}{f_{LL}(K_i, L_i)}dK_i,$$
where $|J_i| \equiv f_{KK}f_{LL} - f_{KL}^2 > 0$. Hence, we have

$$\frac{dK_i}{d(\rho + t_i)} = \frac{f_{LL}(K_i, L_i)}{|J_i|} < 0. \tag{A1}$$

The total differentiation of equation (1) yields
$$\sum_{i=1}^{N} dK_i = 0.$$
From this equation and (A1), we obtain equation (5):

$$\frac{dK_i}{d(\rho + t_i)}\left(\frac{\partial \rho}{\partial t_i} + 1\right) + \frac{\partial \rho}{\partial t_i}\sum_{j \neq i}\frac{dK_j}{d(\rho + t_j)} = 0 \Leftrightarrow \frac{\partial \rho}{\partial t_i} = -\frac{\dfrac{dK_i}{d(\rho + t_i)}}{\sum_i \dfrac{dK_i}{d(\rho + t_i)}} = -\frac{\dfrac{\varepsilon_i K_i}{\rho + t_i}}{\sum_j \dfrac{\varepsilon_j K_j}{\rho + t_j}} < 0.$$

Using equations (2) and (A1) lead to
$$\frac{\partial \rho}{\partial t_i} + 1 = \frac{|J_i|}{f_{LL}(K_i, L_i)}\frac{\partial K_i}{\partial t_i}.$$
Inserting equation (5) into the above equation generates equation (6):

$$-\frac{\dfrac{dK_i}{d(\rho + t_i)}}{\sum_i \dfrac{dK_i}{d(\rho + t_i)}} + 1 = \frac{1}{\dfrac{dK_i}{d(\rho + t_i)}}\frac{\partial K_i}{\partial t_i} \Leftrightarrow \frac{\partial K_i}{\partial t_i} = \frac{dK_i}{d(\rho + t_i)}\frac{\sum_{j \neq i}\dfrac{dK_j}{d(\rho + t_j)}}{\sum_j \dfrac{dK_j}{d(\rho + t_j)}} = \frac{\varepsilon_i K_i}{\rho + t_i}\frac{\sum_{j \neq i}\dfrac{\varepsilon_j K_j}{\rho + t_j}}{\sum_j \dfrac{\varepsilon_j K_j}{\rho + t_j}} < 0.$$

Equations (3) and (6) yield equation (7):

$$\frac{\partial L_i}{\partial t_i} = \frac{dL_i}{dK_i}\frac{\partial K_i}{\partial t_i} = \frac{\varepsilon_i \mu_i K_i}{\rho + t_i}\frac{\sum_{j \neq i}\dfrac{\varepsilon_j K_j}{\rho + t_j}}{\sum_j \dfrac{\varepsilon_j K_j}{\rho + t_j}}.$$

Similarly, equations (2), (3), and (A1) make equations (8) and (9), respectively:

$$\frac{\partial K_j}{\partial t_i} = \frac{dK_j}{d(\rho + t_j)}\frac{\partial \rho}{\partial t_i} = -\frac{\dfrac{dK_j}{d(\rho + t_j)}\dfrac{dK_i}{d(\rho + t_i)}}{\sum_i \dfrac{dK_i}{d(\rho + t_i)}} = -\frac{\dfrac{\varepsilon_i K_i}{\rho + t_i}\dfrac{\varepsilon_j K_j}{\rho + t_j}}{\sum_s \dfrac{\varepsilon_s K_s}{\rho + t_s}} > 0,$$

$$\frac{\partial L_j}{\partial t_i} = \frac{dL_j}{dK_j}\frac{\partial K_j}{\partial t_i} = -\frac{\dfrac{\varepsilon_i \mu_i K_i}{\rho + t_i}\dfrac{\varepsilon_j K_j}{\rho + t_j}}{\sum_s \dfrac{\varepsilon_s K_s}{\rho + t_s}}.$$

B. Derivations of equations (11) and (12)

The first-order partial derivative of the social welfare function in region i with respect to t_i is:
$$\frac{\partial U_i}{\partial t_i} = \bar{w}\frac{\partial L_i}{\partial t_i} - K_i f_{KK}(K_i, L_i)\frac{\partial K_i}{\partial t_i} - K_i f_{KL}(K_i, L_i)\frac{\partial L_i}{\partial t_i} + \theta_i \bar{K}\frac{\partial \rho}{\partial t_i} + v'(G_i)\left[K_i + t_i\frac{\partial K_i}{\partial t_i}\right]$$

$$= \bar{w}\frac{\partial L_i}{\partial t_i} - K_i \frac{\sum_{j \neq i}\frac{\varepsilon_j K_j}{\rho + t_j}}{\sum_j \frac{\varepsilon_j K_j}{\rho + t_j}} + \theta_i \bar{K}\frac{\partial \rho}{\partial t_i} + v'(G_i)\left[K_i + t_i\frac{\partial K_i}{\partial t_i}\right].$$

Inserting $\partial U_i / \partial t_i = 0$ into above equation and after some calculations, we have

$$v'(G_i) = \frac{K_i\frac{\sum_{j \neq i}\frac{\varepsilon_j K_j}{\rho + t_j}}{\sum_j \frac{\varepsilon_j K_j}{\rho + t_j}} - \bar{w}\frac{\partial L_i}{\partial t_i} - \theta_i \bar{K}\frac{\partial \rho}{\partial t_i}}{K_i + t_i\frac{\partial K_i}{\partial t_i}} = \frac{\frac{\sum_{j \neq i}\frac{\varepsilon_j K_j}{\rho + t_j}}{\sum_j \frac{\varepsilon_j K_j}{\rho + t_j}} - \frac{\bar{w}}{K_i}\frac{\partial L_i}{\partial t_i} - \theta_i \frac{\bar{K}}{K_i}\frac{\partial \rho}{\partial t_i}}{1 + \frac{\partial \log K_i}{\partial \log t_i}}. \tag{A2}$$

(a) $\theta = 0$. Using equation (A2), $\theta = 0$, $K_i = \bar{K}/N = n\bar{K} = K$, $t_i = t_j$, and $\varepsilon_i = \varepsilon_j$,

$$v'(G_i) = \frac{\frac{N-1}{N} - \frac{\bar{w}}{K_i}\frac{\partial L_i}{\partial t_i}}{1 + \frac{\partial \log K_i}{\partial \log t_i}} = \frac{1 - n - \frac{\bar{w}}{K_i}\frac{\partial L_i}{\partial t_i}}{1 + \frac{\partial \log K_i}{\partial \log t_i}}.$$

(b) $\theta = n$. Utilizing equation (5), (A2), $\theta = n$, $K_i = \bar{K}/N = n\bar{K} = K$, $t_i = t_j$, and $\varepsilon_i = \varepsilon_j$,

$$v'(G_i) = \frac{1 - n - \frac{\bar{w}}{K_i}\frac{\partial L_i}{\partial t_i} - (-n)}{1 + \frac{\partial \log K_i}{\partial \log t_i}} = \frac{1 - \frac{\bar{w}}{K_i}\frac{\partial L_i}{\partial t_i}}{1 + \frac{\partial \log K_i}{\partial \log t_i}}.$$

C. Derivations of equations (13) through (16)

The partial derivative of the social welfare function in region i with respect to G_i is

$$\frac{\partial U_i}{\partial G_i} = \left[\bar{w}\frac{\partial L_i}{\partial t_i} - K_i f_{KK}(K_i, L_i)\frac{\partial K_i}{\partial t_i} - K_i f_{KL}(K_i, L_i)\frac{\partial L_i}{\partial t_i} + \theta_i \bar{K}\frac{\partial \rho}{\partial t_i}\right]\frac{\partial t_i}{\partial G_i}$$
$$+ \sum_{j \neq i}\left[\bar{w}\frac{\partial L_i}{\partial t_j} - K_i f_{KK}(K_i, L_i)\frac{\partial K_i}{\partial t_j} - K_i f_{KL}(K_i, L_i)\frac{\partial L_i}{\partial t_j} + \theta_i \bar{K}\frac{\partial \rho}{\partial t_j}\right]\frac{\partial t_j}{\partial G_i} + v'(G_i).$$

From this equation, the first-order condition becomes

$$v'(G_i)$$
$$= \left[K_i\frac{\sum_{j \neq i}\frac{\varepsilon_j K_j}{\rho + t_j}}{\sum_j \frac{\varepsilon_j K_j}{\rho + t_j}} - \bar{w}\frac{\partial L_i}{\partial t_i} - \theta_i \bar{K}\frac{\partial \rho}{\partial t_i}\right]\frac{\partial t_i}{\partial G_i} - \sum_{j \neq i}\left[\bar{w}\frac{\partial L_i}{\partial t_j} + K_i\frac{\frac{\varepsilon_j K_j}{\rho + t_j}}{\sum_s \frac{\varepsilon_s K_s}{\rho + t_s}} + \theta_i \bar{K}\frac{\partial \rho}{\partial t_j}\right]\frac{\partial t_j}{\partial G_i}. \tag{A3}$$

(a) $\theta = 0$. Inserting $\theta = 0$, $K_i = \bar{K}/N = n\bar{K} = K$, $t_i = t_j$, and $\varepsilon_i = \varepsilon_j$ into (A3), we obtain equation (13).

(b) $\theta = n$. Using equation (5), (A3), $\theta = n$, $K_i = \bar{K}/N = n\bar{K} = K$, $t_i = t_j$, and $\varepsilon_i = \varepsilon_j$,

$$v'(G_i) = \left[(1-n)K_i - \bar{w}\frac{\partial L_i}{\partial t_i} - \theta_i \bar{K}\frac{\partial \rho}{\partial t_i}\right]\frac{\partial t_i}{\partial G_i} - \sum_{j \neq i}\left[\bar{w}\frac{\partial L_i}{\partial t_j} + nK_i + \theta_i \bar{K}\frac{\partial \rho}{\partial t_j}\right]\frac{\partial t_j}{\partial G_i}$$
$$= \left[K_i - \bar{w}\frac{\partial L_i}{\partial t_i}\right]\frac{\partial t_i}{\partial G_i} - \sum_{j \neq i}\bar{w}\frac{\partial L_i}{\partial t_j}\frac{\partial t_j}{\partial G_i}.$$

From equation (4), we have

$$1 = K_j(\tau)\frac{\partial t_j}{\partial G_i} + t_i\sum_s \frac{\partial K_j(\tau)}{\partial t_s}\frac{\partial t_s}{\partial G_i}, \tag{A4}$$

$$0 = K_j(\tau)\frac{\partial t_j}{\partial G_i} + t_j\sum_{s \neq i}\frac{\partial K_j(\tau)}{\partial t_s}\frac{\partial t_s}{\partial G_i} + t_j\frac{\partial K_j(\tau)}{\partial t_i}\frac{\partial t_i}{\partial G_i}. \tag{A5}$$

Note that the subscripts have been dropped because of symmetry.

Equations (A4) and (A5) lead to

$$
\begin{pmatrix}
K_i(\tau) + t_i \dfrac{\partial K_i(\tau)}{\partial t_i} & (N-1)t_i \dfrac{\partial K_i(\tau)}{\partial t_j} \\[2ex]
t_j \dfrac{\partial K_j(\tau)}{\partial t_i} & K_j(\tau) + t_j \dfrac{\partial K_j(\tau)}{\partial t_j} + (N-2)t_j \dfrac{\partial K_j(\tau)}{\partial t_i}
\end{pmatrix}
\begin{pmatrix}
\dfrac{\partial t_i}{\partial G_i} \\[2ex]
\dfrac{\partial t_j}{\partial G_i}
\end{pmatrix}
= \begin{pmatrix} 1 \\ 0 \end{pmatrix}.
\tag{A6}
$$

In (A6), the determinant of the coefficient matrix is

$$
D \equiv \left[K_i(\tau) + t_i \frac{\partial K_i(\tau)}{\partial t_i}\right]\left[K_j(\tau) + t_j \frac{\partial K_j(\tau)}{\partial t_j} + (N-2)t_j \frac{\partial K_j(\tau)}{\partial t_i}\right] - (N-1)t_i t_j \frac{\partial K_j(\tau)}{\partial t_i} \frac{\partial K_i(\tau)}{\partial t_j}.
$$

If $D \neq 0$, by Cramer's rule, we obtain

$$
\frac{\partial t_i}{\partial G_i} = D^{-1}\left[K_j(\tau) + t_j \frac{\partial K_j(\tau)}{\partial t_j} + (N-2)t_j \frac{\partial K_j(\tau)}{\partial t_i}\right]
$$

$$
= D^{-1}K\left[1 + \frac{n\varepsilon t}{f_K(K,L)}\right],
$$

and

$$
\frac{\partial t_j}{\partial G_i} = -D^{-1}t_j \frac{\partial K_j(\tau)}{\partial t_i} = D^{-1}K\frac{n\varepsilon t}{f_K(K,L)}.
$$

D. Proof of Proposition 1

Using $(\varepsilon, \mu, w, K, L, f_K) = (\bar\varepsilon, \bar\mu, \bar w, \bar k, \bar L, \bar f_K)$, equation (17) becomes

$$
P(t) \equiv v'(t\bar k) - \frac{\left[1 - \bar w \dfrac{\bar\varepsilon\bar\mu}{\bar f_K}\right](1-n)}{1 + \dfrac{(1-n)\bar\varepsilon t}{\bar f_K}} = 0.
$$

By the assumption 2, the function P is monotonically decreasing in t $(P'(t) < 0)$ for

$$
t \in \left(0, \frac{-\bar f_K}{(1-n)\bar\varepsilon}\right).
$$

Furthermore, we have $\lim_{t\downarrow 0} P(t) = \infty$ and

$$
P\left(\frac{-\bar f_K}{(1-n)\bar\varepsilon}\right) = v'\left(\frac{-\bar f_K \bar k}{(1-n)\bar\varepsilon}\right) - \lim_{t\uparrow \frac{-\bar f_K}{(1-n)\bar\varepsilon}} \frac{\left[1 - \bar w \dfrac{\bar\varepsilon\bar\mu}{\bar f_K}\right](1-n)}{1 + \dfrac{(1-n)\bar\varepsilon t}{\bar f_K}} = -\infty.
$$

Therefore, there exists a unique value of t^* satisfying equation (17). Similarly, we have $Q'(t) < 0$, $\lim_{t\to +0} Q(t) = \infty$, and

$$
\lim_{t\uparrow \frac{-\bar f_K}{\bar\varepsilon}} Q(t) = -\infty,
$$

where

$$
Q(t) \equiv v'(t\bar k) - \frac{\left[1 - \bar w \dfrac{\bar\varepsilon\bar\mu}{\bar f_K}\right](1-n)}{1 + \dfrac{\bar\varepsilon t}{\bar f_K}} = 0 \text{ for } t \in \left(0, \frac{-\bar f_K}{\bar\varepsilon}\right).
$$

Thus, the unique value of $t^\star$ exists to solve $Q(t) = 0$.

We now move to show $t^* > t^\star$ and $G^* > G^\star$. Taking the difference between equations (17) and (18) yields

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$$T\text{-MCPF} - G\text{-MCPF} = \frac{\left[1 - \overline{w}\dfrac{\varepsilon\mu}{f_K(K,L)}\right](1-n)}{1 + \dfrac{(1-n)\varepsilon t}{f_K(K,L)}} - \frac{\left[1 - \overline{w}\dfrac{\varepsilon\mu}{f_K(K,L)}\right](1-n)}{1 + \dfrac{\varepsilon t}{f_K(K,L)}}$$

$$= \frac{\dfrac{\varepsilon t}{f_K(K,L)}\left[1 - \overline{w}\dfrac{\varepsilon\mu}{f_K(K,L)}\right](1-n)}{\left[1 + \dfrac{(1-n)\varepsilon t}{f_K(K,L)}\right]\left[1 + \dfrac{\varepsilon t}{f_K(K,L)}\right]} < 0.$$

(A7)

Equations (17) and (18) with (A7) lead to $v'(G^*) < v'(G^\star)$. By the concavity of v, $G^* > G^\star$ holds. Equation (4) with $G^* > G^\star$ yields $G^* = t^*K > t^\star K = G^\star$. Hence, we arrive at $t^* > t^\star$.

E. Proof of Proposition 2

Equations (17) and (18) provide

$$T\text{-MCPF} - 1 = \frac{\left[1 - \overline{w}\dfrac{\varepsilon\mu}{f_K(K,L)}\right](1-n)}{1 + \dfrac{(1-n)\varepsilon t}{f_K(K,L)}} - 1 = -\frac{n + \dfrac{(\overline{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)}}{1 + \dfrac{(1-n)\varepsilon t}{f_K(K,L)}},$$

(A8)

$$G\text{-MCPF} - 1 = \frac{\left[1 - \overline{w}\dfrac{\varepsilon\mu}{f_K(K,L)}\right](1-n)}{1 + \dfrac{\varepsilon t}{f_K(K,L)}} - 1 = -\frac{n + \dfrac{(\overline{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)} + \dfrac{n\varepsilon t}{f_K(K,L)}}{1 + \dfrac{\varepsilon t}{f_K(K,L)}}.$$

(A9)

From equations (A8) and (A9), there are a few possible cases. First, we consider when MCPFs are less than unity. Then, we have

$$n + \frac{(\overline{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)} > 0 \Rightarrow T\text{-MCPF} < 1,$$

$$n + \frac{(\overline{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)} + \frac{n\varepsilon t}{f_K(K,L)} \geq 0 \Rightarrow G\text{-MCPF} \leq 1.$$

This is true if

$$n + \frac{(\overline{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)} \geq -\frac{n\varepsilon t}{f_K(K,L)}.$$

Note that the right side of the inequality mentioned above is positive. Under this condition, using Proposition 1, we obtain $t^* > t^\star \geq t^o$. By the concavity of v, $U^* < U^\star \leq U^o$ holds. These results show result (iii) in Proposition 2.

Next, we have

$$n + \frac{(\overline{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)} \leq 0 \Rightarrow n + \frac{(\overline{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)} + \frac{n\varepsilon t}{f_K(K,L)} < 0.$$

Under the above condition, equations (A8) and (A9) lead to

$$G\text{-MCPF} > T\text{-MCPF} \geq 1.$$

Then, we have $t^o \geq t^* > t^\star$ and $U^o \geq U^* > U^\star$. Result (i) holds.

The last case is

$$n + \frac{(\overline{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)} > 0, n + \frac{(\overline{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)} + \frac{n\varepsilon t}{f_K(K,L)} < 0 \Rightarrow T\text{-MCPF} < 1 < G\text{-MCPF}.$$

This means that $t^* > t^o > t^\star$ and $U^o > \max(U^*, U^\star)$. Therefore, result (ii) was proven.

F. Proof of Proposition 3

In a similar manner to the proof of Proposition 1, $R'(t) < 0$, $\lim\limits_{t\downarrow 0} R(t) = \infty$, and

$$\lim_{t\uparrow \frac{-\bar{f}_K}{(1-n)\bar{\varepsilon}}} R(t) = -\infty,$$

where

$$R(t) \equiv v'(t\bar{k}) - \frac{1 - \bar{w}\dfrac{(1-n)\bar{\varepsilon}\bar{\mu}}{\bar{f}_K}}{1 + \dfrac{(1-n)\bar{\varepsilon}t}{\bar{f}_K}} = 0 \text{ for } t \in \left(0, \frac{-\bar{f}_K}{(1-n)\bar{\varepsilon}}\right).$$

Hence, we obtain the unique value of t^* from the solution of $R(t) = 0$. We now introduce the following function S:

$$S(t) \equiv v'(t\bar{k}) - \frac{1 - \bar{w}\dfrac{(1-n)\bar{\varepsilon}\bar{\mu}}{\bar{f}_K} + \dfrac{n\bar{\varepsilon}t}{\bar{f}_K}}{1 + \dfrac{\bar{\varepsilon}t}{\bar{f}_K}} = 0.$$

We have $\lim\limits_{t\downarrow 0} S(t) = \infty$,

$$S'(t) = v''\bar{k} - \frac{\dfrac{n\bar{\varepsilon}}{\bar{f}_K}\left[1 + \dfrac{\bar{\varepsilon}t}{\bar{f}_K}\right] - \left[1 - \bar{w}\dfrac{(1-n)\bar{\varepsilon}\bar{\mu}}{\bar{f}_K} + \dfrac{n\bar{\varepsilon}t}{\bar{f}_K}\right]\dfrac{\bar{\varepsilon}}{\bar{f}_K}}{\left(1 + \dfrac{\bar{\varepsilon}t}{\bar{f}_K}\right)^2}$$

$$= v''\bar{k} - \frac{(1-n)(\bar{w}\mu + t)\left(\dfrac{\bar{\varepsilon}}{\bar{f}_K}\right)^2}{\left(1 + \dfrac{\bar{\varepsilon}t}{\bar{f}_K}\right)^2}.$$

Note that G-MCPF satisfies

$$G\text{-MCPF} - 1 = \frac{1 - \bar{w}\dfrac{(1-n)\varepsilon\mu}{f_K(K,L)} + \dfrac{n\varepsilon t}{f_K(K,L)}}{1 + \dfrac{\varepsilon t}{f_K(K,L)}} - 1 = -\frac{\dfrac{(\bar{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)}}{1 + \dfrac{\varepsilon t}{f_K(K,L)}} \gtreqless 0 \qquad (A10)$$

$$\Leftrightarrow \bar{w}\mu + t \lesseqgtr 0.$$

Three cases should be considered: *(i)* $\bar{w}\mu + t > 0$. It leads to $S'(t) < 0$ and

$$\bar{w}\mu + t > 0 \Rightarrow 1 - \bar{w}\frac{(1-n)\bar{\varepsilon}\bar{\mu}}{\bar{f}_K} + \frac{n\bar{\varepsilon}t}{\bar{f}_K} > 1 + \frac{\bar{\varepsilon}t}{\bar{f}_K} > 0.$$

The domain of S is given as

$$0 < t < \frac{-\bar{f}_K}{\bar{\varepsilon}}.$$

Then, we obtain

$$\lim_{t\uparrow \frac{-\bar{f}_K}{\bar{\varepsilon}}} S(t) = -\infty.$$

These properties of $S(t)$ show that $S(t)$ has a unique value of t^* such that $S(t^*) = 0$.

(ii) $\bar{w}\mu + t = 0$. Equation (20) becomes $v'(t\bar{k}) = 1$. By the monotonicity and continuity of v', the unique solution is derived as

$$t^* = \frac{v'^{-1}(1)}{\bar{k}}.$$

(iii) $\bar{w}\mu + t < 0$. We have

$$\bar{w}\mu + t < 0 \Rightarrow 1 - \bar{w}\frac{(1-n)\bar{\varepsilon}\bar{\mu}}{\bar{f}_K} + \frac{n\bar{\varepsilon}t}{\bar{f}_K} < 1 + \frac{\bar{\varepsilon}t}{\bar{f}_K}$$

The domain of $S(t)$ is

$$T = \left\{ t \middle| t \in \left(0, \frac{-\bar{f}_K}{n\bar{\varepsilon}} \left[1 - \bar{w} \frac{(1-n)\bar{\varepsilon}\bar{\mu}}{\bar{f}_K} \right] \right) \right\}.$$

Taking this result into account, we obtain $\lim\limits_{t\uparrow \sup T} S(t) = \lim\limits_{t\uparrow \sup T} v''(t\bar{k}) < 0$, where

$$\sup T = \frac{-\bar{f}_K}{n\bar{\varepsilon}} \left[1 - \bar{w} \frac{(1-n)\bar{\varepsilon}\bar{\mu}}{\bar{f}_K} \right].$$

Therefore, there exists at least one root of $S(t) = 0$. To ensure the uniqueness of the root, we need to impose

$$v''\bar{k} - \frac{(1-n)(\bar{w}\mu + t)\left(\frac{\bar{\varepsilon}}{\bar{f}_K} \right)^2}{\left(1 + \frac{\bar{\varepsilon}t}{\bar{f}_K} \right)^2} < 0.$$

If $\bar{w}\bar{\mu} + t$ is sufficiently large, a unique value of $t^\star$ such that $S(t^\star) = 0$.

Equations (19) and (18) yield

$$T\text{-MCPF} - G\text{-MCPF} = \frac{1 - \bar{w}\dfrac{(1-n)\varepsilon\mu}{f_K(K,L)}}{1 + \dfrac{(1-n)\varepsilon t}{f_K(K,L)}} - \frac{1 - \bar{w}\dfrac{(1-n)\varepsilon\mu}{f_K(K,L)} + \dfrac{n\varepsilon t}{f_K(K,L)}}{1 + \dfrac{\varepsilon t}{f_K(K,L)}}$$

$$= -\frac{(\bar{w}\mu + t)\dfrac{(1-n)n\varepsilon^2 t}{[f_K(K,L)]^2}}{\left[1 + \dfrac{(1-n)\varepsilon t}{f_K(K,L)} \right]\left[1 + \dfrac{\varepsilon t}{f_K(K,L)} \right]} \lessgtr 0 \Leftrightarrow \bar{w}\mu + t \gtrless 0. \quad \text{(A11)}$$

G. Proof of Proposition 4

From equation (19), we have

$$T\text{-MCPF} - 1 = \frac{1 - \bar{w}\dfrac{(1-n)\varepsilon\mu}{f_K(K,L)}}{1 + \dfrac{(1-n)\varepsilon t}{f_K(K,L)}} - 1 = -\frac{(\bar{w}\mu + t)(1-n)\varepsilon}{f_K(K,L)}}{1 + \dfrac{(1-n)\varepsilon t}{f_K(K,L)}} \gtrless 0 \Leftrightarrow \bar{w}\mu + t \gtrless 0. \quad \text{(A12)}$$

Equations (A10) and (A12) derive $G\text{-MCPF} \gtreqless T\text{-MCPF} \gtreqless 1 \Leftrightarrow \bar{w}\mu + t \gtreqless 0$.

(i) $\bar{w}\mu + t > 0$. Then, $G\text{-MCPF} > T\text{-MCPF} > 1$ holds. By the concavity of v, we obtain $G^o > G^* > G^\star$. Equation (4) and $K_i = \bar{k}$ lead to $t^o > t^* > t^\star$. Furthermore, it shows $U^o > U^* > U^\star$.

(ii) $\bar{w}\mu + t < 0$. It shows $G\text{-MCPF} < T\text{-MCPF} < 1$. Then, by the similar way to derive the results in the case (i), we obtain $G^o < G^* < G^\star$, $t^o < t^* < t^\star$, and $U^\star < U^* < U^o$.

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Figures
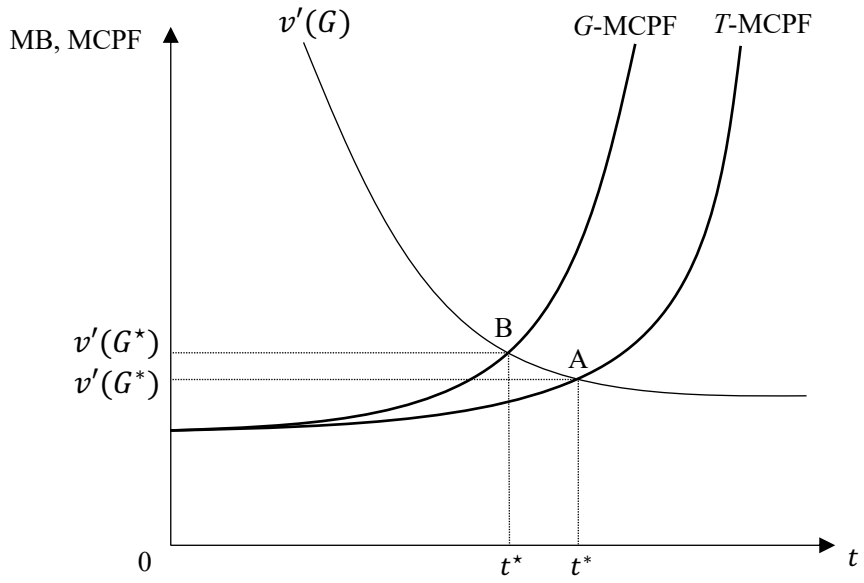
Figure 1. T-equilibrium and G-equilibrium
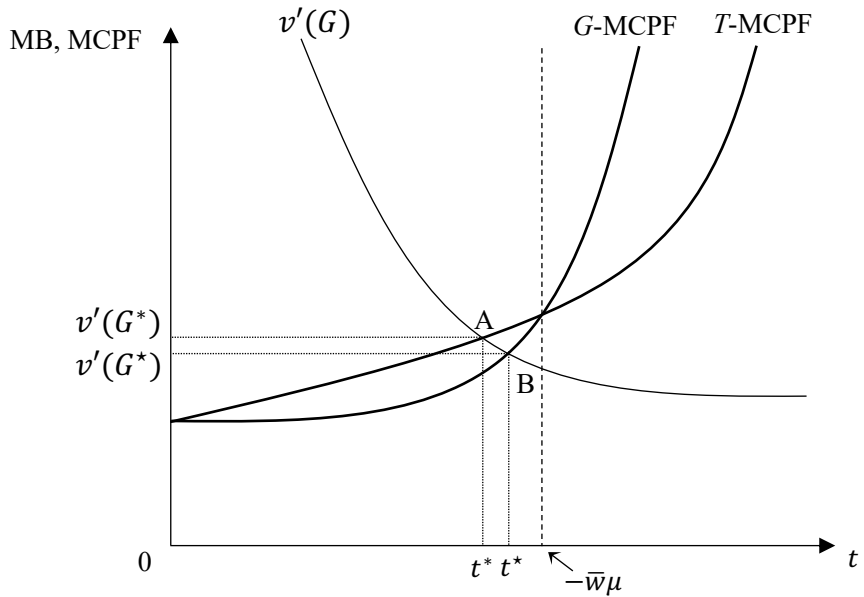
Figure 2. T-equilibrium and G-equilibrium in the case (b)-(ii)