A Two-country Model of Public Infrastructure Capital: Trade Patterns and Trade Gains in the Long Run

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Abstract

This study develops a two-country dynamic trade model with public infrastructure capital that has a positive effect on private sector productivity. Under the assumptions that welfare-maximizing national governments determine the paths of production taxes for financing public investment and that infrastructure has an "unpaid factor" property, this study examines the economy's trade pattern and the long-run effects of trade. If governments take world prices as given when they make public investments, a country with a smaller labor endowment, a lower depreciation rate for infrastructure capital stock, and/or a lower rate of time preference will become an exporter of a good that is more dependent on public infrastructure and will unambiguously gain from trade, whereas its trading partner may lose from trade. In the case of strategic governments that recognize the effect on the terms of trade when they determine their policy paths, the country exporting (importing) the good that is more dependent on the stock of public infrastructure will underaccumulate (overaccumulate) infrastructure in the long run.

Key Words: Public infrastructure capital; Two-country trade; Trade pattern; Gains/losses from trade; Terms-of-trade effect

JEL classification: F11; H41

1 Introduction

In economic activities, public infrastructure such as the electricity network, water supply, transportation, telecommunications, fundamental education, and the legal system plays a key role in facilitating increased production and smooth economic transactions. In light of the recent rapid globalization of the world economy, there is a growing need to consider the role of public infrastructure in connection with international trade, as documented by recent empirical studies such as Limão and Venables (2001), Bougheas et al. (2003), and Francois and Manchin (2013).

In international trade theory, productive public infrastructure has long been analyzed by incorporating public intermediate goods in traditional (Ricardian or Heckscher–Ohlin–Samuelson) trade models (Manning and McMillan, 1979; Tawada and Okamoto, 1983; Tawada and Abe, 1984; Ishizawa, 1988; Abe, 1990; Altenburg, 1992; Suga and Tawada, 2007). There are also studies on cost-reducing transport infrastructure and international trade (Martin and Rogers, 1995; Bougheas et al., 1999; Bond, 2006; Mun and Nakagawa, 2010; Tsubuku, 2016). However, most of these studies use a static framework. In reality, many public intermediate goods have the characteristics of durable goods or capital stock; research and development activities, national defense, and transportation and communication infrastructure are typical examples.

An exceptional study is McMillan (1978), who considers the stock effect of a public intermediate good in an open economy. He considers a three-sector (two private goods and a public intermediate good), one-factor (labor) small open economy with optimal supply of the public intermediate good. It is shown that the stock of the public intermediate good determines the slope of the production possibility frontier, and thus determines the pattern of international trade. Yanase and Tawada (2012) reexamine McMillan's model and show the possibility of multiple steady states and history-dependent dynamic paths. Moreover, they discuss whether trade is beneficial or not in McMillan's model.

In both McMillan (1978) and Yanase and Tawada (2012), the public intermediate good is assumed to have an impact similar to the "creation of atmosphere"-type externality classified by Meade (1952). That is, in their models, the technology in each private sector exhibits constant returns to scale in primary inputs alone. In a one-factor model, this assumption implies that for a given stock of the public intermediate good, the production possibility

frontier becomes linear, as in the standard Ricardian model, and thus the economy hardly diversifies production.

There is another class of public intermediate good that can be interpreted as "unpaid factors of production" according to Meade's (1952) terminology. If the public good is of this type, the production function of each private sector is characterized by constant returns to scale in all inputs, including the public intermediate good.[1] In the case of an unpaid factor type, congestion arises within an industry, although not among industries. For example, public transportation systems like highways may be included in this type. The agriculture industry uses the highway system in a certain season or region, while the manufacturing industry uses it in a different season or region. This highway system can be considered as public infrastructure of the unpaid factor type. Similarly, the communication system can be assumed to be of this type. In this study, we focus on this kind of public intermediate good and present a dynamic trade model in which the stock of a public good has a positive effect on private sector production. Following McMillan (1978) and Yanase and Tawada (2012), we consider an open economy in which two tradable private consumption goods and one nontradable public intermediate good are produced by one primary input, that is, labor. However, in contrast with McMillan (1978) and Yanase and Tawada (2012), we suppose that the private goods are produced under constant returns to scale with respect to the stock of the public intermediate good, as well as labor. Thus, the production possibility frontier becomes strictly concave to the origin, even in the case of one primary factor. Therefore, in general, diversified production is possible even after trade.

A dynamic trade model with public infrastructure of the unpaid factor type has already been analyzed by Yanase and Tawada (2017). However, their analysis is confined to a small open economy in which commodity prices are constant. In addition, they focus on how the labor endowment affects a country's comparative advantage and trade pattern. In the present study, we consider a two-country model, and thus commodity prices are endogenously determined in the international market under free trade. We also examine other determinants of trade patterns in addition to the labor endowment.

We begin with a dynamic two-country model in which the national governments, taking the prices of tradables as given, make infrastructure investments. The cost of public investment is

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[1]Tawada (1980) and Tawada and Okamoto (1983) used the alternative term "semi-public intermediate good" to describe this kind of good, compared with the "pure" public intermediate goods that have "creation of atmosphere"-type externalities.

financed by production taxes imposed on each private sector. We show that the competitive equilibrium path with the optimal production taxes coincides with the optimal path determined by a social planner who directly controls the economy's resource allocation. We also show that there exists a unique saddle-point stable steady state if the economic fundamentals of the two countries are slightly different. We then proceed to analyze the trade pattern of each country. It is shown that if the economy is initially at the autarkic steady state, after commencing trade, a country with a smaller (larger) labor endowment tends to be an exporter of a good that is more (less) dependent on the stock of public infrastructure. We also discuss whether each country gains or loses from trade by comparing the steady-state welfare level under free trade with that under autarky. It is shown that in comparison with the autarkic steady state, free trade increases (reduces) the steady-state stock of public infrastructure and thereby expands (diminishes) the long-run production possibility in a country exporting a good that is more (less) dependent on public infrastructure. An interesting result is that a smaller country unambiguously gains from trade in the long run, whereas a larger country may lose from trade.

We also consider an alternative scenario in which the governments strategically determine their provision of public infrastructure, taking into consideration the effect of each country's choice on the world market. If each country recognizes the effect of its decisions on the world price, namely the terms-of-trade effect, the governments' decision-making in relation to public investment can involve, through the pursuit of national interests, an improvement of its terms of trade. Because of this strategic behavior undertaken by national governments, it is shown that the steady-state stock of public infrastructure in each country differs from that in the case where the governments have no such strategic incentives. That is, in comparison with the case of nonstrategic governments, the country exporting (importing) a good that is more dependent on the infrastructure stock would underaccumulate (overaccumulate) public infrastructure in the long run.

The interdependence of public goods supply among countries in the presence of international trade has already been discussed in the literature. Devereux and Mansoorian (1992) and Figuières et al. (2013) extend Barro's (1990) model of endogenous growth driven by productive public expenditure to an open-economy context. Their main focus is on the strategic distortions caused by the lack of cooperation in determining tax rates to finance the production of public goods, and the growth consequences of such noncooperative policy-making. The

inefficiency of noncooperative supply of infrastructure that reduces transportation costs in the presence of trade is also discussed in Bougheas et al. (2003), who develop a static two-country, two-good model. They also conduct an empirical analysis using data from 16 European countries over the period 1987–1995 and find evidence consistent with their theoretical predictions. One shortcoming of these studies is that they assume that only one of two traded goods is produced in each country. In other words, trade patterns are exogenously determined in these studies. By contrast, although we focus on the optimal policy for the provision of public infrastructure from a single country's viewpoint, we take a closer look at the basic theorems in trade theory, namely the patterns of trade and the gains from trade, in a dynamic context.

The rest of the paper is organized as follows. Section 2 describes our two-country dynamic model with public infrastructure. In Sections 3 and 4, we consider a situation in which the government in each country makes a decision on public investment without taking into account its effect on world commodity prices and international trade. The properties of the dynamic equilibrium path and the steady state are analyzed in Section 3, and each country's trade pattern and the possibility of trade gains are examined in Section 4. In Section 5, we consider a situation in which the governments act strategically in the sense that they make public infrastructure investments while recognizing the effect they have on the terms of trade and compare the equilibrium conditions with those in the case of nonstrategic governments. Section 6 concludes.

2   Model

We consider a world economy consisting of two countries, home and foreign, in which two private and one public production sectors and one primary factor exist. The primary factor is assumed to be labor. The public sector produces an investment good using nonincreasing returns to scale technology with respect to labor. The investment good can be accumulated, and its accumulated stock, public infrastructure, serves production in the private sectors as a positive external effect without congestion between sectors. The two private sectors are denoted by sectors 1 and 2, where goods 1 and 2, respectively, are produced using constant returns to scale technology with respect to labor and the stock of public infrastructure. Total labor endowment is assumed to be constant over time. The private goods are traded between countries, while the public good for infrastructure investment is assumed to be nontradable.

2.1 Production side

Let us focus on the home country. The foreign country, whose variables are denoted by an asterisk (*), has a similar economic structure.

The production function of each private sector is assumed to take the following form:

$$Y_i = R^{\alpha_i} L_i^{1-\alpha_i}, \quad 0 \leq \alpha_i < 1, \quad i = 1, 2, \tag{1}$$

where Y_i is the output of good i, R is the stock of public infrastructure, and L_i is the labor input in sector i. It is clear that the labor productivity in each private sector is given by $\partial Y_i / \partial L_i = (1 - \alpha_i) R^{\alpha_i} L_i^{-\alpha_i}$ and is dependent on the stock of public infrastructure R.

In the following analysis, we make the following assumption regarding the impact of the stock of public infrastructure on industries:

Assumption 1 $\alpha_1 > \alpha_2$.

Because α_i is the production elasticity of the infrastructure stock in sector i, i.e., $\alpha_i = (\partial Y_i / \partial R) \cdot (R / Y_i)$, Assumption 1 can be interpreted as stating that *sector 1 is more dependent on the stock of public infrastructure than sector 2*. At the same time, Assumption 1 implies that sector 2 is more labor intensive than sector 1, as is usual in the standard Heckscher–Ohlin–Samuelson model.

The production function of the investment good in the public sector is expressed as $r = f(L_R)$, where r is the output and L_R is the labor input in the public sector. Concerning $f(L_R)$, we assume that this function is increasing and strictly concave ($f' > 0 > f''$), and $f(0) = 0$. Given the initial stock $R_0 > 0$, the infrastructure capital is assumed to accumulate over time according to[2]

$$\dot{R} = f(L_R) - \beta R, \tag{2}$$

where $\beta > 0$ is the depreciation rate of the stock of public infrastructure.

The full-employment condition of labor at each moment in time is given by

$$L_1 + L_2 + L_R = L, \tag{3}$$

where $L > 0$ is labor endowment and is assumed to be constant over time.

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[2] A dot over a variable denotes a time derivative. To reduce the complexity of notation, we omit time arguments when no confusion is caused by doing so.

We consider a competitive economy and assume that the cost of public investment is financed by production taxes:

$$wL_R = \tau_1 pY_1 + \tau_2 Y_2, \tag{4}$$

where w is the wage rate, p is the world price of good 1 (good 2 is assumed to be numeraire), and τ_i is the production tax rate imposed in sector $i = 1, 2$.[3] Then, a representative firm's profit in sector 1 is given by $\Pi_1 = (1 - \tau_1)pY_1 - wL_1$, and that in sector 2 is given by $\Pi_2 = (1 - \tau_2)Y_2 - wL_2$. The first-order conditions for profit maximization of the respective private firms are therefore given by

$$(1 - \tau_1)(1 - \alpha_1)p\frac{Y_1}{L_1} = w, \tag{5}$$

$$(1 - \tau_2)(1 - \alpha_2)\frac{Y_2}{L_2} = w. \tag{6}$$

The adjustment of wages in the labor market means that the full-employment condition of labor (3) is satisfied. Therefore, the government's budget constraint (4), profit maximization conditions (5) and (6), production function (1), and market-clearing condition of labor (3) jointly determine the equilibrium values for Y_1, Y_2, L_1, L_2, L_R, and w as a function of tax rates τ_1 and τ_2, as well as R, p, and L.

For the subsequent analysis, it is of interest to present the properties of Y_1, Y_2, and L_R. The comparative static results are shown in Appendix A.1, from which we can state that an increase in the tax rate in each sector reduces output in that sector, i.e., $\partial Y_i / \partial \tau_i < 0$, $i = 1, 2$, and that the law of supply holds, i.e., $\partial Y_1 / \partial p > 0 > \partial Y_2 / \partial p$.

2.2 Consumption side

The consumption side of the economy is described by a representative household whose lifetime utility is given by:

$$U = \int_0^\infty e^{-\rho t} \left[\gamma \ln C_1 + (1 - \gamma) \ln C_2 \right] dt, \tag{7}$$

where C_i is consumption of good i ($i = 1, 2$), ρ is the rate of time preference, and $\gamma \in (0, 1)$ is a parameter.

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[3]Kalaitzidakis and Tzouvelekas (2011) develop an endogenous growth model of a decentralized market economy with N different types of productive public capital, the investment costs of which are financed by tax revenue. They show that the growth-maximizing tax rate and the growth-maximizing share of public investment also maximize welfare.

Assuming that no borrowing or lending is permitted, the household's utility maximization behavior yields the optimal consumption for each good as $C_1 = \gamma I / p$ and $C_2 = (1 - \gamma)I$, where I is the household's income. I consists of firm profits Π_1 and Π_2, and labor income $w(L_1 + L_2 + L_R)$. In light of (4), it is verified that $I = pY_1 + Y_2$.

Substituting the household's optimal consumption into the lifetime utility (7), the household's indirect lifetime utility is derived as

$$V = \int_0^\infty e^{-\rho t} \left\{ \ln[pY_1(\tau_1, \tau_2, R, p, L) + Y_2(\tau_1, \tau_2, R, p, L)] - \gamma \ln p + \Gamma \right\} dt, \qquad (8)$$

where $\Gamma \equiv \gamma \ln \gamma + (1 - \gamma) \ln(1 - \gamma)$.

3 Trading Equilibrium under Nonstrategic Behavior of Governments

In this and the next sections, we assume that governments in both countries take the world price p as given when they determine the time paths of their stock of public infrastructure by controlling the tax rates.

3.1 Optimal policy

Let us focus on the home country. The home government's dynamic optimization problem is to maximize the household's indirect lifetime utility (8) subject to (2), where $L_R = L_R(\tau_1, \tau_2, R, p, L)$, by choosing the time paths of τ_1 and τ_2. The current-value Hamiltonian is

$$H = \ln[pY_1(\tau_1, \tau_2, R, p, L) + Y_2(\tau_1, \tau_2, R, p, L)] - \gamma \ln p + \Gamma + \theta\{f(L_R(\tau_1, \tau_2, R, p, L)) - \beta R\}.$$

The optimality conditions are derived as follows:

$$\frac{\partial H}{\partial \tau_1} = \frac{1}{pY_1 + Y_2} \left(p \frac{\partial Y_1}{\partial \tau_1} + \frac{\partial Y_2}{\partial \tau_1} \right) + \theta f'(L_R) \frac{\partial L_R}{\partial \tau_1} = 0, \qquad (9)$$

$$\frac{\partial H}{\partial \tau_2} = \frac{1}{pY_1 + Y_2} \left(p \frac{\partial Y_1}{\partial \tau_2} + \frac{\partial Y_2}{\partial \tau_2} \right) + \theta f'(L_R) \frac{\partial L_R}{\partial \tau_2} = 0, \qquad (10)$$

$$\dot{\theta} = \rho\theta - \frac{\partial H}{\partial R} = \theta \left\{ \rho + \beta - f'(L_R) \frac{\partial L_R}{\partial R} \right\} - \frac{1}{pY_1 + Y_2} \left(p \frac{\partial Y_1}{\partial R} + \frac{\partial Y_2}{\partial R} \right), \qquad (11)$$

$$\lim_{t \to \infty} e^{-\rho t} \theta(t) R(t) = 0. \qquad (12)$$

Substituting the comparative statics results (A.4) in the Appendix into the first-order conditions (9) and (10) and rearranging terms, it follows that

$$\tau_1 = \tau_2 = 1 - \frac{w}{(pY_1 + Y_2)\theta f'(L_R)}. \qquad (13)$$

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Substituting (13) into the private firms' profit maximization conditions (5) and (6) and rearranging terms, we have

$$(1 - \alpha_1)p\frac{Y_1}{L_1} = (1 - \alpha_2)\frac{Y_2}{L_2} = (pY_1 + Y_2)\theta f'(L_R). \tag{14}$$

In addition, substituting the comparative statics results (A.4) into the adjoint equation (11) and rearranging terms, we have

$$\dot{\theta} = (\rho + \beta)\theta - \frac{\alpha_1 pY_1 + \alpha_2 Y_2}{R(pY_1 + Y_2)}. \tag{15}$$

It is convenient to characterize the abovementioned set of equilibrium conditions with optimal output taxes by using the GDP function defined by

$$G(p, R, l) = \max_{L_1, L_2} \left\{ pR^{\alpha_1}L_1^{1-\alpha_1} + R^{\alpha_2}L_2^{1-\alpha_2} \quad \text{s.t.} \quad L_1 + L_2 = l \right\},$$

where $l \equiv L - L_R$ is the sum of labor inputs in the private sectors. Applying the envelope theorem to the GDP function, we obtain the following derivatives:[4]

$$G_p = Y_1, \qquad G_R = \frac{\alpha_1 pY_1 + \alpha_2 Y_2}{R}, \qquad G_l = (1 - \alpha_1)\frac{pY_1}{L_1} = (1 - \alpha_2)\frac{Y_2}{L_2}. \tag{16}$$

Appendix A.2 provides other properties of the GDP function.

Lemma 1 *Suppose that the government in each country determines the production taxes so as to maximize national welfare, taking the world price as given, in a competitive world economy. Then, the equilibrium path with optimal production taxes coincides with the optimal path in which a social planner determines the path of L_R so as to maximize $\int_0^\infty e^{-\rho t}[\ln G(p, R, L - L_R) - \gamma \ln p + \Gamma]dt$ subject to (2).*

Proof. Let us define the current-value Hamiltonian associated with the social planner's dynamic optimization problem as

$$\tilde{H}(L_R, R, \theta) = \ln[G(p, R, L - L_R)] - \gamma \ln p + \Gamma + \tilde{\theta}\left\{f(L_R) - \beta R\right\}.$$

Then, the optimal control must satisfy

$$\frac{\partial \tilde{H}}{\partial L_R} = 0 \quad \Rightarrow \quad \tilde{\theta}f'(L_R) = \frac{G_l(p, R, L - L_R)}{G(p, R, L - L_R)}. \tag{17}$$

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[4]The subscripts denote partial derivatives: $G_p = \partial G/\partial p$, and so on.

Moreover, the adjoint equation and the transversality condition are expressed, respectively, as

$$\dot{\theta} = \rho\tilde{\theta} - \frac{\partial \tilde{H}}{\partial R} = (\rho + \beta)\tilde{\theta} - \frac{G_R(p, R, L - L_R)}{G(p, R, L - L_R)}, \tag{18}$$

$$\lim_{t \to \infty} e^{-\rho t}\tilde{\theta}(t)R(t) = 0. \tag{19}$$

In light of (16), the optimality conditions (17), (18), and (19) are identical to (14), (15), and (12), respectively, if we set $\tilde{\theta} = \theta$. □

The optimality conditions (17) and (18), which are equivalent to (14) and (15), respectively, provide the following familiar interpretations. Eq.(17) indicates that the value of the marginal product of labor should be equalized across the public and private sectors. Eq.(18) indicates that the sum of the capital gain (or loss if it is negative) of the infrastructure capital $\dot{\theta}$ and the marginal social benefit of an increase in infrastructure G_R/G should be equal to the marginal cost $(\rho + \beta)\theta$, which is the sum of the intertemporal cost implied by the discount rate and the replacement cost of depreciated infrastructure capital.[5]

3.2 Dynamic equilibrium path

At each moment in time, the world market for each private good has to hold along an equilibrium path. We assume that the two countries share identical production technologies, meaning that the GDP function in the foreign country is given by $G(p, R^*, L^* - L_R^*)$, and have the same felicity function, meaning that $\gamma = \gamma^*$. Then, the world market-clearing condition for good 1, $C_1 + C_1^* = Y_1 + Y_1^*$, can be rewritten as

$$\frac{\gamma}{p}\{G(p, R, L - L_R) + G(p, R^*, L^* - L_R^*)\} = G_p(p, R, L - L_R) + G_p(p, R^*, L^* - L_R^*). \tag{20}$$

The dynamic equilibrium of the world economy is characterized by the home country's static optimality condition (17) and dynamic equations (2) and (18), their foreign counterparts, and the world market-clearing condition (20).

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[5]Needless to say, this is a dynamic version of Kaizuka's condition for optimal resource allocation in an economy with public intermediate goods (Kaizuka, 1965).

3.3   Steady state

At the steady state, it holds that $\dot{R} = \dot{R}^* = \dot{\theta} = \dot{\theta}^* = 0$. In light of (2), (17), and (18), conditions for the free-trade, steady-state equilibrium are given by

$$f(L_R) = \beta R, \tag{21}$$

$$\rho + \beta = \frac{G_R(p, R, L - L_R)}{G_l(p, R, L - L_R)} f'(L_R), \tag{22}$$

$$f(L_R^*) = \beta^* R^*, \tag{23}$$

$$\rho^* + \beta^* = \frac{G_R(p, R^*, L^* - L_R^*)}{G_l(p, R^*, L^* - L_R^*)} f'(L_R^*), \tag{24}$$

and (20).

From (21) and (22), the steady-state level of L_R can be represented as a function of p (and the parameters β, ρ, and L): $L_R = \psi(p)$. Under Assumption 1, the following lemma is obtained.

Lemma 2 *(i)* $\psi'(p) > 0$. *(ii) There exist* $\overline{L}_R$ *and* $\underline{L}_R$ *such that* $0 < \underline{L}_R < \overline{L}_R < L$, $\lim_{p \to 0} \psi(p) = \underline{L}_R$, *and* $\lim_{p \to \infty} \psi(p) = \overline{L}_R$.

Proof. See Appendix A.3.

From (21), the steady-state stock of public infrastructure is denoted as $R = f(\psi(p))/\beta$. Analogously for the foreign country, from (23) and (24), we have $L_R^* = \psi^*(p)$, which has similar properties to Lemma 2, and $R^* = f(\psi^*(p))/\beta^*$. Substituting these expressions into (20), the steady-state, market-clearing condition is given by

$$\frac{\gamma}{p} \left\{ G\left(p, \frac{f(\psi(p))}{\beta}, L - \psi(p)\right) + G\left(p, \frac{f(\psi^*(p))}{\beta^*}, L^* - \psi^*(p)\right) \right\}$$

$$= G_p\left(p, \frac{f(\psi(p))}{\beta}, L - \psi(p)\right) + G_p\left(p, \frac{f(\psi^*(p))}{\beta^*}, L^* - \psi^*(p)\right). \tag{25}$$

Let us denote the solution of (25), i.e., the steady-state equilibrium solution for the relative price of good 1 in the world market by p_{ss}. The steady-state stocks of public infrastructure in the home and foreign country are then derived as $R_{ss} = f(\psi(p_{ss}))/\beta$ and $R_{ss}^* = f(\psi^*(p_{ss}))/\beta^*$, respectively. The existence, uniqueness, and stability of the steady state are characterized by the following theorem.

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Theorem 1 *There exists at least one steady-state, free-trade equilibrium. If the differences in discount rates, depreciation rates, and labor endowments between the two countries are not very large, there exists a unique saddle-point stable steady state in which production is diversified in both countries.*

Proof. See Appendix A.4.

4 Trade Patterns and Trade Gains in the Long Run

4.1 Trade patterns

Let us focus on the home country's excess demand for good 1 in the steady state:

$$ED(p) \equiv \frac{\gamma}{p} G\left(p, \frac{f(\psi(p))}{\beta}, L - \psi(p)\right) - G_p\left(p, \frac{f(\psi(p))}{\beta}, L - \psi(p)\right). \qquad (26)$$

The foreign country's excess demand in the steady state, $ED^*(p)$, can be analogously defined. Let us denote the autarkic steady-state prices in the home country and foreign country by p_a and p_a^*, respectively. Then, in the autarkic steady-state equilibrium, it holds that $ED(p_a) = 0 = ED^*(p_a^*)$. It is verified that $ED(p)$ and $ED^*(p)$ are downward sloping in the neighborhood of the steady-state, autarkic equilibrium price.

Labor endowments   Evaluating at the autarkic steady-state solutions, we obtain the shift in excess demand in response to a change in the labor endowment L as follows:

$$\left.\frac{\partial ED(p)}{\partial L}\right|_{p=p_a} = \left(\frac{G_p G_R}{G} - G_{pR}\right)\frac{f'}{\beta}\frac{\partial \psi(p)}{\partial L} + \left(\frac{G_p G_l}{G} - G_{pl}\right)\left(1 - \frac{\partial \psi(p)}{\partial L}\right), \qquad (27)$$

where

$$\frac{\partial \psi}{\partial L} = \frac{\left(G_{Rl} - \frac{G_R G_{ll}}{G_l}\right)f'}{\left(\frac{G_R G_{lR}}{G_l} - G_{RR}\right)\frac{(f')^2}{\beta} + \left(G_{lR} - \frac{G_R G_{ll}}{G_l}\right)f' - G_R f''} > 0$$

from (21) and (22).[6] Using the calculations given in the Appendix, it can be verified that the sign of $\partial ED(p)/\partial L$ is equal to that of $(\alpha_2 - \alpha_1)f''$. Therefore, $\partial ED(p)/\partial L$ is positive under Assumption 1. Suppose that the home country has a smaller labor endowment than the foreign country: $L < L^*$. Then, it holds that $ED(p) < ED^*(p)$ in the neighborhood of
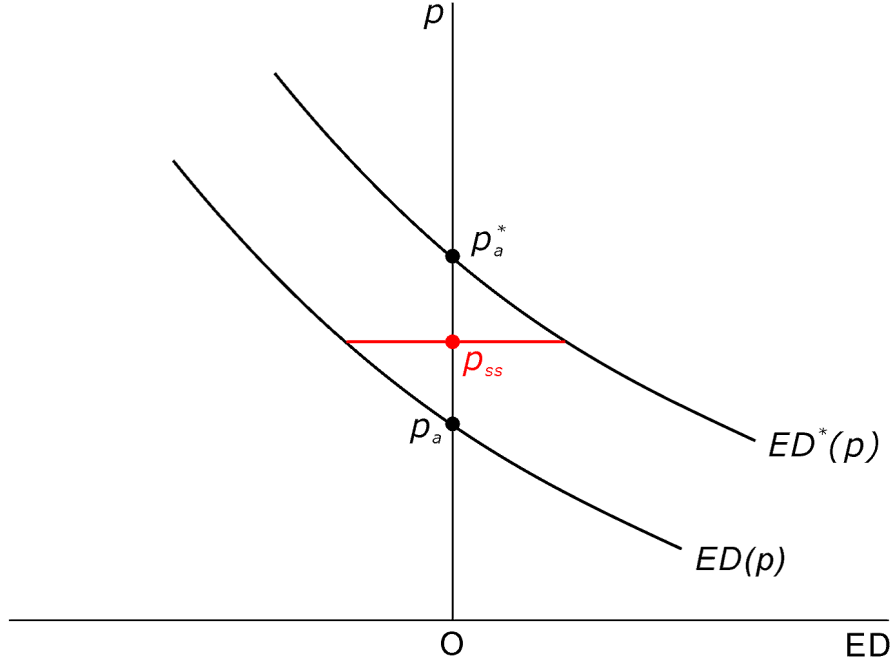
Figure 1: Steady-state equilibrium prices in the case where $ED(p) < ED^*(p)$

autarkic equilibrium prices, as illustrated in Figure 1, and thus $p_a < p_a^*$. Hence, the home country exports good 1 in the steady state.

McMillan (1978) and Yanase and Tawada (2012) consider a dynamic model of a small open economy with a stock of a public intermediate good in which the production technology of each private good exhibits a Ricardian property, i.e., constant returns to scale with respect to labor. They show that if the labor endowment is sufficiently large (small), a small open country specializes in a good whose productivity is more (less) sensitive to the public intermediate good. This implies that after commencing trade, a country with a higher labor endowment becomes an exporter of a good whose productivity is more sensitive to the public intermediate good. However, in the present model with constant-returns technology with respect to labor and the public good stock, the result is reversed.

Intuitively, the difference between the present model and McMillan's model can be interpreted as follows. In McMillan (1978) and Yanase and Tawada (2012), the private sectors' production function is given by $Y_i = g_i(R)L_i$, where $g_i(R)$ is increasing and concave in R, and thus each country's comparative advantage depends on the current stock of the public

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[6]See (A.22) in Appendix A.5 for the derivation.

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intermediate good. The labor endowment affects comparative advantage only indirectly, via the accumulation of the public good, and a country with a higher labor endowment will have a higher R. Hence, a larger country tends to specialize in a good that is more dependent on R. In the present model, by contrast, each country's comparative advantage depends on both R and L in a manner such that a higher R (L) increases Y_1 to a greater (lesser) extent than Y_2. Although a higher labor endowment implies a higher steady-state stock of R, the steady-state relative factor endowment is decreasing in L. This implies that a country with a higher L has comparative advantage in the labor-intensive good, which in turn is less dependent on R.

Depreciation rates The shift in excess demand in response to a change in the depreciation rate of the public good stock β is derived as

$$
\left.\frac{\partial ED(p)}{\partial \beta}\right|_{p=p_a} = \left(\frac{G_p G_R}{G} - G_{pR}\right)\left(\frac{f'}{\beta}\frac{\partial \psi(p)}{\partial \beta} - \frac{f}{\beta^2}\right) - \left(\frac{G_p G_l}{G} - G_{pl}\right)\frac{\partial \psi(p)}{\partial \beta}, \quad (28)
$$

where

$$
\frac{\partial \psi}{\partial \beta} = \frac{\left(\frac{G_R G_{lR}}{G_l} - G_{RR}\right)\frac{ff'}{\beta^2} - G_l}{\left(\frac{G_R G_{lR}}{G_l} - G_{RR}\right)\frac{(f')^2}{\beta} + \left(G_{lR} - \frac{G_R G_{ll}}{G_l}\right)f' - G_R f''}.
$$

Using the calculations given in the Appendix, it can be verified that $\partial ED(p)/\partial \beta > 0$ under Assumption 1. Suppose that the home country has a lower depreciation rate for its public good stock than the foreign country: $\beta < \beta^*$. In this case, it holds that $ED(p) < ED^*(p)$ in the neighborhood of autarkic equilibrium prices, and thus $p_a < p_a^*$. The home country exports good 1 in the steady state. The intuition behind this result is straightforward. Other things being equal, a smaller β implies a higher steady-state stock of public infrastructure, which leads to a higher relative supply of good 1 under Assumption 1, and thus a lower relative price of good 1 under autarky.

Discount rates The shift in excess demand in response to a change in the discount rate ρ is derived as

$$
\left.\frac{\partial ED(p)}{\partial \rho}\right|_{p=p_a} = \left(\frac{G_p G_R}{G} - G_{pR}\right)\frac{f'}{\beta}\frac{\partial \psi(p)}{\partial \rho} - \left(\frac{G_p G_l}{G} - G_{pl}\right)\frac{\partial \psi(p)}{\partial \rho}, \quad (29)
$$

where

$$
\frac{\partial \psi}{\partial \rho} = -\frac{G_l}{\left(\frac{G_R G_{lR}}{G_l} - G_{RR}\right)\frac{(f')^2}{\beta} + \left(G_{lR} - \frac{G_R G_{ll}}{G_l}\right)f' - G_R f''} < 0.
$$

14

From (A.15) and (A.16) in the Appendix, it holds that $G_p G_R / G - G_{pR} < 0$ and $G_p G_l / G - G_{pl} > 0$ under Assumption 1. Therefore, it follows that $\partial ED(p)/\partial \rho > 0$, i.e., an increase in ρ shifts $ED(p)$ upwards. Suppose that the home consumer is more patient than the foreign consumer: $\rho < \rho^*$. In this case, it holds that $ED(p) < ED^*(p)$ in the neighborhood of autarkic equilibrium prices, and thus $p_a < p_a^*$. The home country exports good 1 in the steady state.

The intuition behind this result is as follows. Other things being equal, a smaller ρ implies that people are more patient and put more weight on future consumption than on current consumption. Future consumption can be enhanced by the accumulation of public infrastructure, which augments outputs of final goods in the future.

To summarize, we establish the following theorem regarding each country's trade pattern.

Theorem 2 *Other things being equal, (i) the country with a smaller (larger) labor endowment, (ii) the country with a lower (higher) depreciation rate for its public good stock, and/or (iii) the patient (impatient) country exports (imports) the good that is more dependent on the public good stock in the steady state.*

4.2 Gains from trade

Let us denote the autarkic steady-state stock of public infrastructure in the home and foreign countries by R_a and R_a^*, respectively. Suppose that the home country exports good 1 in the free-trade, steady-state equilibrium, i.e., $p_a^* > p_{ss} > p_a$. Then, from Lemma 2, it holds that $R_{ss} > R_a$ and $R_{ss}^* < R_a^*$. That is, in comparison with the autarkic steady state, trade liberalization increases the steady-state stock of public infrastructure in the country exporting a good that is more dependent on the public good stock, while it reduces the stock of public infrastructure in the other country.

Theorem 3 *The country exporting a good that is more dependent on the stock of public infrastructure unambiguously gains from trade in the long run, in the sense that the country enjoys higher steady-state welfare under free trade than under autarky.*

Proof. Let us define the expenditure function as

$$E(p, u) = \min_{C_1, C_2} \{ pC_1 + C_2 \quad \text{s.t.} \quad \gamma \ln C_1 + (1 - \gamma) \ln C_2 \geq u \}.$$

It is easily verified that $E_u > 0$. Let us also denote the steady-state utility level under autarky and free trade by u_a and u_{ss}, respectively. In light of the budget constraint $pY_1 + Y_2 = pC_1 + C_2$, we have the following expression:

$$E(p_{ss}, u_{ss}) - E(p_{ss}, u_a)$$
$$= G(p_{ss}, R_{ss}, l_{ss}) - (p_{ss}Y_{1a} + Y_{2a}) + (p_{ss}C_{1a} + C_{2a}) - E(p_{ss}, u_a), \tag{30}$$

where l_{ss} is the free-trade, steady-state level of l, and Y_{ia} and C_{ia} are the autarkic steady-state levels of output and consumption, respectively, of good $i = 1, 2$. From the definition of the expenditure function, it holds that $p_{ss}C_{1a} + C_{2a} \geq E(p_{ss}, u_a)$. Using the GDP function, the second term on the right-hand side of (30) can be rewritten as $G(p_{ss}, R_a, l_a)$, where l_a is the autarkic steady-state level of l. Because the stead-state labor input in the public sector is rewritten as $L_R = f^{-1}(\beta R)$, the effect of an increase in R on the maximized GDP for a given price level can be derived as

$$G_R(p, R, L - f^{-1}(\beta R)) - \frac{\beta}{f'} G_l(p, R, L - f^{-1}(\beta R))$$
$$= G_R(p, R, L - f^{-1}(\beta R)) - \frac{\beta}{\rho + \beta} G_R(p, R, L - f^{-1}(\beta R))$$
$$= \frac{\rho}{\rho + \beta} G_R(p, R, L - f^{-1}(\beta R)) > 0, \tag{31}$$

where we use (22). Eq.(31) implies that $G(p_{ss}, R_{ss}, l_{ss}) > p_{ss}Y_{1a} + Y_{2a}$. To summarize, it follows that the sign of (30) is unambiguously positive, and thus $u_{ss} > u_a$. $\square$

For the country importing a good that is more dependent on the stock of public infrastructure, international trade reduces the steady-state stock of public infrastructure, and thus the steady-state national income evaluated at post-trade prices. If this reduction in national income outweighs the efficiency gains from specialization and exchange, the country will suffer steady-state losses from trade.

Note that the above discussion on gains/losses from trade is from a long-run viewpoint; we have focused on a comparison of steady-state welfare levels. Let us now discuss the welfare effects of trade along the transition path. Suppose that the home country exports good 1 and that both countries are initially under the autarkic steady state and open international trade. Then, in the short run, both countries enjoy welfare improvement because their consumption possibilities expand. However, along the transition path, the stock of public infrastructure

increases over time in the home country and decreases in the foreign country. This implies that the home country continues to improve welfare, but in the foreign country, instantaneous welfare decreases over time and may be lower than autarkic welfare after a certain point in time.

4.3 Possibility of loss from trade: an example

As indicated in the previous subsection, a country importing a good that is less dependent on the stock of public infrastructure may suffer lower steady-state welfare than under autarky. In this subsection, we demonstrate this possibility by considering a simplified version of the model.

Let us assume that $\alpha_1 = \alpha > 0 = \alpha_2$, $L = L^* = 1$, and $\beta^* = \lambda\beta$ and $\rho^* = \lambda\rho$, where $\lambda > 1$. That is, we assume that only good 1 is dependent on the stock of public infrastructure and the two countries are identical except for their rates of depreciation and time preferences. In addition, let us specify the production function in the public sector as $f(L_R) = L_R - L_R^2/2$.

From Theorem 2 (ii) and (iii), the home (foreign) country becomes an exporter of good 1 (good 2).[7] This assertion is verified in this example. Let us define $q \equiv (1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha p^{\frac{1}{\alpha}}$. Then, as shown in Appendix A.6, we obtain[8]

$$q_a = \sqrt{\frac{(\rho+\beta)\{(1+\alpha\gamma)\beta + (1-\alpha\gamma)\rho\}}{1-\alpha\gamma}}, \quad q_a^* = \lambda q_a, \quad q_{ss} = \sqrt{\lambda}q_a, \tag{32}$$

and thus $q_a < q_{ss} < q_a^*$.

Remark Substituting $q = q_{ss}$ into each country's steady-state equilibrium level of labor input in the public sector, we can conclude that both $L_R > 0$ and $L_R^* > 0$ hold only if

$$\frac{(1+\alpha\gamma)\beta + (1-\alpha\gamma)\rho}{(1-\alpha\gamma)(\rho+\beta)} > \lambda > 1. \tag{33}$$

In the following analysis, we assume that this condition is satisfied.

---

[7]Alternatively, we can consider a situation in which the labor endowments differ between the countries (e.g., $L = 1$ and $L^* = 1 + \epsilon$, where $\epsilon > 0$. Theorem 2 (i) and 3 can be verified under this alternative specification, although the calculation becomes more complicated.

[8]Because both α and γ are between 0 and 1, $1 - \alpha\gamma > 0$.

Denoting the steady-state welfare level by $u \equiv \gamma \ln C_1 + (1 - \gamma) \ln C_2$, we obtain the welfare comparison result in the home country:

$$u_{ss} - u_a = \ln \left\{ 1 + \frac{(\lambda - 1)[(1 + \alpha\gamma)\beta + (1 - \alpha\gamma)\rho]}{2\beta} \right\} - \frac{1 + \alpha\gamma}{2} \ln \lambda. \tag{34}$$

Theorem 3 purports that the home country obtains unambiguously higher welfare under free trade than under autarky. This assertion is also verified because $u_{ss} > u_a$ holds.[9] However, for the foreign country, the following expression holds:

$$u_{ss}^* - u_a^* = \ln \left\{ 1 + \frac{(\lambda - 1)(1 - \alpha\gamma)(\beta - \rho)}{2\beta} \right\} - \frac{1 - \alpha\gamma}{2} \ln \lambda, \tag{35}$$

the sign of which is ambiguous. In particular, if ρ becomes higher (i.e., people are more impatient), the sign of (35) is more likely to be negative.

Yanase and Tawada (2012) show that in McMillan's (1978) model with a pure public intermediate good, a country unambiguously gains from trade in the long run only if it has a comparative advantage in a good with productivity that is more sensitive to the public intermediate good; if the country has a comparative advantage in a good with productivity that is less sensitive to the public intermediate good, the economy may lose from trade in the long run. In the present model, we obtain a similar result. However, in their model, the country that gains (may lose) from trade is the larger (smaller) country, measured in terms of labor endowments. By contrast, in the present model, as implied by Theorem 2, the country that gains (may lose) from trade is the smaller (larger) country. In this sense, we can conclude that the results in terms of gains/losses from trade obtained in Yanase and Tawada (2012) are not robust, and are dependent on the property of the public intermediate good.

5   Trading Equilibrium under Dynamic Nash Provision of Public Goods

So far, we have assumed that the national government in each country is a price taker in the world commodity markets and public investment is undertaken by the government without strategic motives to manipulate the country's terms of trade. In this section, we consider a situation in which the governments act strategically in providing the public good, i.e., the

---

[9]The right-hand side of (34) is continuous in λ, becomes zero if $\lambda = 1$, and is strictly increasing in λ.

governments noncooperatively make an investment in public infrastructure, recognizing that it can affect international prices.[10]

As discussed in Section 2, the competitive equilibrium outputs of private goods are derived from (1), (3), (4), (5), and (6), and the equilibrium consumption levels are $C_1 = \gamma(pY_1 + Y_2)/p$ and $C_2 = (1-\gamma)(pY_1 + Y_2)$. Thus, the market-clearing condition for good 1 in the international market under free trade is given by[11]

$$
\begin{aligned}
&Y_1(\tau_1, \tau_2, R, p) + Y_1(\tau_1^*, \tau_2^*, R^*, p) \\
&= \frac{\gamma \{p\left[Y_1(\tau_1, \tau_2, R, p) + Y_1(\tau_1^*, \tau_2^*, R^*, p)\right] + Y_2(\tau_1, \tau_2, R, p) + Y_2(\tau_1^*, \tau_2^*, R^*, p)\}}{p}.
\end{aligned}
\tag{36}
$$

From (36), the equilibrium world price of good 1 can be derived as a function of tax rates and the stock of public infrastructure in both countries, i.e., $p = P(\tau_1, \tau_1^*, \tau_2, \tau_2^*, R, R^*)$, with the following properties:

$$
\frac{\partial P}{\partial x} = \frac{1}{\Omega} \left\{ \gamma \frac{\partial Y_2}{\partial x} - (1-\gamma)p \frac{\partial Y_1}{\partial x} \right\}, \quad x = \tau_1, \tau_2, R,
\tag{37a}
$$

$$
\frac{\partial P}{\partial x^*} = \frac{1}{\Omega} \left\{ \gamma \frac{\partial Y_2^*}{\partial x^*} - (1-\gamma)p \frac{\partial Y_1^*}{\partial x^*} \right\}, \quad x^* = \tau_1^*, \tau_2^*, R^*,
\tag{37b}
$$

where

$$
\Omega \equiv (1-\gamma)(Y_1 + Y_1^*) + (1-\gamma)p \left(\frac{\partial Y_1}{\partial p} + \frac{\partial Y_1^*}{\partial p} \right) - \gamma \left(\frac{\partial Y_2}{\partial p} + \frac{\partial Y_2^*}{\partial p} \right) > 0
$$

and the derivatives of outputs Y_1 and Y_2 with respect to τ_i, R, and p are given by (A.4).

The governments no longer take the world price p as given; rather, they take into account the effects of the change in the tax rate at each moment in time and the stock of public infrastructure on the world price. The home government's problem is to maximize welfare (8) subject to the dynamics of domestic public infrastructure (2) and $p = P(\tau_1, \tau_1^*, \tau_2, \tau_2^*, R, R^*)$. Each government recognizes that the rival country's choice of tax rates and stock of public infrastructure also affect the world price. In this sense, there exists dynamic and strategic interaction, and thus the trading equilibrium is characterized as a differential game between national governments that determine the appropriate levels of their domestic public infrastructure.

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[10]In a static model, Shimomura (2007) proves that if governments determine the level of public goods noncooperatively, free trade is beneficial to all countries.

[11]For simplicity of exposition, we omit the labor endowments L and L^* from the arguments of $Y_1(\cdot)$ and $Y_2(\cdot)$.

Two equilibrium concepts are frequently employed in dynamic game analysis in economics; the open-loop Nash equilibrium, in which each player's equilibrium strategy is a simple function independent of the current state of the system, and the Markov perfect Nash equilibrium, in which each player designs their optimal strategy as a feedback decision rule dependent only on the state variable. Both concepts satisfy time consistency, but only the Markov perfect Nash equilibrium satisfies subgame perfectness (Long, 2010). Because of its tractability, we focus on the open-loop Nash equilibrium. This strategy concept requires that governments commit themselves to particular strategy paths at the beginning of the game, and we simply assume that the commitment is credible. Formally, the open-loop Nash equilibrium for this dynamic game is defined as a pair of time paths in relation to the home government's tax rates $\{\tau_1(t), \tau_2(t)\}_{t=0}^{\infty}$ and the foreign government's tax rates $\{\tau_1^*(t), \tau_2^*(t)\}_{t=0}^{\infty}$ such that $\{\tau_1(t), \tau_2(t)\}_{t=0}^{\infty}$ maximizes the home country's national welfare subject to the dynamics of R, taking $\{\tau_1^*(t), \tau_2^*(t)\}_{t=0}^{\infty}$ as given, and $\{\tau_1^*(t), \tau_2^*(t)\}_{t=0}^{\infty}$ maximizes the foreign country's national welfare subject to the dynamics of R^*, taking $\{\tau_1(t), \tau_2(t)\}_{t=0}^{\infty}$ as given.

5.1 The Nash equilibrium conditions

The current-value Hamiltonian for the home government's problem is now defined as

$$H = \ln\left[P(\boldsymbol{\tau}, \boldsymbol{R})Y_1(\tau_1, \tau_2, R, P(\boldsymbol{\tau}, \boldsymbol{R})) + Y_2(\tau_1, \tau_2, R, P(\boldsymbol{\tau}, \boldsymbol{R}))\right]$$
$$- \gamma \ln\left[P(\boldsymbol{\tau}, \boldsymbol{R})\right] + \Gamma + \theta\left\{f(L_R(\tau_1, \tau_2, R, P(\boldsymbol{\tau}, \boldsymbol{R}))) - \beta R\right\},$$

where $\boldsymbol{\tau} \equiv (\tau_1, \tau_1^*, \tau_2, \tau_2^*)$ and $\boldsymbol{R} \equiv (R, R^*)$. The optimality conditions are presented by

$$\frac{\partial H}{\partial \tau_1} = \frac{1}{pY_1 + Y_2}\left[\left(Y_1 + p\frac{\partial Y_1}{\partial p} + \frac{\partial Y_2}{\partial p}\right)\frac{\partial P}{\partial \tau_1} + p\frac{\partial Y_1}{\partial \tau_1} + \frac{\partial Y_2}{\partial \tau_1}\right] - \frac{\gamma}{p}\frac{\partial P}{\partial \tau_1}$$
$$+ \theta f'(L_R)\left(\frac{\partial L_R}{\partial \tau_1} + \frac{\partial L_R}{\partial p}\frac{\partial P}{\partial \tau_1}\right) = 0, \tag{38}$$

$$\frac{\partial H}{\partial \tau_2} = \frac{1}{pY_1 + Y_2}\left[\left(Y_1 + p\frac{\partial Y_1}{\partial p} + \frac{\partial Y_2}{\partial p}\right)\frac{\partial P}{\partial \tau_2} + p\frac{\partial Y_1}{\partial \tau_2} + \frac{\partial Y_2}{\partial \tau_2}\right] - \frac{\gamma}{p}\frac{\partial P}{\partial \tau_2}$$
$$+ \theta f'(L_R)\left(\frac{\partial L_R}{\partial \tau_2} + \frac{\partial L_R}{\partial p}\frac{\partial P}{\partial \tau_2}\right) = 0, \tag{39}$$

$$\dot{\theta} = \rho\theta - \frac{\partial H}{\partial R} = \theta\left\{\rho + \beta - f'(L_R)\left(\frac{\partial L_R}{\partial R} + \frac{\partial L_R}{\partial p}\frac{\partial P}{\partial R}\right)\right\} + \frac{\gamma}{p}\frac{\partial P}{\partial R}$$
$$- \frac{1}{pY_1 + Y_2}\left\{\left(Y_1 + p\frac{\partial Y_1}{\partial p} + \frac{\partial Y_2}{\partial p}\right)\frac{\partial P}{\partial R} + p\frac{\partial Y_1}{\partial R} + \frac{\partial Y_2}{\partial R}\right\}, \tag{40}$$

and the transversality condition. The optimality conditions for the foreign government's problem can be analogously derived.

Since $C_1 = \gamma(pY_1 + Y_2)/p$, (38) and (39) can be rewritten as

$$\left\{ Y_1 - C_1 + p\frac{\partial Y_1}{\partial p} + \frac{\partial Y_2}{\partial p} + (pY_1 + Y_2)\theta f'(L_R)\frac{\partial L_R}{\partial p} \right\} \frac{\partial P}{\partial \tau_1}$$
$$+ p\frac{\partial Y_1}{\partial \tau_1} + \frac{\partial Y_2}{\partial \tau_1} + (pY_1 + Y_2)\theta f'(L_R)\frac{\partial L_R}{\partial \tau_1} = 0, \tag{41}$$

$$\left\{ Y_1 - C_1 + p\frac{\partial Y_1}{\partial p} + \frac{\partial Y_2}{\partial p} + (pY_1 + Y_2)\theta f'(L_R)\frac{\partial L_R}{\partial p} \right\} \frac{\partial P}{\partial \tau_2}$$
$$+ p\frac{\partial Y_1}{\partial \tau_2} + \frac{\partial Y_2}{\partial \tau_2} + (pY_1 + Y_2)\theta f'(L_R)\frac{\partial L_R}{\partial \tau_2} = 0. \tag{42}$$

From these equations, it holds that

$$\frac{p\frac{\partial Y_1}{\partial \tau_1} + \frac{\partial Y_2}{\partial \tau_1} + (pY_1 + Y_2)\theta f'(L_R)\frac{\partial L_R}{\partial \tau_1}}{\frac{\partial P}{\partial \tau_1}} = \frac{p\frac{\partial Y_1}{\partial \tau_2} + \frac{\partial Y_2}{\partial \tau_2} + (pY_1 + Y_2)\theta f'(L_R)\frac{\partial L_R}{\partial \tau_2}}{\frac{\partial P}{\partial \tau_2}},$$

which in light of (A.4) and (37) can be rewritten as

$$\gamma\tau_1 + (1 - \gamma)\tau_2 = 1 - \frac{w}{\theta f'(L_R)(pY_1 + Y_2)}. \tag{43}$$

Substituting (43) into the first-order condition (41) or (42) again and rearranging terms,[12] we obtain

$$\frac{\tau_1 - \tau_2}{1 - \gamma\tau_1 - (1 - \gamma)\tau_2} \left\{ (1 - \gamma)(Y_1 + Y_1^*) + (1 - \gamma)p\frac{\partial Y_1^*}{\partial p} - \gamma\frac{\partial Y_2^*}{\partial p} \right\} = Y_1 - C_1. \tag{44}$$

Eqs. (43) and (44) jointly determine the optimal tax rates for the home government, τ_1 and τ_2, as a function of the tax rates in the foreign country τ_1^* and τ_2^*,[13] as well as the state and co-state variables in the home country, R and θ.

The foreign government's optimality conditions can be analogously derived. That is, the optimal tax rates for the foreign country, τ_1^* and τ_2^*, satisfy the following conditions:

$$\gamma\tau_1^* + (1 - \gamma)\tau_2^* = 1 - \frac{w^*}{\theta^* f'(L_R^*)(pY_1^* + Y_2^*)}, \tag{45}$$

$$\frac{\tau_1^* - \tau_2^*}{1 - \gamma\tau_1^* - (1 - \gamma)\tau_2^*} \left\{ (1 - \gamma)(Y_1 + Y_1^*) + (1 - \gamma)p\frac{\partial Y_1}{\partial p} - \gamma\frac{\partial Y_2}{\partial p} \right\} = Y_1^* - C_1^*. \tag{46}$$

From (44) and (46), we immediately obtain the following proposition:

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[12]Either operation yields the same outcome.
[13]Note that $\partial Y_1^*/\partial p$ and $\partial Y_2^*/\partial p$ depend on τ_1^* and τ_2^*.

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Proposition 1 *Suppose that the national governments determine their production taxes strategically, in the sense that they take into account the effects on the international price. If the home country exports good 1, which is more dependent on the stock of public infrastructure, and the foreign country imports that good, the home government chooses $\tau_1 > \tau_2$, whereas the foreign government chooses $\tau_1^* < \tau_2^*$.*

Proposition 1 highlights the national government's incentive to improve its terms of trade in determining the tax rates for funding public investment. Higher production taxes add more costs to firms, and thus dampen their outputs. The reduction in outputs leads to excess demand for the good in the world market, which increases the relative price of that good. If the home country exports good 1, its terms of trade improve when the relative price of good 1, p, increases, making the home country better off. Therefore, the home government has an incentive to impose a higher tax rate on sector 1 than on sector 2, thereby reducing the relative output of good 1.

In light of (37), (41), (42), and the comparative statics results (A.4), the home government's adjoint equation (40) can be rewritten as

$$\dot{\theta} = (\rho + \beta)\theta - \frac{(1-\tau_1)\alpha_1 pY_1 + (1-\tau_2)\alpha_2 Y_2}{[1 - \gamma\tau_1 - (1-\gamma)\tau_2](pY_1 + Y_2)R}. \tag{47}$$

Eq.(47) can be interpreted in a similar manner to that in the case of nonstrategic governments; it indicates that it is optimal for the home government to choose the resource allocation in the home country that balances the sum of the capital gain/loss and the marginal benefit of an increase in public infrastructure with the marginal cost of providing the infrastructure. However, the marginal social benefit of public infrastructure is distorted by the strategic use of production taxes. Comparing the second term on the right-hand side of (47) with that of (15) and making use of the fact that $\gamma = p(Y_1 + Y_1^*)/[p(Y_1 + Y_1^*) + Y_2 + Y_2^*]$ from the world market-clearing condition (36), we obtain

$$\frac{(1-\tau_1)\alpha_1 pY_1 + (1-\tau_2)\alpha_2 Y_2}{[1 - \gamma\tau_1 - (1-\gamma)\tau_2](pY_1 + Y_2)R} - \frac{\alpha_1 pY_1 + \alpha_2 Y_2}{(pY_1 + Y_2)R}$$
$$= -\frac{(\tau_1 - \tau_2)p[\alpha_1 Y_1(Y_2 + Y_2^*) - \alpha_2 Y_2(Y_1 + Y_1^*)]}{R(pY_1 + Y_2)[(1-\tau_1)p(Y_1 + Y_1^*) + (1-\tau_2)(Y_2 + Y_2^*)]}. \tag{48}$$

If the home country exports good 1, $Y_1(Y_2 + Y_2^*) > Y_2(Y_1 + Y_1^*)$ must hold.[14] Given this fact and Assumption 1, it follows that $\alpha_1 Y_1(Y_2 + Y_2^*) > \alpha_2 Y_2(Y_1 + Y_1^*)$. In addition, from

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[14] Since $C_1 = \gamma(pY_1 + Y_2)/p$, it follows that $Y_1 - C_1 = (1-\gamma)Y_1 - \gamma Y_2/p = [Y_1(Y_2 + Y_2^*) - Y_2(Y_1 + Y_1^*)]/[p(Y_1 + Y_1^*) + Y_2 + Y_2^*]$. Therefore, the sign of $Y_1 - C_1$ is the same as that of $Y_1(Y_2 + Y_2^*) - Y_2(Y_1 + Y_1^*)$.

Proposition 1, $\tau_1 > \tau_2$ holds in the case where the home country exports good 1. Therefore, the sign of (48) is negative if the home country exports good 1. In the foreign country, which imports good 1 and in which the government sets $\tau_1^* < \tau_2$, the opposite result occurs. Thus, the following proposition can be established.

Proposition 2 *Suppose that the national governments determine their production taxes strategically. Then, in comparison with the case where the governments act nonstrategically, the marginal social benefit of public infrastructure becomes smaller (larger) in a country that exports (imports) good 1, which is more dependent on the stock of public infrastructure.*

5.2 Steady state

The steady state of the open-loop Nash equilibrium must satisfy (21), (23), and the following two equations:

$$\rho + \beta = \frac{(1 - \tau_1)\alpha_1 pY_1 + (1 - \tau_2)\alpha_2 Y_2}{w} \cdot \frac{f'(L_R)}{R}, \tag{49}$$

$$\rho^* + \beta^* = \frac{(1 - \tau_1^*)\alpha_1 pY_1^* + (1 - \tau_2^*)\alpha_2 Y_2^*}{w^*} \cdot \frac{f'(L_R^*)}{R^*}, \tag{50}$$

where we made use of (43) and (47) in deriving (49) and the foreign counterpart (50) can be analogously derived.

The left-hand sides of (49) and (50) are the sum of the discount and depreciation rates in the home country and foreign country, respectively, and can be interpreted as the long-run marginal cost of providing the stock of public infrastructure in the respective countries. The right-hand sides of these equations denote the shadow value of the public good stock measured in terms of labor, and can be interpreted as the long-run marginal social benefit of public infrastructure in these countries. Other things being equal, the existence of the terms-of-trade effect changes the marginal benefit. As demonstrated in Proposition 2, if the home country exports good 1, which is more intensive to R, the marginal benefit will be smaller compared with the nonstrategic case. Because the marginal benefit is decreasing in L_R (see the proof of Lemma 2), this implies that in comparison with the nonstrategic case, the home government will have an incentive to underaccumulate the public good stock. It is also verified in a similar manner that the foreign country will tend to overaccumulate the public good stock compared with the nonstrategic case if it imports good 1. This under- and overaccumulation is the result of the terms-of-trade effect; because an increase in the stock of

public infrastructure augments the relative output of good 1, its relative price will fall. The exporting country will suffer a deterioration in its terms of trade, and thus has an incentive to reduce its public good provision to reduce the welfare loss.

6   Concluding Remarks

In this paper, we have developed a dynamic two-country model of international trade with productive public infrastructure. In the case where the government in each country acts as a price taker in the world commodity market, we revealed that if the economy is initially at the autarky steady state, a country with a smaller (larger) labor endowment, a lower (higher) depreciation rate of its stock of infrastructure capital, and/or a lower (higher) rate of time preference exports (imports) a good that is more dependent on the stock of public infrastructure. We also showed that the country exporting the good that is more dependent on the stock of public infrastructure unambiguously gains from trade in the long run, but the country importing that good may lose from trade in the long run.

We also considered an alternative situation in which the governments strategically determine their provision of public infrastructure, taking into consideration the effect of government policy on the world commodity market. Because of the terms-of-trade effect, in comparison with the nonstrategic case, the country exporting (importing) the good that is more dependent on public infrastructure would underaccumulate (overaccumulate) public infrastructure.

We should note that, with a few exceptions such as Bougheas et al. (2003) and Figuiéres et al. (2013) that consider spillover effects of infrastructure across countries, most of the existing studies on infrastructure and trade have focused on national public infrastructure, and our analysis is also confined to national public infrastructure. However, international public goods are receiving increasing attention. For example, railway and highway construction is carried out across national boundaries, and various communication systems are also available across boundaries. Thus, it is of increasing importance to accommodate infrastructure that has the characteristics of international public goods into the present model and to reexamine the trade theorems. There is much work to be done once our attention is focused on this area.

Appendix

A.1 Comparative statics of the competitive equilibrium solutions

From (5) and (6), it holds that $L_1 = (1-\tau_1)(1-\alpha_1)pY_1/w$ and $L_2 = (1-\tau_2)(1-\alpha_2)Y_2/w$. Substituting these equations into (1) and (3), we obtain

$$Y_1 = R^{\alpha_1} \left[\frac{(1-\tau_1)(1-\alpha_1)pY_1}{w} \right]^{1-\alpha_1}, \tag{A.1}$$

$$Y_2 = R^{\alpha_2} \left[\frac{(1-\tau_2)(1-\alpha_2)Y_2}{w} \right]^{1-\alpha_2}, \tag{A.2}$$

$$\frac{(1-\tau_1)(1-\alpha_1)pY_1 + (1-\tau_2)(1-\alpha_2)Y_2}{w} + L_R = L. \tag{A.3}$$

Totally differentiating (4), (A.1), (A.2), and (A.3), and solving for dY_1, dY_2, dL_R, and dw, we can derive the comparative statics results as follows:

$$\frac{\partial Y_1}{\partial \tau_1} = -\frac{(1-\alpha_1)Y_1\{\alpha_2 pY_1 + [1-\alpha_2(1-\tau_2)]Y_2\}}{\Delta(1-\tau_1)}, \tag{A.4a}$$

$$\frac{\partial Y_1}{\partial \tau_2} = \frac{(1-\alpha_1)Y_1 Y_2[1-\alpha_2-\alpha_2(1-\tau_2)]}{\Delta(1-\tau_2)}, \tag{A.4b}$$

$$\frac{\partial Y_1}{\partial R} = \frac{Y_1}{\Delta R}\{\alpha_1\alpha_2[1-\alpha_1(1-\tau_1)]pY_1 + [1-\alpha_2(1-\tau_2)][\alpha_1-(1-\alpha_1)\alpha_2]Y_2\}, \tag{A.4c}$$

$$\frac{\partial Y_1}{\partial p} = \frac{(1-\alpha_1)Y_1 Y_2[1-\alpha_2(1-\tau_2)]}{\Delta p}, \tag{A.4d}$$

$$\frac{\partial Y_2}{\partial \tau_1} = \frac{(1-\alpha_2)pY_1 Y_2[1-\alpha_1-\alpha_1(1-\tau_1)]}{\Delta(1-\tau_1)}, \tag{A.4e}$$

$$\frac{\partial Y_2}{\partial \tau_2} = -\frac{(1-\alpha_2)Y_2\{[1-\alpha_1(1-\tau_1)]pY_1 + \alpha_1 Y_2\}}{\Delta(1-\tau_2)}, \tag{A.4f}$$

$$\frac{\partial Y_2}{\partial R} = \frac{Y_2}{\Delta R}\{[1-\alpha_1(1-\tau_1)][\alpha_2-\alpha_1(1-\alpha_2)]pY_1 + \alpha_1\alpha_2[1-\alpha_2(1-\tau_2)]Y_2\}, \tag{A.4g}$$

$$\frac{\partial Y_2}{\partial p} = -\frac{(1-\alpha_2)Y_1 Y_2[1-\alpha_1(1-\tau_1)]}{\Delta}, \tag{A.4h}$$

$$\frac{\partial L_R}{\partial \tau_1} = \frac{(1-\alpha_1)pY_1}{\Delta w}\left\{\alpha_2 pY_1 + \left[\tau_2 + \frac{(1-\alpha_2)(1-\tau_2)}{(1-\alpha_1)(1-\tau_1)}(\alpha_1-\tau_1)\right]Y_2\right\}, \tag{A.4i}$$

$$\frac{\partial L_R}{\partial \tau_2} = \frac{(1-\alpha_2)Y_2}{\Delta w}\left\{\left[\tau_1 + \frac{(1-\alpha_1)(1-\tau_1)}{(1-\alpha_2)(1-\tau_2)}(\alpha_2-\tau_2)\right]pY_1 + \alpha_1 Y_2\right\}, \tag{A.4j}$$

$$\frac{\partial L_R}{\partial R} = \frac{(\alpha_1-\alpha_2)pY_1 Y_2}{\Delta w R}[(1-\alpha_1\tau_2-\alpha_2)\tau_1 - (1-\alpha_1-\alpha_2\tau_1)\tau_2], \tag{A.4k}$$

$$\frac{\partial L_R}{\partial p} = \frac{Y_1 Y_2\{\tau_1[1-\alpha_2(1-\tau_2)] - \tau_2[1-\alpha_1(1-\tau_1)]\}}{\Delta w}, \tag{A.4l}$$

$$\frac{\partial L_R}{\partial \tau_1} = -\frac{wpY_1\alpha_2[1-\alpha_1-\alpha_1(1-\tau_1)]}{\Delta(1-\tau_1)}, \tag{A.4m}$$

$$\frac{\partial w}{\partial \tau_2} = -\frac{wY_2\alpha_1[1-\alpha_2-\alpha_2(1-\tau_2)]}{\Delta(1-\tau_2)}, \tag{A.4n}$$

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$$\frac{\partial w}{\partial R} = \frac{w\alpha_1\alpha_2\{[1-\alpha_1(1-\tau_1)]pY_1 + [1-\alpha_2(1-\tau_2)]Y_2\}}{\Delta R}, \tag{A.4o}$$

$$\frac{\partial w}{\partial p} = \frac{wY_1\alpha_2[1-\alpha_1(1-\tau_1)]}{\Delta}, \tag{A.4p}$$

where $\Delta \equiv \alpha_2[1-\alpha_1(1-\tau_1)]pY_1 + \alpha_1[1-\alpha_2(1-\tau_2)]Y_2 > 0$.

A.2 Properties of the GDP function

Let us define the Lagrangian as follows: $\mathcal{L} = pR^{\alpha_1}L_1^{1-\alpha_1} + R^{\alpha_2}L_2^{1-\alpha_2} + \omega(l-L_1-L_2)$. The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial L_1} = (1-\alpha_1)pR^{\alpha_1}L_1^{-\alpha_1} - \omega = (1-\alpha_1)p\frac{Y_1}{L_1} - \omega = 0, \tag{A.5}$$

$$\frac{\partial \mathcal{L}}{\partial L_2} = (1-\alpha_2)R^{\alpha_2}L_2^{-\alpha_2} - \omega = (1-\alpha_2)\frac{Y_2}{L_2} - \omega = 0, \tag{A.6}$$

$$\frac{\partial \mathcal{L}}{\partial \omega} = l - L_1 - L_2 = 0. \tag{A.7}$$

From (A.5) and (A.6), we obtain $L_1 = (1-\alpha_1)pY_1/\omega$ and $L_2 = (1-\alpha_2)Y_2/\omega$, respectively. Substituting these into (A.7) and solving for ω, it follows that $\omega = [(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2]/l$. Therefore, (1) can be rewritten as

$$Y_1 = R^{\alpha_1}\left[\frac{(1-\alpha_1)pY_1 l}{(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2}\right]^{1-\alpha_1}, \tag{A.8}$$

$$Y_2 = R^{\alpha_2}\left[\frac{(1-\alpha_2)Y_2 l}{(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2}\right]^{1-\alpha_2}. \tag{A.9}$$

Totally differentiating (A.8) and (A.9), and solving for dY_1 and dY_2, we obtain

$$dY_1 = \frac{(1-\alpha_1)(1-\alpha_2)Y_1Y_2}{\Delta' p}dp + \frac{\{(1-\alpha_1)\alpha_1\alpha_2 pY_1 + (1-\alpha_2)[\alpha_1 - (1-\alpha_1)\alpha_2]Y_2\}Y_1}{\Delta' R}dR$$
$$+ \frac{(1-\alpha_1)\alpha_2 Y_1[(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2]}{\Delta' l}dl, \tag{A.10}$$

$$dY_2 = -\frac{(1-\alpha_1)(1-\alpha_2)Y_1Y_2}{\Delta'}dp + \frac{\{(1-\alpha_1)[\alpha_2 - \alpha_1(1-\alpha_2)]pY_1 + (1-\alpha_2)\alpha_1\alpha_2 Y_2\}Y_2}{\Delta' R}dR$$
$$+ \frac{\alpha_1(1-\alpha_2)Y_2[(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2]}{\Delta' l}dl, \tag{A.11}$$

where $\Delta' \equiv (1-\alpha_1)\alpha_2 pY_1 + (1-\alpha_2)\alpha_1 Y_2$.

Since $G = pY_1 + Y_2$, (A.10) and (A.11) yield the partial derivatives of the GDP function as follows:

$$G_p = Y_1 + p\frac{\partial Y_1}{\partial p} + \frac{\partial Y_2}{\partial p} = Y_1, \tag{A.12a}$$

$$G_R = p\frac{\partial Y_1}{\partial R} + \frac{\partial Y_2}{\partial R} = \frac{\alpha_1 pY_1 + \alpha_2 Y_2}{R}, \tag{A.12b}$$

$$G_l = p\frac{\partial Y_1}{\partial l} + \frac{\partial Y_2}{\partial l} = \frac{(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2}{l} = \omega. \tag{A.12c}$$

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Thus, we have the properties of the GDP function (16). In addition, since $G_{pp} = \partial Y_1/\partial p$, $G_{pR} = \partial Y_1/\partial R$, and $G_{pl} = \partial Y_1/\partial l$, (A.10) derives

$$G_{pp} = \frac{(1-\alpha_1)(1-\alpha_2)Y_1Y_2}{\Delta' p} > 0, \tag{A.13a}$$

$$G_{pR} = \frac{\{(1-\alpha_1)\alpha_1\alpha_2 pY_1 + (1-\alpha_2)[\alpha_1 - (1-\alpha_1)\alpha_2]Y_2\}Y_1}{\Delta' R}, \tag{A.13b}$$

$$G_{pl} = \frac{(1-\alpha_1)\alpha_2 Y_1[(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2]}{\Delta' l} > 0. \tag{A.13c}$$

Moreover, from (A.12b) and (A.12c), it follows that

$$G_{RR} = -\frac{\alpha_1\alpha_2[(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2]^2}{\Delta' R^2} < 0, \tag{A.14a}$$

$$G_{ll} = -\frac{\alpha_1\alpha_2[(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2]^2}{\Delta' l^2} < 0, \tag{A.14b}$$

$$G_{Rl} = \frac{\alpha_1\alpha_2[(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2]^2}{\Delta' Rl} > 0. \tag{A.14c}$$

Finally, we present some results that are useful in the subsequent analysis:

$$G_{pR} - \frac{G_p G_R}{G} = \frac{(\alpha_1 - \alpha_2)[(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2]Y_1Y_2}{\Delta'(pY_1 + Y_2)R}, \tag{A.15}$$

$$G_{pl} - \frac{G_p G_l}{G} = -\frac{(\alpha_1 - \alpha_2)[(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2]Y_1Y_2}{\Delta'(pY_1 + Y_2)l}, \tag{A.16}$$

$$G_{Rl} - \frac{G_R G_l}{G} = -\frac{(\alpha_1 - \alpha_2)^2[(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2]pY_1Y_2}{\Delta'(pY_1 + Y_2)Rl} < 0, \tag{A.17}$$

$$G_{pR} - \frac{G_R G_{lp}}{G_l} = \frac{(\alpha_1 - \alpha_2)Y_1Y_2}{\Delta' R}, \tag{A.18}$$

$$G_{RR} - \frac{G_R G_{Rl}}{G_l} = -\frac{\alpha_1\alpha_2[(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2](pY_1 + Y_2)}{\Delta' R^2} < 0, \tag{A.19}$$

$$G_{Rl} - \frac{G_R G_{ll}}{G_l} = \frac{\alpha_1\alpha_2[(1-\alpha_1)pY_1 + (1-\alpha_2)Y_2](pY_1 + Y_2)}{\Delta' Rl} > 0. \tag{A.20}$$

Under Assumption 1, the signs of (A.15) and (A.18) are positive, whereas the sign of (A.16) is negative.

A.3 Proof of Lemma 2

(i) Substituting (21) into (22) yields

$$\rho + \beta = \frac{G_R(p, f(L_R)/\beta, L - L_R)}{G_l(p, f(L_R)/\beta, L - L_R)} f'(L_R). \tag{A.21}$$

Totally differentiating (A.21) and rearranging, we have

$$\left\{ \left(\frac{G_R G_{lR}}{G_l} - G_{RR} \right) \frac{(f')^2}{\beta} + \left(G_{Rl} - \frac{G_R G_{ll}}{G_l} \right) f' - G_R f'' \right\} dL_R$$

$$= \left(G_{Rp} - \frac{G_R G_{lp}}{G_l} \right) f' dp + \left(G_{Rl} - \frac{G_R G_{ll}}{G_l} \right) f' dL$$

$$+ \left\{ \left(\frac{G_R G_{lR}}{G_l} - G_{RR} \right) \frac{ff'}{\beta^2} - G_l \right\} d\beta - G_l d\rho. \tag{A.22}$$

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From (A.19), (A.20), and $f'' \leq 0$, the coefficient of dL_R in (A.22) is positive. In addition, from (A.18), the coefficient of dp is positive if $\alpha_1 > \alpha_2$. Therefore, $dL_R/dp > 0$ holds under Assumption 1.

(ii) In light of (16) and (A.12c), (22) can be rewritten as

$$\rho + \beta = \frac{G_R}{G_l} f'(L_R) = \frac{(\alpha_1 pY_1 + \alpha_2 Y_2)\beta f'(L_R)(L - L_R)}{\{(1 - \alpha_1)pY_1 + (1 - \alpha_2)Y_2\}f(L_R)}, \tag{A.23}$$

from which we have:

$$\lim_{p \to 0} \frac{G_R}{G_l} f'(L_R) = \frac{\alpha_2}{1 - \alpha_2} \frac{\beta f'(L_R)(L - L_R)}{f(L_R)},$$

$$\lim_{p \to \infty} \frac{G_R}{G_l} f'(L_R) = \frac{\alpha_1}{1 - \alpha_1} \frac{\beta f'(L_R)(L - L_R)}{f(L_R)}.$$

Under Assumption 1, it holds that $\lim_{p \to 0} \frac{G_R}{G_l} f'(L_R) < \lim_{p \to \infty} \frac{G_R}{G_l} f'(L_R)$. In addition, both $\lim_{p \to 0} \frac{G_R}{G_l} f'(L_R)$ and $\lim_{p \to \infty} \frac{G_R}{G_l} f'(L_R)$ are decreasing in L_R:

$$\frac{d[\beta f'(L_R)(L - L_R)/f(L_R)]}{dL_R} = \frac{\beta\{[(L - L_R)f'' - f']f - (L - L_R)(f')^2\}}{f^2} < 0.$$

Therefore, the solution for L_R that satisfies (A.23) in each limiting case (i.e., $p \to 0$ and $p \to \infty$) is uniquely determined, as illustrated in Figure A1. □

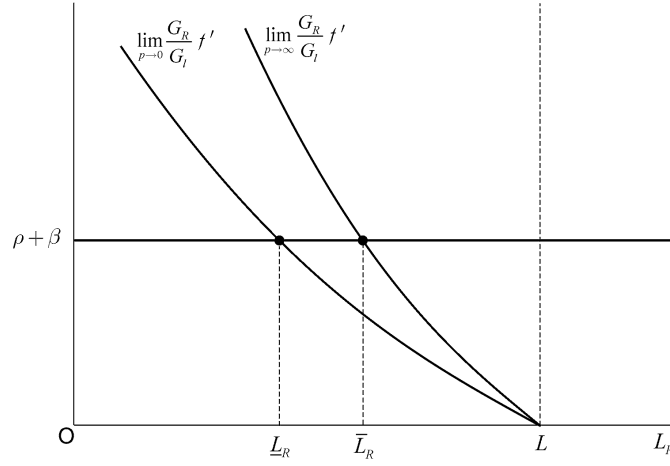Figure A1: Determination of $\overline{L}_R$ and $\underline{L}_R$

A.4 Proof of Theorem 1

Let us define the left-hand side of (25) by $\zeta_L(p)$ and the right-hand side by $\zeta_R(p)$. It is easily verified that

$$\lim_{p \to 0} \zeta_L(p) = \infty, \quad \lim_{p \to \infty} \zeta_L(p) = 0,$$

$$\lim_{p \to 0} \zeta_R(p) = 0, \quad \lim_{p \to \infty} \zeta_R(p) = \left[\frac{f(\bar{L}_R)}{\beta}\right]^{\alpha_1} (L - \bar{L}_R)^{1-\alpha_1} + \left[\frac{f(\bar{L}_R^*)}{\beta^*}\right]^{\alpha_1} (L^* - \bar{L}_R^*)^{1-\alpha_1} > 0.$$

Because both $\zeta_L(p)$ and $\zeta_R(p)$ are continuous in p, it follows that there exists at least one solution for p in $(0, \infty)$. Differentiating $\zeta_L(p)$ and $\zeta_R(p)$, and rearranging in light of (A.21), we have, respectively,

$$\zeta_L'(p) = \frac{\gamma}{p^2} \left\{ p \left(\frac{\rho}{\beta} G_l \psi' + \frac{\rho^*}{\beta^*} G_l^* \psi^{*'} \right) - (Y_2 + Y_2^*) \right\}, \tag{A.24}$$

$$\zeta_R'(p) = G_{pp} + G_{pp}^* + \left(\frac{\rho + \beta}{\beta} \frac{G_l G_{pR}}{G_R} - G_{pl} \right) \psi' + \left(\frac{\rho^* + \beta^*}{\beta^*} \frac{G_l^* G_{pR}^*}{G_R^*} - G_{pl}^* \right) \psi^{*'}. \tag{A.25}$$

From (A.13a), (A.18), and Lemma 2, it follows that $\zeta_R'(p) > 0$. Therefore, if $\zeta_L'(p) < \zeta_R'(p)$ holds at the equilibrium price such that $\zeta_L(p) = \zeta_R(p)$, the equilibrium is unique. From (A.24) and (A.25), we have

$$\zeta_L'(p) - \zeta_R'(p) = \left(\frac{\gamma}{p} \frac{\rho}{\beta} G_l - \frac{\rho + \beta}{\beta} \frac{G_l G_{pR}}{G_R} + G_{pl} \right) \psi' + \left(\frac{\gamma}{p} \frac{\rho^*}{\beta^*} G_l^* - \frac{\rho^* + \beta^*}{\beta^*} \frac{G_l^* G_{pR}^*}{G_R^*} + G_{pl}^* \right) \psi^{*'}$$

$$- \frac{\gamma^2}{p} (Y_2 + Y_2^*) - (G_{pp} + G_{pp}^*). \tag{A.26}$$

Let us assume that the two countries are identical: $L = L^*$, $\beta = \beta^*$, and $\rho = \rho^*$. Then, the two countries share an identical GDP function, and thus the world market-clearing condition $\zeta_L(p) = \zeta_R(p)$ implies that $\gamma/p = G_p/G$. Substituting this into the first term on the right-hand side of (A.26), we have

$$\left(\frac{\gamma}{p} \frac{\rho}{\beta} G_l - \frac{\rho + \beta}{\beta} \frac{G_l G_{pR}}{G_R} + G_{pl} \right) \psi' = \left\{ \frac{\rho}{\beta} \left(\frac{G_p}{G} - \frac{G_{pR}}{G_R} \right) G_l + \left(G_{pl} - \frac{G_l G_{pR}}{G_R} \right) \right\} \psi'.$$

From (A.15), (A.18), and Lemma 2, the above expression is shown to be negative, as is the second term in (A.26). Therefore, $\zeta_L'(p) < \zeta_R'(p)$, and thus there exists a unique solution p_{ss} in the symmetric case.

We next turn to the stability of the steady state. The dynamic system of the world economy is described as

$$\dot{R} = f(L_R) - \beta R,$$

$$\dot{R}^* = f(L_R^*) - \beta^* R^*,$$

$$\dot{\theta} = (\rho + \beta)\theta - \frac{G_R(p, R, L - L_R)}{G(p, R, L - L_R)},$$

$$\dot{\theta}^* = (\rho^* + \beta^*)\theta^* - \frac{G_R(p, R^*, L^* - L_R^*)}{G(p, R^*, L^* - L_R^*)},$$

$$0 = \frac{\gamma}{p} \{ G(p, R, L - L_R) + G(p, R^*, L^* - L_R^*) \} - G_p(p, R, L - L_R) - G_p(p, R^*, L^* - L_R^*).$$

Note that L_R is dependent on R, θ, and p, and given (17), the following derivatives are obtained:

$$\frac{\partial L_R}{\partial R} = \frac{G_{lR} - \theta f' G_R}{\theta G f'' - \theta f' G_l + G_{ll}} = \frac{G_{lR} - \frac{G_l G_R}{G}}{\theta G f'' - \theta f' G_l + G_{ll}} > 0,$$

$$\frac{\partial L_R}{\partial \theta} = -\frac{f' G}{\theta G f'' - \theta f' G_l + G_{ll}} > 0,$$

$$\frac{\partial L_R}{\partial p} = \frac{G_{lp} - \theta f' G_p}{\theta G f'' - \theta f' G_l + G_{ll}} = \frac{G_{lp} - \frac{G_l G_p}{G}}{\theta G f'' - \theta f' G_l + G_{ll}} > 0 \quad \text{if} \quad \alpha_1 > \alpha_2.$$

Assuming that the two countries are identical and linearizing the dynamic system around the symmetric steady state, we obtain

$$
\begin{bmatrix} \dot{R} \\ \dot{R}^* \\ \dot{\theta} \\ \dot{\theta}^* \\ 0 \end{bmatrix} = \begin{bmatrix} f'\frac{\partial L_R}{\partial R} - \beta & 0 & f'\frac{\partial L_R}{\partial \theta} & 0 & f'\frac{\partial L_R}{\partial p} \\ 0 & f'\frac{\partial L_R}{\partial R} - \beta & 0 & f'\frac{\partial L_R}{\partial \theta} & f'\frac{\partial L_R}{\partial p} \\ \delta_R & 0 & \delta_\theta & 0 & \delta_p \\ 0 & \delta_R & 0 & \delta_\theta & \delta_p \\ \chi_R & \chi_R & \chi_\theta & \chi_\theta & \chi_p \end{bmatrix} \begin{bmatrix} R - \bar{R} \\ R^* - \bar{R}^* \\ \theta - \bar{\theta} \\ \theta^* - \bar{\theta}^* \\ 0 \end{bmatrix}, \tag{A.27}
$$

where

$$
\delta_R \equiv -\frac{1}{G}\left\{ G_{RR} - \frac{G_R^2}{G} + \left(\frac{G_R G_l}{G} - G_{Rl} \right) \frac{\partial L_R}{\partial R} \right\},
$$

$$
\delta_\theta \equiv \rho + \beta - \frac{1}{G}\left(\frac{G_R G_l}{G} - G_{Rl} \right) \frac{\partial L_R}{\partial \theta},
$$

$$
\delta_p \equiv -\frac{1}{G}\left\{ G_{Rp} - \frac{G_R G_p}{G} + \left(\frac{G_R G_l}{G} - G_{Rl} \right) \frac{\partial L_R}{\partial p} \right\},
$$

$$
\chi_R \equiv \frac{G_p G_R}{G} - G_{pR} + \left(G_{pl} - \frac{G_p G_l}{G} \right) \frac{\partial L_R}{\partial R},
$$

$$
\chi_\theta \equiv \left(G_{pl} - \frac{G_p G_l}{G} \right) \frac{\partial L_R}{\partial \theta},
$$

$$
\chi_p \equiv 2\left\{ -\frac{Y_1 Y_2}{pG} - G_{pp} + \left(G_{pl} - \frac{G_p G_l}{G} \right) \frac{\partial L_R}{\partial p} \right\}.
$$

Let us denote the above matrix by J and the corresponding eigenvalues by z, which is determined by the characteristic equation

$$
\Omega(z) \equiv \left| J - z \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \right| = 0.
$$

After some calculations, the characteristic equation can be rewritten as $\Omega(z) = 2\Omega_1(z)\Omega_2(z) = 0$, where

$$
\Omega_1(z) \equiv z^2 - \rho z + \left(f'\frac{\partial L_R}{\partial R} - \beta \right) \delta_\theta - f'\frac{\partial L_R}{\partial \theta}\delta_R,
$$

$$
\Omega_2(z) \equiv \left\{ \left(G_{pl} - \frac{G_p G_l}{G} \right) \frac{\partial L_R}{\partial p} - \left(\frac{Y_1 Y_2}{pG} + G_{pp} \right) \right\} z^2
$$

$$
+ \left\{ \left(G_{pl} - \frac{G_p G_l}{G} \right) \left[\frac{\partial L_R}{\partial p} \left(f'\frac{l}{R} + \beta - \delta_\theta \right) + \frac{\partial L_R}{\partial \theta}\delta_p \right] + \left(\frac{Y_1 Y_2}{pG} + G_{pp} \right) \rho \right\} z
$$

$$
+ \left(G_{pl} - \frac{G_p G_l}{G} \right) \left(\frac{\partial L_R}{\partial \theta}\delta_p - \frac{\partial L_R}{\partial p}\delta_\theta \right) f'\left(\frac{l}{R} + \frac{\beta}{f'} \right)
$$

$$
- \left(\frac{Y_1 Y_2}{pG} + G_{pp} \right) \left[\left(f'\frac{\partial L_R}{\partial R} - \beta \right) \delta_\theta - f'\frac{\partial L_R}{\partial \theta}\delta_R \right].
$$

The constant term of $\Omega_1(z)$ can be rewritten as

$$\left(f'\frac{\partial L_R}{\partial R} - \beta\right)\delta_\theta - f'\frac{\partial L_R}{\partial \theta}\delta_R$$
$$= \frac{f}{R}\left(\frac{f'L_R}{f}\frac{\partial L_R}{\partial R}\frac{R}{L_R} - 1\right)(\rho + \beta) + \frac{1}{G}\frac{\partial L_R}{\partial \theta}\left(f'G_{RR} - \beta G_{Rl} - \rho\frac{G_R G_l}{G}\right), \qquad \text{(A.28)}$$

which is unambiguously negative because of the concavity of f. This indicates that the two solutions to $\Omega_1(z) = 0$ have opposite signs. Next, the z^2 term of $\Omega_2(z)$ is negative:

$$\left(G_{pl} - \frac{G_p G_l}{G}\right)\frac{\partial L_R}{\partial p} - \left(\frac{Y_1 Y_2}{pG} + G_{pp}\right)$$
$$= \frac{1}{\theta G f'' - \theta f' G_l + G_{ll}}\left(G_{pl} - \frac{G_p G_l}{G}\right)^2 - \left(\frac{Y_1 Y_2}{pG} + G_{pp}\right) < 0.$$

Moreover, since the sign of (A.28) is negative and

$$\left(G_{pl} - \frac{G_p G_l}{G}\right)\left(\frac{\partial L_R}{\partial \theta}\delta_p - \frac{\partial L_R}{\partial p}\delta_\theta\right)$$
$$= \left(G_{pl} - \frac{G_p G_l}{G}\right)\left\{\frac{\partial L_R}{\partial \theta}\frac{1}{G}\left(G_{Rp} - \frac{G_R G_p}{G}\right) + (\rho + \beta)\frac{\partial L_R}{\partial p}\right\} > 0,$$

the constant term of $\Omega_2(z)$ is positive. Therefore, the two solutions to $\Omega_2(z) = 0$ also have opposite signs. To summarize, there are two positive characteristic roots and two negative roots. Because there are two state variables, R and R^*, it follows that the steady state is a saddle point.

Finally, we can check that $\Omega(0) \neq 0$. It follows that the implicit function theorem ensures that even if the economic fundamentals of the two countries are slightly different, the existence, uniqueness, and stability of the steady state are established (Chen et al., 2008). □

A.5 Signs of $\partial ED(p)/\partial L$ and $\partial ED(p)/\partial \beta$

Substituting the expression for $\partial \psi/\partial L$ into (27), we have

$$\left.\frac{\partial ED(p)}{\partial L}\right|_{p=p_a} = \frac{\left\{\left(G_{Rl} - \frac{G_R G_{ll}}{G_l}\right)\left(\frac{G_p G_R}{G} - G_{pR}\right) + \left(\frac{G_p G_l}{G} - G_{pl}\right)\left(\frac{G_R G_{lR}}{G_l} - G_{RR}\right)\right\}\frac{(f')^2}{\beta}}{\left(\frac{G_R G_{lR}}{G_l} - G_{RR}\right)\frac{(f')^2}{\beta} + \left(G_{lR} - \frac{G_R G_{ll}}{G_l}\right)f' - G_R f''}$$
$$- \frac{\left(\frac{G_p G_l}{G} - G_{pl}\right)G_R f''}{\left(\frac{G_R G_{lR}}{G_l} - G_{RR}\right)\frac{(f')^2}{\beta} + \left(G_{lR} - \frac{G_R G_{ll}}{G_l}\right)f' - G_R f''}. \qquad \text{(A.29)}$$

However, from (A.15), (A.16), (A.19), and (A.20), it is verified that the first term in the above equation becomes zero. Moreover, from (A.16), the term $\left(\frac{G_p G_l}{G} - G_{pl}\right)$ is positive under Assumption 1. Then, it follows that the sign of $\partial ED(p)/\partial L$ becomes nonnegative.

Substituting the expression for $\partial\psi/\partial\beta$ into (A.16), we have

$$\left.\frac{\partial ED(p)}{\partial\beta}\right|_{p=p_a} = -\frac{\left\{\left(G_{Rl} - \frac{G_R G_{ll}}{G_l}\right)\left(\frac{G_p G_R}{G} - G_{pR}\right) + \left(\frac{G_p G_l}{G} - G_{pl}\right)\left(\frac{G_R G_{lR}}{G_l} - G_{RR}\right)\right\}\frac{ff'}{\beta^2}}{\left(\frac{G_R G_{lR}}{G_l} - G_{RR}\right)\frac{(f')^2}{\beta} + \left(G_{lR} - \frac{G_R G_{ll}}{G_l}\right)f' - G_R f''}$$
$$+ \frac{\left\{\frac{G_p G_l}{G} - G_{pl} - \frac{f'}{\beta}\left(\frac{G_p G_R}{G} - G_{pR}\right)\right\}G_l + \left(\frac{G_p G_R}{G} - G_{pR}\right)G_R\frac{ff''}{\beta^2}}{\left(\frac{G_R G_{lR}}{G_l} - G_{RR}\right)\frac{(f')^2}{\beta} + \left(G_{lR} - \frac{G_R G_{ll}}{G_l}\right)f' - G_R f''}. \tag{A.30}$$

Again, the first term in the above equation becomes zero. Moreover, from (A.15) and (A.16), the terms $\left(\frac{G_p G_l}{G} - G_{pl}\right)$ and $\left(\frac{G_p G_R}{G} - G_{pR}\right)$ are positive and negative, respectively, under Assumption 1. Then, it follows that the sign of $\partial ED(p)/\partial\beta$ becomes positive.

A.6 Analysis of the simplified model in Section 4.3

The GDP function in the simplified model is derived as

$$G(p, R, l) \equiv \max_{L_1, L_2}\left\{pR^\alpha L_1^{1-\alpha} + L_2 \quad \text{s.t.} \quad L_1 + L_2 = l\right\} = (1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha p^{\frac{1}{\alpha}}R + l. \tag{A.31}$$

The first- and second-order derivatives of the GDP function are

$$G_p = (1-\alpha)^{\frac{1-\alpha}{\alpha}}p^{\frac{1-\alpha}{\alpha}}R, \quad G_R = (1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha p^{\frac{1}{\alpha}}, \quad G_l = 1,$$
$$G_{pp} = (1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha p^{\frac{1-\alpha}{\alpha}}R, \quad G_{pR} = (1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha p^{\frac{1}{\alpha}}, \quad G_{pl} = G_{RR} = G_{Rl} = G_{ll} = 0.$$

Using (A.31) and $f(L_R) = L_R - L_R^2/2$, we obtain the steady-state conditions of the home country's optimal resource allocation:

$$L_R - \frac{L_R^2}{2} = \beta R,$$
$$\rho + \beta = q(1 - L_R),$$

where we use the variable transformation $q \equiv (1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha p^{\frac{1}{\alpha}}$. Solving the above equations, we have

$$L_R = 1 - \frac{\rho + \beta}{q}, \quad R = \frac{1}{2\beta}\left(1 - \frac{\rho + \beta}{q}\right)\left(1 + \frac{\rho + \beta}{q}\right). \tag{A.32}$$

Similarly for the foreign country, solving the optimal steady-state conditions

$$L_R^* - \frac{L_R^{*2}}{2} = \lambda\beta R^*,$$
$$\lambda(\rho + \beta) = q(1 - L_R^*),$$

we obtain

$$L_R^* = 1 - \frac{\lambda(\rho + \beta)}{q}, \quad R^* = \frac{1}{2\lambda\beta}\left\{1 - \frac{\lambda(\rho + \beta)}{q}\right\}\left\{1 + \frac{\lambda(\rho + \beta)}{q}\right\}. \tag{A.33}$$

The autarkic equilibrium condition in the home country is

$$\alpha\gamma(qR + 1 - L_R) = qR.$$

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Substituting (A.32) into the above equation and solving for q, we obtain

$$q_a = \sqrt{\frac{(\rho + \beta)\{(1 + \alpha\gamma)\beta + (1 - \alpha\gamma)\rho\}}{1 - \alpha\gamma}}. \tag{A.34}$$

Similarly, in light of (A.33), the autarkic steady-state solution for q in the foreign country is derived as

$$q_a^* = \lambda\sqrt{\frac{(\rho + \beta)\{(1 + \alpha\gamma)\beta + (1 - \alpha\gamma)\rho\}}{1 - \alpha\gamma}}. \tag{A.35}$$

The steady-state equilibrium condition under free trade, (25), is rewritten as

$$\alpha\gamma(qR + 1 - L_R) + \alpha\gamma(qR^* + 1 - L_R^*) = qR + qR^*.$$

Substituting (A.32) and (A.33) into the above equation and solving for q, we obtain

$$q_{ss} = \sqrt{\frac{\lambda(\rho + \beta)\{(1 + \alpha\gamma)\beta + (1 - \alpha\gamma)\rho\}}{1 - \alpha\gamma}}. \tag{A.36}$$

The steady-state welfare is

$$u = \gamma \ln C_1 + (1 - \gamma) \ln C_2 = \ln(qR + 1 - L_R) - \alpha\gamma \ln q + A, \tag{A.37}$$

where $A \equiv \gamma\{\alpha \ln \alpha + (1 - \alpha) \ln(1 - \alpha)\} + \gamma \ln \gamma + (1 - \gamma) \ln(1 - \gamma)$. Substituting (A.32) and (A.34) into (A.37), we obtain the autarkic steady-state welfare as follows:

$$u_a = \frac{1 - \alpha\gamma}{2} \ln \frac{\rho + \beta}{1 - \alpha\gamma} - \frac{1 + \alpha\gamma}{2} \ln\{(1 + \alpha\gamma)\beta + (1 - \alpha\gamma)\rho\} + A. \tag{A.38}$$

Substituting (A.32) and (A.36) into (A.37), we obtain the steady-state welfare under free trade as follows:

$$\begin{aligned} u_{ss} = &\frac{1 - \alpha\gamma}{2} \ln \frac{\rho + \beta}{1 - \alpha\gamma} - \frac{1 + \alpha\gamma}{2} \ln\{\lambda[(1 + \alpha\gamma)\beta + (1 - \alpha\gamma)\rho]\} \\ &+ \ln\left\{1 + \frac{(\lambda - 1)[(1 + \alpha\gamma)\beta + (1 - \alpha\gamma)\rho]}{2\beta}\right\} + \Gamma. \end{aligned} \tag{A.39}$$

In a similar manner, from (A.33), (A.35), (A.36), and (A.37), the steady-state welfare under autarky and free trade, respectively, in the foreign country is derived as

$$u_a^* = \frac{1 - \alpha\gamma}{2} \ln \frac{\rho + \beta}{1 - \alpha\gamma} - \frac{1 + \alpha\gamma}{2} \ln\{(1 + \alpha\gamma)\beta + (1 - \alpha\gamma)\rho\} - \alpha\gamma \ln \lambda + A, \tag{A.40}$$

$$\begin{aligned} u_{ss}^* = &\frac{1 - \alpha\gamma}{2} \ln \frac{\rho + \beta}{1 - \alpha\gamma} - \frac{1 + \alpha\gamma}{2} \ln\{\lambda[(1 + \alpha\gamma)\beta + (1 - \alpha\gamma)\rho]\} \\ &+ \ln\left\{1 + \frac{(\lambda - 1)(1 - \alpha\gamma)(\beta - \rho)}{2\beta}\right\} + A. \end{aligned} \tag{A.41}$$

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