ECONOMIC RESEARCH CENTER
DISCUSSION PAPER

E-Series

No.E19-3

Unemployment, Tax Competition, and Tax Transfer Policy

by

Toshiki Tamai
Gareth Myles

May 2019

ECONOMIC RESEARCH CENTER
GRADUATE SCHOOL OF ECONOMICS
NAGOYA UNIVERSITY

Unemployment, Tax Competition, and Tax Transfer Policy*

Toshiki Tamai† and Gareth Myles‡

May 31, 2019

Abstract

The reduction of capital tax rates witnessed over the past wo decades has been motivated by the wish to boost investment and reduce unemployment. Previous models of tax competition have explored the consequences for capital allocation in great detail, but have mostly been silent on the employment effects due to the assumption of a perfect labor market. To address the impact of tax competition on employment and public good provision this paper reconsiders the analysis in the presence of a labor market imperfection that generates unemployment. We incorporate a wage rigidity and intergovernmental transfers financed by the labor income tax into the standard tax competition model to explore the outcome of federal-state policy interaction. In this setting there is an employment externality of taxation in addition to the standard fiscal externality. The key factors in determining the efficiency are the substitutability/complementary between capital and labor in production (which determines the magnitude of the employment externality) and the cost of the efficient level of public good supply relative to the ability of the labor income to generate revenue. If there is complementarity the equilibrium is Pareto efficient when the labor income tax can finance the public good. When the labor income tax is not sufficient for financing, there can only be efficiency if the aggregate externality is negative - but this outcome is only one of multiple equilibria. We also that federal government leadership generally improves (and does not worsen) social welfare in comparison to the equilibrium policies under simultaneous policy choice.

Keywords: H21; H71; J64
JEL classification: Tax competition; Unemployment; Transfer policy

*This work was supported by JSPS KAKENHI Grant Number 16K03726 and 16KK0077.

†Address: Graduate School of Economics, Nagoya University, Furo-cho, Chikusa-ku, Nagoya, Aichi, 464-8601, Japan. Tel: +81-52-789-4933. E-mail: tamai@soec.nagoya-u.ac.jp.

‡Address: School of Economics, The University of Adelaide, Adelaide, South Australia 5005, Australia. Tel: +61-8-8313-4768. E-mail: gareth.myles@adelaide.edu.au.

1 Introduction

International and interregional tax competition have pressured regional and central governments into decreasing taxes over the past two decades. In particular, the statutory corporate income tax rate has decreased significantly in many countries. According to OECD (2019), the average combined (central and sub-central government) statutory tax rate for all covered jurisdictions was 21.4% in 2018, compared to 28.6% in 2000. The motivation for the tax reforms in OECD countries was the promotion of investment and employment via reductions in the corporate income tax rate, and many of the countries that lowered the tax rate did so via a series of cuts over several years (OECD 2017). The reforms are supported by empirical evidence which shows that the corporate income tax rate affects the employment rate (e.g. Feld and Kirchgassner 2002; Feldmann 2011).

The conventional theory of tax competition assumes perfect capital and labor markets, and predicts a race to the bottom. When capital is fully mobile among regions, an increase in the capital tax rate by the government of one region causes capital flight from that region to other regions. This expands the tax bases of other governments and constitutes the *fiscal externality* that drives tax competition. As a consequence of the fiscal externality, all governments have an incentive to levy an inefficiently low capital tax rate which results in underprovision of public goods (Wilson 1986; Zodrow and Mieszkowski 1986).[1] Hence, a policy such as a Pigouvian tax/subsidy at the federal level is necessary to correct for the fiscal externality (Wildasin 1989; DePeter and Myers 1994).

The standard description of the tax competition process does not address the employment consequences. This is because previous models make the assumption of a perfect labor market which precludes them from exploring whether the reforms can have an employment-boosting effect. To investigate this issue requires the incorporation of a labor market imperfection into the analysis in order to capture the stated motivation behind observed policy choices. An imperfect labor market introduces a new dimension into the analysis because the allocation of mobile capital determined by tax rates influences employment levels. The direction in which employment changes is determined by the complementarity/substitutability between capital and labor: when they are substitutes (complements) a loss of mobile capital raises (reduces) employment. Hence, if there is an imperfect labor market, a state government can affect the employment level through choice of the capital tax rate. If one state loses mobile capital then the other gains, so an imperfect labor market creates an *employment externality* of capital taxation (Ogawa et al. 2006a).[2] This employment externality is additional to the standard fiscal externality. Unlike the fiscal externality which is always positive (a tax increase boosts the revenue of other jurisdictions due to capital flight), the employment externality can be positive or negative. It is negative if capital is a substitute for labor so more capital reduces employment and positive if capital and labor are complements. This second externality is an additional source of inefficiency in tax setting.

Two recent studies have analyzed the efficiency of public good/public input provision in the presence of labor market imperfections.[3] Gillet and Pauser (2018) show that public inputs financed entirely by lump-sum tax are inefficiently overprovided because of the employment and fiscal externalities. They also demonstrate that an additional tax (subsidy) on capital will improve efficiency through its strategic influence on the interest rate. Kikuchi and Tamai (2019) investigate the effects of an equalizing transfer on the efficiency of public good supply. They focus on some representative equalization transfers (e.g., tax base equalization, revenue equalization) without treating the federal government as an explicit decision maker. In contrast, this paper explores the interaction between federal and state policies, with the federal government as a strategic actor. Within this framework we clarify the effectiveness of intergovernmental transfers at resolving the inefficiency resulting from tax competition. Furthermore, we consider the nature of interaction between federal and state governments (e.g. Boadway and Keen

[1] Numerous empirical studies support that the governments compete each other in the tax rates (For instance, Devereux et al.2008).

[2] They refer it as the unemployment-exporting externality (Ogawa et al. 2006a, b;Sato 2009). Following Gillet and Pauser (2018), we call it employment externality.

[3] Some studies also examine the relation between unemployment and tax competition (e.g., Aronsson and Wehke 2008; Eichner and Upmann 2012; Exbrayat et al. 2012).

1996)[4] since the timing of policy decisions affects the equilibrium outcome. Hayashi and Boadway (2001) provide empirical support for the federal government acting as a Stackelberg leader, and federal government leadership is commonly assumed in the existing literature (Keen and Konrad 2013).[5] Therefore, it is insightful to contrast the outcome with simultaneous policy choice to the outcome with centralized leadership when considering the effects of interregional transfers with a labor market imperfection.

We assume that there are two state governments and a federal government in the economy. The state governments posses the right to levy a tax on capital and have the responsibility for providing a public good. The federal government levies a tax on the labor incomes of all residents in the country and distributes the tax revenue between the two states as an intergovernmental transfer. Capital is perfectly mobile between states, while labor is immobile. A labor market imperfection is introduced by assuming that the wage rate is fixed above the competitive level which creates underemployment in both states. The labor income tax has no direct distortionary effect since we assume labor is supplied inelastically. The mobility of capital implies that any policy instrument that affects the return to capital will have a distortionary effect due to capital relocation between states. The production side of our model is similar to the setting developed by Köthenbüerger (2008), except for the assumptions about labor supply and the fixed wage: Köthenbüerger assumes elastic supply of divisible labor whereas we assume inelastic supply. However,in our model capital taxes affect the amount of employment as they do in Köthenbüerger (2008) because a change in the tax rate influences the factor demand for labor via a change in the factor price.

The main results of our analysis are as follows: First, we derive the equilibrium policies under simultaneous policy choice and characterize how the tax rates and public goods supply are determined by the fiscal and employment externalities. If the employment externality is positive, the equilibrium transfer to the states (and the labor income tax) is positive and enhances social welfare. It is noteworthy that the equilibrium policy is Pareto efficient when the total cost of supplying the public good can be covered by the labor income tax alone. However, a non-negative transfer policy does not improve social welfare in all cases. This is because a positive transfer gives state governments an incentive to decrease their capital tax rates. Hence, it worsens the equilibrium inefficiency when there is a negative employment externality since this decreases the capital tax rates chosen by the state governments. Second, we analyze the equilibrium policies under federal government leadership. In this case, the federal government chooses the intergovernmental transfers, as well as the tax rate on labor income, taking the state governments' responses into account. Surprisingly, when the employment externality is positive, the outcomes are equivalent to those obtained with simultaneous policy choice. In contrast, the efficiency of public good supply is improved for some combinations of parameter values if the employment externality is negative. Federal government leadership enables a negative transfer from the federal government to the state governments. Therefore, the federal government sets an appropriately negative transfer and succeeds in ensuring the optimal level of public goods by the state governments.

The remainder of this paper is organized as follows. The basic model is developed in Section 3. Section 4 analyzes the equilibrium outcomes when federal and state governments simultaneously choose their policies. The economic performance under federal government leadership is investigated in Section 5. Finally, Section 6 provides some conclusions.

[4] Boadway and Keen (1996) and Boadway et al. (1998) analyze the federal-state policy interaction at the early stage. These studies focus on an overlapping tax base and shed light on the presence of vertical externality through it. However, this paper do not treat the issue of overlapping tax base.

[5] Altshuler and Goodspeed (2015) find that smaller countries follow the beggest country when they choose the tax policy. It seems to be true on the relation between federal and state governments. Numerous studies investigate the decentralized leadership and the efficiency (e.g., Köethenbüerger 2004, 2008; Breuille et al. 2010; Caputo and Silva 2014, 2015, 2017). It is natural to analyze the impact of the decentralized leadership on the efficiency because a positive transfer has the incentive effect to reduce the state governments' tax rates. We can rely on their contributions.

2 The basic model

Consider an economy which is composed of two states. Each state has a representative firm and a continuum of size one of residents. The production technology in state i, $i = 1, 2$, is described by $y_i = F(k_i, l_i, z_i)$ where F is a constant-returns-to-scale function, y_i is the output of single good, k_i is the capital input, l_i is the labor input, and z_i is the land input. Assuming that land size is fixed at unity, the production function can be rewritten as

$$y_i = f(k_i, l_i), \tag{1}$$

where $f(k_i, l_i) \equiv F(k_i, l_i, 1)$. It is assumed that the function f is strictly concave and at least twice continuously differentiable. Profit maximization yields

$$f_k^i \equiv \frac{\partial f(k_i, l_i)}{\partial k_i} = r + t_i, \tag{2}$$

$$f_l^i \equiv \frac{\partial f(k_i, l_i)}{\partial l_i} = \overline{w}, \tag{3}$$

where r is the interest rate, t_i is the source-based capital tax rate, and $\overline{w}$ is the fixed wage rate. Furthermore, the return to land, π_i, is given by

$$\pi_i = f(k_i, l_i) - (r + t_i)k_i - \overline{w}l_i. \tag{4}$$

Following Ogawa et al. (2006a), we assume that the fixed wage rate is higher than the wage rate determined in a competitive labor market.

Each resident in state i has a capital endowment of $\sigma_i\overline{k}$ units, a labor endowment of one unit, and a land endowment of one unit. Note that $\overline{k}$ is the entire capital endowment of the economy and σ_i is the share for the residents in state i. The residents supply these endowments inelastically. The utility function is $U_i = x_i + v(g_i)$ where x_i and g_i denotes private good and public good consumption respectively. We specify $v(g_i) = \alpha \log g_i$, where $\alpha > 0$. Since $\overline{w}$ exceeds the competitive level, there is unemployment, so the residents can be divided into two groups: the employed and the unemployed. An employed resident must pay a labor income tax at rate τ and so has the budget constraint $x_i = r\sigma_i\overline{k} + \pi_i + (1-\tau)\,\overline{w}$. In contrast, an unemployed resident does not pay the labor income tax so their budget constraint is $x_i = r\sigma_i\overline{k} + \pi_i$. Consequently, the utilities of the employed (e) and the unemployed (u) are respectively

$$U_i^e = r\sigma_i\overline{k} + \pi_i + (1-\tau)\,\overline{w} + v(g_i), \tag{5}$$

$$U_i^u = r\sigma_i\overline{k} + \pi_i + v(g_i). \tag{6}$$

To ensure that the residents of both states want to work, the following condition is imposed:

$$U_i^e - U_i^u = (1-\tau)\,\overline{w} \geq 0,\ \ i = 1, 2. \tag{7}$$

The capital market clearing condition is

$$\sum_{i=1}^{2} k_i = \overline{k}. \tag{8}$$

On the other hand, there is unemployment in the labor market in each state, so $l_i < 1$, $i = 1, 2$.

Equations (2)–(4) and (8) lead to an equilibrium described by $r = r(\boldsymbol{t})$, $k_i = k_i(\boldsymbol{t})$, and $l_i = l_i(\boldsymbol{t})$ where $\boldsymbol{t} = (t_1, t_2)$. The responses of r, k_i, and l_i to a change in t_i are given by (see Appendix A for derivation):

$$\frac{\partial k_i}{\partial t_i} = \frac{f_{ll}^1 f_{ll}^2}{\Delta} < 0, \frac{\partial k_i}{\partial t_j} = -\frac{f_{ll}^1 f_{ll}^2}{\Delta} > 0, \tag{9}$$

$$\frac{\partial l_i}{\partial t_i} = \frac{f_{lk}^i f_{ll}^j}{\Delta} \gtreqless 0 \Leftrightarrow f_{lk}^i \gtreqless 0, \frac{\partial l_i}{\partial t_j} = -\frac{f_{lk}^i f_{ll}^j}{\Delta} \lesseqgtr 0 \Leftrightarrow f_{lk}^i \gtreqless 0 \tag{10}$$

$$\frac{\partial r}{\partial t_i} = -\frac{f_{ll}^i \Delta_j}{\Delta} < 0, (i \neq j) \tag{11}$$

where

$$
\begin{aligned}
f_{kk}^i &\equiv \frac{\partial^2 f(k_i,l_i)}{\partial k_i^2} < 0, f_{ll}^i, \equiv \frac{\partial^2 f(k_i,l_i)}{\partial l_i^2} < 0, f_{kl}^i \equiv \frac{\partial^2 f(k_i,l_i)}{\partial k_i \partial l_i}, f_{lk}^i \equiv \frac{\partial^2 f(k_i,l_i)}{\partial l_i \partial k_i}, \\
\Delta_i &\equiv f_{kk}^i f_{ll}^i - \left(f_{lk}^i\right)^2 > 0, \Delta \equiv f_{ll}^2 \Delta_1 + f_{ll}^1 \Delta_2 < 0.
\end{aligned}
$$

Using equations (9)–(11), we obtain

$$
\frac{\partial \pi_i}{\partial t_i} = -\left(1 + \frac{\partial r}{\partial t_i}\right) k_i, \frac{\partial \pi_i}{\partial t_j} = -\frac{\partial r}{\partial t_j} k_i. \tag{12}
$$

From (10) the sign of $\frac{\partial l_i}{\partial t_i}$ is the same as that of the cross derivative f_{kl}.[6] As reported in OECD (2017), both the public and politicians seem to believe that reductions in the corporate tax will reduce unemployment, which implies an increase in capital tax rate must have a negative impact on employment. This belief is only valid in the model if $f_{lk}^i < 0$. However, empirical evidence on the relation of employment to the corporate tax rate is controversial: Feld and Kirchgassner (2002), Bettendorf et al. (2009), Zirgulis and Šarapovas (2017) found that a rise in the corporate tax rate significantly increased unemployment levels while Feldmann (2011) showed that higher corporate taxes may lower the unemployment rate. Hence, we need to consider both possibilities in the analysis below.

Integrating the utility function over the continuum of residents, the welfare function for region i is

$$
V_i = r\sigma_i \overline{k} + \pi_i + (1-\tau)\, \overline{w} l_i + v(g_i). \tag{13}
$$

Equation (13) is the objective function for state government i when choosing its tax policy. Each state government taxes the return to the capital, receives a tax transfer from the federal government and provides a public good for its residents. Consequently, the budget constraint for state government i is

$$
g_i = t_i k_i + s_i, \tag{14}
$$

where s_i denotes the tax transfer from the federal government. The federal government income tax is imposed on all residents in the country so the budget constraint for the federal government is

$$
\tau \overline{w} \sum_{i=1}^{2} l_i = \sum_{i=1}^{2} s_i \tag{15}
$$

The Pareto efficient allocations for the economy are used as a comparison point in the analysis that follows. To obtain the characterization consider the maximization of state 1's welfare subject to equations (1)–(4), (14) and (15) for given a given level of state 2's welfare. Solving the problem yields (see Appendix B)

$$
v'(g_i) = 1 \Leftrightarrow g^* = \alpha, \tag{16}
$$

where g^* stands for the efficient level of public good supply. The optimal tax rates at this equilibrium are

$$
\begin{aligned}
(t^*, \tau^*) &= \left(0, \frac{\alpha}{\overline{w} l}\right) \text{ for } \alpha \leq \overline{w} l, \qquad (17) \\
(t^*, \tau^*) &= \left(\frac{\alpha - \overline{w} l}{k}, 1\right) \text{ for } \alpha > \overline{w} l. \qquad (18)
\end{aligned}
$$

Since the labor income tax has no distortionary effect, it can be treated as a lump-sum tax.[7] When $\alpha \leq \overline{w} l$ the public good can be financed entirely through the income tax, so the distortionary capital tax is not used. In contrast, when $\alpha > \overline{w} l$ the labor income tax is set at its maximum and the capital tax finances the remainder of the public good.

[6] One example form of the production function that satisfies $f_{kl} < 0$ is $y = \sin(kl)$, where $k, l \in [1, 1.5]$. Another example is $y = [1 - \exp(-k - l - 1)]$.

[7] If the labor supply generates a (fixed unit of) disutility δ, the upper bound of labor income tax will be lowered. Hence, the planner needs to impose the capital tax even if $\alpha < \overline{w} l$.

3 Simultaneous policy choice

The analysis in this section focuses on the equilibrium outcome with simultaneous policy choice. The state and federal governments simultaneously choose their policy instruments to maximize their objective functions, subject to constraints, under the assumption of Nash behavior. The game has the following two stages. At the first stage, each state government chooses their policy instruments to maximize their objective function subject to their budget equation taking the firm's and residents' behavior into account. At the second stage, firms choose their factor demands to maximize profit given the policy instruments $\{t_1, t_2, s_1, s_2, \tau\}$.

The behavior of the firms is characterized by the factor demand functions (9)–(11). Taking these demands into account, state government i faces the maximization problem:

$$\max_{\{t_i \geq 0\}} V_i = r\sigma_i \overline{k} + \pi_i + (1-\tau)\,\overline{w} l_i + v(g_i), \tag{19}$$

subject to (4), (14), $r = r(\boldsymbol{t})$, $k_i = k_i(\boldsymbol{t})$, and $l_i = l_i(\boldsymbol{t})$ for given t_j, s_i, s_j, τ ($j \neq i$). Exploiting the symmetry of the two states, the partial derivative of V_i with respect to τ_i is

$$\frac{\partial V_i}{\partial t_i} = -k_i + (1-\tau)\,\overline{w}\frac{\partial l_i}{\partial t_i} + v'(g_i)\left[k_i + t_i \frac{\partial k_i}{\partial t_i}\right]. \tag{20}$$

From (20), the first-order condition for an interior solution of (19) is

$$v'(g_i) = \frac{1 + (1-\tau)\,\gamma_i}{1+\epsilon_i}, \tag{21}$$

where

$$\begin{aligned}
\epsilon_i &\equiv \frac{t_i}{k_i}\frac{\partial k_i}{\partial t_i} \lesseqgtr 0 \Leftrightarrow t_i \gtreqless 0, \\
\gamma_i &\equiv -\frac{\overline{w}}{k_i}\frac{\partial l_i}{\partial t_i} \gtreqless 0 \Leftrightarrow f^i_{lk} \gtreqless 0.
\end{aligned}$$

The terms ϵ_i and γ_i represent the *fiscal externality* and *employment externality* of the capital tax respectively. The fiscal externality is always positive when the tax rate is positive (Wildasin 1989). The employment externality can be positive or negative (Ogawa et al. 2006a) depending on the cross-derivative f_{kl} of the production function.[8] In contrast, the first-order condition for a corner solution ($t_i = 0$) can be obtained from (20) as

$$v'(g_i) \leq 1 + (1-\tau)\,\gamma_i. \tag{22}$$

We impose the following assumption to ensure $v'(g_i) > 0$ at the optimum irrespective of whether it is interior or at a corner.

Assumption 1. *$\epsilon_i > -1$ and $\gamma_i > -1$.*

Equations (21) and (22) determine the best response functions for the state governments. Again appealing to the symmetry of the states, the equilibrium properties of the reaction function are summarized as follows.

Lemma 1. *Suppose that the regions are symmetric. With $t \geq 0$, the equilibrium capital tax function for each state government is given by*

$$t = \max\left[0, \frac{\alpha - [1 + (1-\tau)\gamma]\,\tau\overline{w}l}{\alpha\beta + [1+(1-\tau)\gamma]\,k}\right],$$

where

$$\beta \equiv -\frac{\epsilon}{t} = -\frac{1}{k}\frac{\partial k}{\partial t} > 0.$$

[8] Sato (2009) also shows the unemployment-exporting externality (positive employment externality) using job-search model.

Proof. For the interior solution, equations (14), (15), and (21) yield

$$\frac{\alpha}{tk + \tau \overline{w} l} = \frac{1 + (1-\tau)\gamma}{1 - \beta t} \Rightarrow t = \frac{\alpha - [1 + (1-\tau)\gamma]\, \tau \overline{w} l}{\alpha\beta + [1 + (1-\tau)\gamma]\, k}.$$

Note that k and l are independent of both t and τ in the symmetric equilibrium. The capital tax rate derived above can be negative; imposing the lower bound of 0 for the tax rate gives the tax function in Lemma 1.

The terms β and γ that appear in the tax function of Lemma 1 are associated with the fiscal externality and employment externality, respectively. An increase in the labor income tax has two different impacts on the resident's utility level: it has a negative impact via a decrease in the level of disposable labor income and a positive impact via the increase in the transfer from the federal government for public good provision. The balance of these two impacts determines the capital tax rates chosen by the state governments. In the absence of a labor income tax ($\tau = 0$) and, consequently, no transfers from the federal government ($s_i = 0$), the capital tax rate is

$$t^O = \frac{\alpha}{\alpha\beta + (1+\gamma)\, k} > 0.$$

An increase in γ decreases the capital tax rate because a large positive value of γ indicates a significant negative effect of a rise in the capital tax rate on employment. In contrast, with the maximum labor income tax ($\tau = 1$), the terms involving γ are eliminated from the tax function. A labor income tax of this magnitude eliminates the effect of the employment externality.

As shown in Ogawa et al. (2006a), the equilibrium supply of public good is affected by the presence of the employment externality. When there are no transfers from the federal government to the states, the following result applies:

Lemma 2. *Without transfers ($\tau = 0 \Longrightarrow s_i = 0, i = 1, 2$), the supply of public good in each state is equal to*

$$g^O = \frac{\alpha k}{\alpha\beta + (1+\gamma)\, k} \gtreqless g^* \Leftrightarrow \alpha\beta + \gamma k \lesseqgtr 0.$$

Proof. Equations (14) and (15) with $s = \tau = 0$ lead to $g^O = tk$. Hence, we obtain

$$g^O - g^* = tk - \alpha = -\frac{(\alpha\beta + \gamma k)\,\alpha}{\alpha\beta + (1+\gamma)\, k} \gtreqless 0 \Leftrightarrow \alpha\beta + \gamma k \lesseqgtr 0.$$

The term $\alpha\beta + \gamma k$ captures the aggregate external effect derived from the fiscal and employment externalities. The public good is overprovided (underprovided) if the aggregate externality is negative (positive). In the standard setting without the employment externality, the positive fiscal externality always results in underprovision. The employment externality can reverse this conclusion if it is sufficiently negative. The consequence of Lemma 2 is that there will be inefficiency unless the fiscal and employment externalities neutralize each other. When there is inefficiency, there is a potential role for transfers from the federal government to provide a resolution and achieve efficiency.

The federal government chooses the transfers and the labor income tax to maximize a utilitarian welfare function defined on state welfare levels subject to equations (4), (7), (14), (15), $r = r(\boldsymbol{t})$, $k_i = k_i(\boldsymbol{t})$, and $l_i = l_i(\boldsymbol{t})$ for given t_i. Formally, the optimization of the federal government is

$$\max_{\{s_1, s_2, \tau\}} \mathcal{L} = \sum_{i=1}^{2} V_i + \left[\tau \overline{w} \sum_{i=1}^{2} l_i - \sum_{i=1}^{2} s_i\right] \mu + s_i \zeta_i + (1-\tau)\, \overline{w} \xi, \tag{23}$$

where μ is the Lagrange multiplier for the constraint (15), ζ_i is the Kuhn-Tucker multiplier for non-negativity of transfers, and ξ is the Kuhn-Tucker multiplier for the constraint (7). The first-order conditions are

$$s_i \quad : \quad v'(g_i) + \zeta_i - \mu = 0, s_i \geq 0, \zeta_i \geq 0, s_i\zeta_i = 0, \tag{24}$$

$$\tau \quad : \quad \left[(\mu - 1)\sum_{i=1}^{2} l_i - \xi\right]\overline{w} = 0, \tag{25}$$

$$\xi_i \quad : \quad (1-\tau)\,\overline{w}\xi = 0, \xi \geq 0, \tau \leq 1. \tag{26}$$

Equations (14), (15), (21), and (24)–(26) constitute the description of the Nash equilibrium for simultaneous choice. We use the notation g^N for the supply of public goods, t^N for the capital tax rate, and τ^N for the labor income tax at the Nash equilibrium. The best response functions for the state governments were stated in Lemma 1. The best response functions for the federal government are subject to the boundary conditions requiring non-negative tax rates and a tax rate on labor income that does not exceed 1. The presence of the boundary conditions create kinks in the graphs of the best response functions. The full set of possible cases is illustrated in Figure 1. The nature of the equilibrium is dependent on the value of α relative to $\overline{w}l$, so we treat the cases $\alpha < \overline{w}l$ and $\alpha > \overline{w}l$ separately.

First, the equilibrium with $\alpha < \overline{w}l$ is characterized in the following result.

Proposition 1. *When* $\alpha < \overline{w}l$,
(i) If $f_{kl} \geq 0$ *the unique Nash equilibrium given by*

$$\left(t^N, \tau^N, g^N\right) = \left(0, \frac{\alpha}{\overline{w}l}, g^*\right).$$

(ii) If $f_{kl} < 0$ *the unique Nash equilibrium given by*

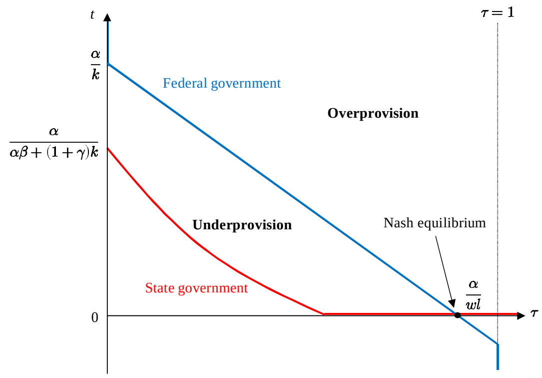
$$\begin{aligned}
(a) \quad \left(t^N, \tau^N, g^N\right) &= \left(\frac{(\alpha - \overline{w}l)\,\gamma}{\beta\overline{w}l + \gamma k}, \frac{\alpha\beta + \gamma k}{\beta\overline{w}l + \gamma k}, g^*\right) \text{ for } |\gamma k| < \alpha\beta, \\
(b) \quad \left(t^N, \tau^N, g^N\right) &= \left(\frac{\alpha}{\alpha\beta + (1+\gamma)\,k}, 0, g^O\right) \text{ for } |\gamma k| > \alpha\beta.
\end{aligned}$$

Proof. See Appendix C.1.

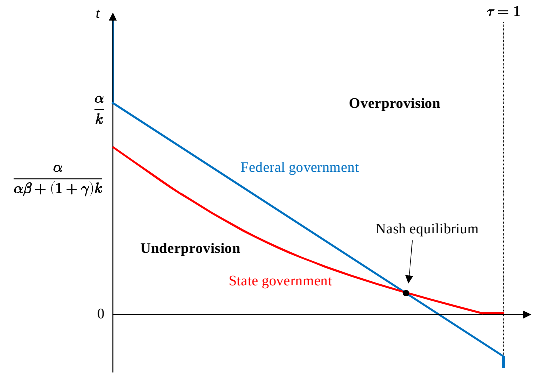
If $f_{kl} \geq 0$, the aggregate effect of the fiscal and employment externalities is positive. Hence, the state governments have an individual incentive to reduce their tax rates to attract capital. In response, the federal government increases the transfer to the state governments via an increase in the labor income tax. The transfer from the federal government to the state governments provides an additional incentive to decrease the capital tax rate because the transfer covers the decrease in the capital tax revenue. Therefore, the equilibrium policy is a zero capital tax with a positive transfer (Figure 1a).

When $f_{kl} < 0$, there are two possible cases: $|\gamma k| < \alpha\beta$ (the aggregate external effect is positive) and $|\gamma k| > \alpha\beta$ (the aggregate external effect is negative). For $|\gamma k| < \alpha\beta$ the same mechanism as described above is at work. However, the unemployment-importing externality (i.e., a negative employment externality) partially offsets the fiscal externality and the incentive to reduce the capital tax rates is diminished. Therefore, both the capital tax rate and the transfer are positive in equilibrium (Figure 1b). If $|\gamma k| > \alpha\beta$ each state government wishes to increase its capital tax rate above the level when $|\gamma k| < \alpha\beta$. In response, the federal government reduces the labor income tax and the transfer to state governments. The equilibrium capital tax rates are positive without a transfer (Figure 1c). In particular, the state governments overprovide public goods under the equilibrium policies. The transfer from federal to state governments fails to decrease the capital tax rate which implies that a transfer from state governments to federal government would be efficient, if it were permitted.

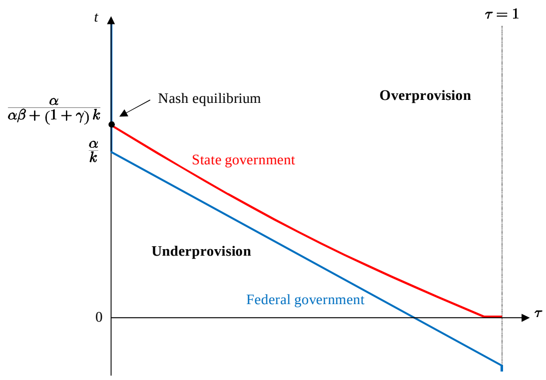
Second, we characterize the equilibrium for $\alpha > \overline{w}l$:



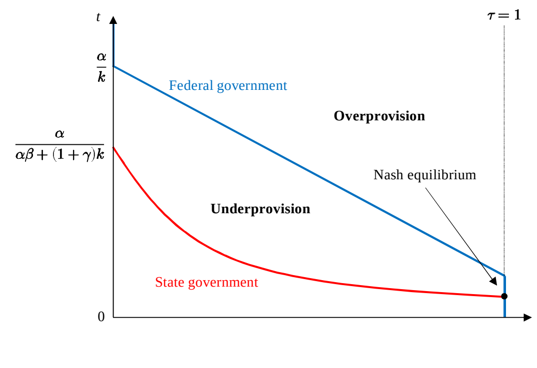
a. $\gamma > 0, \alpha\beta > |\gamma k|, \alpha < \overline{w}l$



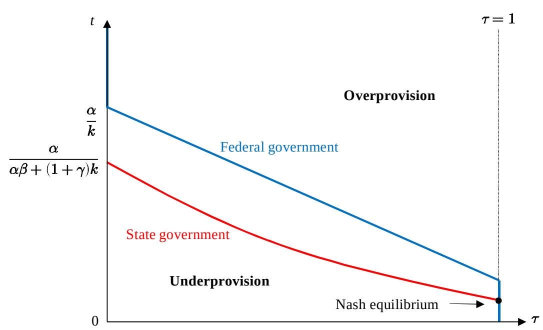
b. $\gamma < 0, \alpha\beta > |\gamma k|, \alpha < \overline{w}l$



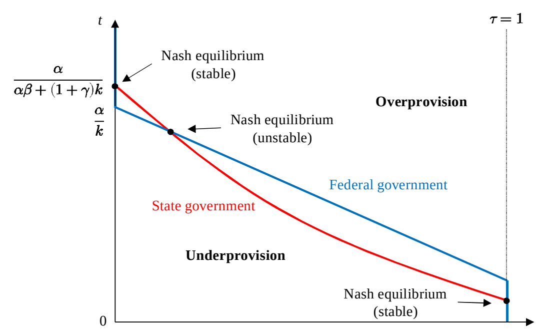
c. $\gamma < 0, \alpha\beta < |\gamma k|, \alpha < \overline{w}l$



d. $\gamma > 0, \alpha\beta > |\gamma k|, \alpha > \overline{w}l$



e. $\gamma < 0, \alpha\beta > |\gamma k|, \alpha > \overline{w}l$



f. $\gamma < 0, \alpha\beta < |\gamma k|, \alpha > \overline{w}l$

Figure 1.

Proposition 2. *When $\alpha > \overline{w}l$,*
(i) If $f_{kl} \geq 0$ the unique Nash equilibrium is given by

$$\left(t^N, \tau^N, g^N\right) = \left(\frac{\alpha - \overline{w}l}{\alpha\beta + k}, 1, \frac{\left(\beta\overline{w}l + k\right)\alpha}{\alpha\beta + k}\right).$$

(ii) If $f_{kl} < 0$
(a) When $|\gamma k| < \alpha\beta$, the unique Nash equilibrium is given by

$$\left(t^N, \tau^N, g^N\right) = \left(\frac{\alpha - \overline{w}l}{\alpha\beta + k}, 1, \frac{\left(\beta\overline{w}l + k\right)\alpha}{\alpha\beta + k}\right).$$

(b) When $|\gamma k| > \alpha\beta$, there exist three Nash equilibria given by:

$$\left(t^N, \tau^N, g^N\right) = \left\{\left(\frac{\alpha - \overline{w}l}{\alpha\beta + k}, 1, \frac{\left(\beta\overline{w}l + k\right)\alpha}{\alpha\beta + k}\right), \left(\frac{\left(\alpha - \overline{w}l\right)\gamma}{\beta\overline{w}l + \gamma k}, \frac{\alpha\beta + \gamma k}{\beta\overline{w}l + \gamma k}, g^*\right), \left(\frac{\alpha}{\alpha\beta + \left(1 + \gamma\right)k}, 0, g^O\right)\right\}.$$

Proof. See Appendix C.2.

The mechanisms through which tax policies operate are similar to those of Proposition 1, and only the presence of the labor income tax wedge creates a difference to the analysis with $\alpha < \overline{w}l$. If $|\gamma k| < \alpha\beta$ (which includes the case $f_{kl} \geq 0$), the upper bound for the labor income tax rate constrains the federal government's choice of tax policy. The equilibrium policy requires both a positive tax and a transfer (Figure 1d and 1e). The case of $|\gamma k| > \alpha\beta$ is significantly different with three Nash equilibria. For a low (high) tax rate on labor income, the state governments choose high (low) tax rates on capital, and then the federal government responds with a lower (higher) tax rate on labor income. The process converges either to one of the two equilibria at the boundary for the income tax, or to an interior equilibrium. The interesting observation is that the efficient provision of public good is attained at the interior equilibrium (Figure 1f). The condition in case (ii) of Proposition 2 can be rewritten as[9]

$$|\gamma k| \gtreqless \alpha\beta \Leftrightarrow \frac{\left|\frac{t}{l}\frac{\partial l}{\partial t}\right|}{\left|\frac{t}{k}\frac{\partial k}{\partial t}\right|} = \frac{\text{Elasticity of employment rate to capital tax rate}}{\text{Elasticity of capital to capital tax rate}} \gtreqless \frac{\alpha}{\overline{w}l}. \tag{27}$$

Since the proposition applies when $\alpha < \overline{w}l$, the multiple equilibria arise when the cross derivative of the production function is negative and the elasticity of employment is large relative to the elasticity of capital.

Focusing on the capital tax rate and the level of public good supply, the results discussed above for $\alpha < \overline{w}l$ are summarized in the next proposition.

Proposition 3. *When $\alpha < \overline{w}l$,*
(i) If $f_{kl} \geq 0$, then:
$t^N = t^* < t^O$ *and* $g^O < g^N = g^*$.
(ii) If $f_{kl} < 0$, then:
(a) $|\gamma k| < \alpha\beta$ *implies* $t^* < t^N < t^O$ *and* $g^O < g^N = g^*$.
(b) $|\gamma k| > \alpha\beta$ *implies* $t^* < t^N = t^O$ *and* $g^* < g^N = g^O$.

Proof. See Appendix D.1.

The corresponding results for $\alpha > \overline{w}l$ are as follows.

[9] This relation is derived from

$$|\gamma k| \gtreqless \alpha\beta \Leftrightarrow -\left(-\frac{\overline{w}}{k}\frac{\partial l}{\partial t}k\right) \gtreqless -\alpha\frac{1}{k}\frac{\partial k}{\partial t} \Leftrightarrow \overline{w}\frac{\partial l}{\partial t} \gtreqless -\alpha\frac{1}{k}\frac{\partial k}{\partial t} \Leftrightarrow \overline{w}l\frac{t}{l}\frac{\partial l}{\partial t} \gtreqless -\alpha\frac{t}{k}\frac{\partial k}{\partial t}.$$

Table 1: Equilibrium tax policy under simultenous policy choice

<table>
<tr><td>(a) $\alpha < \overline{w}l$</td><td>$f_{kl} \geq 0$</td><td>$f_{kl} < 0, |\gamma k| < \alpha\beta$</td><td colspan="3">$f_{kl} < 0, |\gamma k| > \alpha\beta$</td></tr>
<tr><td>g^N</td><td colspan="2">$g^O < g^N = g^*$</td><td colspan="3">$g^N = g^O > g^*$</td></tr>
<tr><td>t^N</td><td>$t^N = t^* = 0$</td><td>$0 < t^N < t^O$</td><td colspan="3">$t^N = t^O > 0$</td></tr>
<tr><td>τ^N</td><td>$\tau^N = \tau^* < 1$</td><td>$0 < \tau^N < \tau^*$</td><td colspan="3">$\tau^N = 0 < \tau^*$</td></tr>
<tr><td>(b) $\alpha > \overline{w}l$</td><td>$f_{kl} \geq 0$</td><td>$f_{kl} < 0, |\gamma k| < \alpha\beta$</td><td colspan="3">$f_{kl} < 0, |\gamma k| > \alpha\beta$</td></tr>
<tr><td>g^N</td><td>$g^O < g^N < g^*$</td><td>$g^N \gtreqless g^O (g^N < g^*)$</td><td>$g^N < g^* < g^O$</td><td>$g^N = g^* < g^O$</td><td>$g^N = g^O > g^*$</td></tr>
<tr><td>t^N</td><td>$t^N \gtreqless t^O$</td><td>$t^N < t^O$</td><td>$t^N < t^O$</td><td>$t^N < t^O$</td><td>$t^N = t^O > 0$</td></tr>
<tr><td>τ^N</td><td colspan="2">$\tau^N = \tau^* = 1$</td><td>$\tau^N = \tau^*$</td><td>$0 < \tau^N < \tau^*$</td><td>$\tau^N = 0 < \tau^*$</td></tr>
</table>

Proposition 4. *When $\alpha > \overline{w}l$,*

(i) If $f_{kl} \geq 0$, then:

$t^* < t^N < t^O$ *or* $t^* < t^O < t^N$ *and* $g^O < g^N < g^*$.

(ii) If $f_{kl} < 0$, then:

(a) $|\gamma k| < \alpha\beta$ implies $t^* < t^N < t^O$ and $g^O < g^N < g^*$ or $g^N \leq g^O < g^*$.

(b) $|\gamma k| > \alpha\beta$ implies $t^* < t^N \leq t^O$ and $g^N \leq g^* < g^O$ or $g^* < g^N = g^O$.

Proof. See Appendix D.2.

Equation (17), Lemma 1, Lemma 2, and Proposition 1 enable us to compare the equilibrium tax policies. Table 1 (a) shows the relationship between the state government's policy without transfers and the Nash equilibrium in the case where $\alpha < \overline{w}l$. For $f_{kl} \geq 0$, the outcome is based on the mechanism explained in connection with Proposition 1. If there is an unemployment-importing externality ($f_{kl} < 0$), Kikuchi and Tamai (2019) show that tax base equalization with a negative coefficient improves the efficiency of public good provision. When the governments simultaneously choose the tax rates, the federal government has to set a non-negative transfer because there is a possibility of the state governments choosing non-positive capital tax rates. Thus, a transfer from the state governments to the federal government is impossible when there is simultaneous policy choice.

Table 1 (b) displays the outcomes for $\alpha > \overline{w}l$ derived from Lemma 1, Lemma 2, and Proposition 2. When $f_{kl} \geq 0$, a transfer from the federal government to the state governments improves the efficiency of public good provision even though it is not sufficient to ensure the efficient level of public good supply. Then, the capital tax rate with the transfer can be either higher or lower than the tax rate without the transfer. In particular, the capital tax rate with the transfer is smaller than the tax rate without transfer when γ takes a sufficiently large positive value.[10]

Depending on the overall external effect, two cases are possible when $f_{kl} < 0$. If the fiscal externality dominates the unemployment importing externality ($|\gamma k| < \alpha\beta$), the supply of public good can be either improved or worsened when compared to the case where $\tau = 0$, but the public good is always below the efficient level. In this case, the capital tax rate with transfers is lower than the tax rate without transfers.

4 Federal government leadership

This section investigates the equilibrium outcome with sequential policy choice. We consider federal government (centralized) leadership, which implies pre-commitment to policy by the federal government. The game structure under federal government leadership can be described as follows. At the

[10] Contrasting the income tax rate in Proposition 2 and $\tau^O = 0$, we an see that

$$\tau^N \gtreqless \tau^O \Leftrightarrow \gamma \lesseqgtr \left(\frac{\alpha\beta + k}{k}\right)\left(\frac{\overline{w}l}{\alpha - \overline{w}l}\right).$$

first stage, the federal government takes the best responses of the state governments and the firms into account, and chooses transfers and an income tax, $\{s_1, s_2, \tau\}$, to maximize social welfare subject to the federal budget constraint. At the second stage, each state government chooses its capital tax rate to maximize (13) subject to (14) and the reaction of firms. Finally, at the third stage, firms in each state, i, determine input use, $\{k_i, l_i\}$, to maximize their profits for given $\{t_i, s_i, \tau\}$ $(i = 1, 2)$.

Backward induction is applied to solve the game. The reactions of firms are represented by equations (11)–(12). The optimization problem for each state government yields equation (21). Equations (14) and (21) provide the optimal response of t_i to a change in s_i and τ. The reaction function of state i's government has the following properties (See Appendix E):

$$\frac{\partial t_i}{\partial s_i} = -\frac{(1+\epsilon_i)\, k_i v''(g_i)}{D_i} < 0, \frac{\partial t_i}{\partial s_j} = 0, \frac{\partial t_i}{\partial \tau} = \frac{\overline{w}}{D_i}\frac{\partial l_i}{\partial t_i} \gtreqless 0 \Leftrightarrow f_{kl} \gtreqless 0 \text{ for } t_i > 0, \tag{28}$$

$$\frac{\partial t_i}{\partial s_i} = \frac{\partial t_i}{\partial s_j} = \frac{\partial t_i}{\partial \tau} = 0 \text{ for } t_i = 0, \tag{29}$$

where

$$D_i \equiv \frac{\partial^2 V_i}{\partial t_i^2} < 0.$$

An increase in the transfer to state i has a positive effect on state i's government revenue but no impact on state j's government revenue. The increase in revenue enables the state governments to decrease their capital tax rates. The income tax rate affects the welfare gain/cost of the capital tax through job creation/loss. Hence, the effect of a rise in the income tax rate on the capital tax rate depends upon the effect of the capital tax rate on employment.

Taking the relationships $r = r(\boldsymbol{t})$, $k_i = k_i(\boldsymbol{t})$, $l_i = l_i(\boldsymbol{t})$, $i = 1, 2$, and equation (28) or (29) into account, the federal government solves

$$\max_{\{s_1, s_2, \tau\}} \mathcal{L} = \sum_{i=1}^{2} V_i + \left[\tau\overline{w}\sum_{i=1}^{2} l_i - \sum_{i=1}^{2} s_i\right]\mu + (1-\tau)\,\overline{w}\xi. \tag{30}$$

Using the symmetry of regions, the first-order conditions are composed of equations (26), and

$$s_i \;:\; \sum_{j=1}^{2}\frac{\partial V_j}{\partial t_j}\frac{\partial t_j}{\partial s_i} + \left[(1-\tau)\,\gamma_i - \epsilon_i v'(g_i)\right] k_i \frac{\partial t_i}{\partial s_i} + v'(g_i) - \mu = 0, \tag{31}$$

$$\tau \;:\; \sum_{i=1}^{2}\frac{\partial V_i}{\partial t_i}\frac{\partial t_i}{\partial \tau} + \sum_{i=1}^{2}\left[(1-\tau)\,\gamma_i - \epsilon_i v'(g_i)\right] k_i \frac{\partial t_i}{\partial \tau} + (\mu - 1)\,\overline{w}\sum_{i=1}^{2} l_i - \overline{w}\xi = 0. \tag{32}$$

Solving equations (14), (15), (21), (26), (31), and (32) yields the equilibrium under federal government leadership.

Let t^F be the capital tax rate, g^F be the public good supply, and τ^F be the labor income tax under federal government leadership. The nature of the equilibrium for $\alpha < \overline{w}l$ is summarized as follows.

Proposition 5. *When $\alpha < \overline{w}l$,*

(i) If $f_{kl} \geq 0$, the equilibrium policies are

$$\left(t^F, \tau^F, g^F\right) = \left(0, \frac{\alpha}{\overline{w}l}, g^*\right).$$

(ii) If $f_{kl} < 0$ the equilibrium policies are

$$\left(t^F, \tau^F, g^F\right) = \left(\frac{(\alpha - \overline{w}l)\,\gamma}{\beta\overline{w}l + \gamma k}, \frac{\alpha\beta + \gamma k}{\beta\overline{w}l + \gamma k}, g^*\right).$$

Proof. See Appendix F.1.

Case (i) in Proposition 5 is identical to that in Proposition 1. Case (ii) shows that federal government leadership achieves an efficient provision of public good even though the tax rates differ from the optimal tax rates. In each case, the federal government decides the labor income tax rate to maximize social welfare by considering the reactions of the state governments. Since the federal government is aware of the substitutability between the capital tax and the transfer in the state governments' response functions, the efficient level of public good supply is appropriately chosen. Based on this anticipation, the federal government should pre-commit to set τ^F to be the optimal outcome for $f_{kl} > 0$.

On the other hand, the federal government does not choose the efficient level of labor income tax rate for $f_{kl} < 0$. Since the state governments prefer to choose higher capital tax rates for $f_{kl} < 0$, the federal government has to set the labor income tax rate higher than the efficient level to keep an efficient supply of public good. In the same way as Proposition 1, we obtain[11]

$$|\gamma k| < \beta\overline{w}l \Leftrightarrow \frac{\left|\frac{t}{l}\frac{\partial l}{\partial t}\right|}{\left|\frac{t}{k}\frac{\partial k}{\partial t}\right|} = \frac{\text{Elasticity of employment rate to capital tax rate}}{\text{Elasticity of capital to capital tax rate}} < 1. \tag{33}$$

For the case of $\alpha > \overline{w}l$, the equilibrium under federal government leadership is described in the following proposition.

Proposition 6. *When $\alpha > \overline{w}l$,*

(i) If $f_{kl} \geq 0$, then the equilibrium policies are

$$\left(t^F, \tau^F, g^F\right) = \left(\frac{\alpha - \overline{w}l}{\alpha\beta + k}, 1, \frac{(\beta\overline{w}l + k)\,\alpha}{\alpha\beta + k}\right).$$

(ii)If $f_{kl} < 0$,

(a) For $|\gamma k| < \beta\overline{w}l$, the equilibrium policies are

$$\left(t^F, \tau^F, g^F\right) = \left(\frac{\alpha - \overline{w}l}{\alpha\beta + k}, 1, \frac{(\beta\overline{w}l + k)\,\alpha}{\alpha\beta + k}\right).$$

(b) For$|\gamma k| > \beta\overline{w}l$, the equilibrium policies are

$$\left(t^F, \tau^F, g^F\right) = \left(\frac{(\alpha - \overline{w}l)\,\gamma}{\beta\overline{w}l + \gamma k}, \frac{\alpha\beta + \gamma k}{\beta\overline{w}l + \gamma k}, g^*\right).$$

Proof. See Appendix F.2.

When the overall external effect is positive ((i) and (a) of (ii)), the federal government pre-commits to choose $\tau^F = 1$ because there is no alternative to improve welfare. After that, the state governments prefer to choose the tax rate lower than the level that enables the efficient provision of public good. This is because the state governments have an incentive to reduce their tax rates to attract capital. However, the equilibrium policies are different when the overall external effect is negative. If $\alpha > \overline{w}l$ and $\beta\overline{w}l < |\gamma k| < \alpha\beta$ ((b) of (ii)), the pre-commitment works well. As stated above, the state governments are motivated to increase the tax rates. Hence, federal government accomplishes the efficient level of public good supply by pre-committing to choose appropriate (negative) tax rate on labor income.

From the analysis, we establish the following propositions.

Proposition 7. *When $\alpha < \overline{w}l$,*

(i) If $f_{kl} \geq 0$, the equilibrium policies satisfy $t^F = t^*$ and $g^O < g^F = g^*$.

(ii) If $f_{kl} < 0$. then

(a) When $|\gamma k| < \alpha\beta < \beta\overline{w}l$, the equilibrium policies satisfy $t^* < t^F < t^O$ and $g^O < g^F = g^*$.

(a) When $\alpha\beta < |\gamma k| < \beta\overline{w}l$, the equilibrium policies satisfy $t^* < t^O < t^F$ and $g^F = g^* < g^O$.

[11] The condition is derived from

$$|\gamma k| < \beta\overline{w}l \Leftrightarrow -\left(-\frac{\overline{w}}{k}\frac{\partial l}{\partial t}k\right) < -\overline{w}l\frac{1}{k}\frac{\partial k}{\partial t} \quad (\because \gamma < 0) \Leftrightarrow \overline{w}\frac{\partial l}{\partial t} < -\overline{w}l\frac{1}{k}\frac{\partial k}{\partial t} \Leftrightarrow \frac{t}{l}\frac{\partial l}{\partial t} < -\frac{t}{k}\frac{\partial k}{\partial t}$$

Table 2: Equilibrium tax policy underfederal government leadership

(a) $\alpha < \overline{w}l$	$f_{kl} \geq 0$	$f_{kl} < 0, \lvert\gamma k\rvert < \alpha\beta < \beta\overline{w}l$	$f_{kl} < 0, \alpha\beta < \lvert\gamma k\rvert < \beta\overline{w}l$
g^F	$g^O < g^F = g^*$		$g^F = g^* < g^O$
t^F	$t^F = t^* = 0$	$0 < t^F < t^O$	$0 < t^O < t^F$
τ^F	$\tau^F = \tau^* < 1$	$0 < \tau^F < \tau^*$	$\tau^F < 0 < \tau^*$

(b) $\alpha > \overline{w}l$	$f_{kl} \geq 0$	$f_{kl} < 0, \lvert\gamma k\rvert < \beta\overline{w}l < \alpha\beta$	$f_{kl} < 0, \beta\overline{w}l < \lvert\gamma k\rvert < \alpha\beta$	$f_{kl} < 0, \beta\overline{w}l < \alpha\beta < \lvert\gamma k\rvert$
g^F	$g^O < g^F < g^*$		$g^O < g^F = g^*$	$g^F = g^* < g^O$
t^F	$t^F \lesseqgtr t^O$	$0 < t^F < t^O$	$0 < t^O < t^F$	$0 < t^F < t^O$
τ^F	$\tau^F = \tau^* = 1$		$\tau^F < 0 < \tau^*$	$0 < \tau^F < \tau^*$

Proof. See Appendix G.1.

Proposition 8. *When $\alpha > \overline{w}l$,*
(i) $f_{kl} \geq 0$ the equilibrium policies satisfy $t^F \lesseqgtr t^O$ and $g^O < g^F < g^*$.
(ii) $f_{kl} < 0$.
(a) If $|\gamma k| < \alpha\beta < \beta\overline{w}l$, the equilibrium policies satisfy $t^* < t^F < t^O$ and $g^O < g^F < g^*$.
(b) If $\beta\overline{w}l < |\gamma k| < \alpha\beta$, the equilibrium policies satisfy $t^* < t^O < t^F$ and $g^O < g^F = g^*$.
(c) If $\beta\overline{w}l < \alpha\beta < |\gamma k|$, the equilibrium policies satisfy $t^* < t^F = t^N < t^O$ and $g^F = g^N = g^* < g^O$.

Proof. See Appendix G.2.

Using lemmas 1 and 2, plus propositions 5 and 6, we can provide a summary of the equilibrium policies with federal government leadership. For $\alpha < \overline{w}l$ the results are displayed in Table 2 (a) and for $\alpha > \overline{w}l$ in Table 2 (b). A comparison between the results in tables 1 and 2 reveals some interesting policy implications. The analysis is also helpful for understanding the effectiveness of federal government leadership.

Consider first the case of $\alpha < \overline{w}l$. We discuss the policies for $f_{kl} < 0$ in detail to focus on the effectiveness of federal government leadership.[12] The federal government knows that there is substitutability between the capital tax and the transfer in the state governments' reactions, and that the labor income tax has no distortionary effect. Hence, the efficient provision of public good is attainable with federal government leadership because a negative tax rate on labor income is allowed in this regime. If the federal government pre-commits to an appropriate value of $\tau^F < 0$, the state governments have to choose capital tax rates $t^F > 0$ that result in the efficient provision of the public good because they are willing to increase the tax rates. Therefore, federal government leadership enables the efficient supply of the public good regardless of the unemployment exporting/importing externality.

Now consider $\alpha > \overline{w}l$. If $f_{kl} \geq 0$, or $f_{kl} < 0$ and $|\gamma k| < \alpha\beta < \beta\overline{w}l$, the equilibrium policies under federal government leadership coincide with the Nash equilibrium policies. The federal government has no choice (it has to set $\tau = 1$). Therefore, the interpretation of the results is identical to those of Proposition 4. The interesting points of Proposition 8 are that the efficient supply of the public good is attainable even if $|\gamma k| < \alpha\beta$ (it cannot be under simultaneous policy choice), and that three Nash equilibrium policies are reduced to one equilibrium policy if $|\gamma k| > \alpha\beta$.

As explained above, the federal government can choose a negative tax rate on labor income under centralized leadership. This is the key to deriving the efficient supply of the public good for $\beta\overline{w}l < |\gamma k| < \alpha\beta$. With simultaneous policy choice, the (repeated) anticipation of the governments split into two opposite directions at the middle equilibrium as the boundary for $|\gamma k| > \alpha\beta$: two extreme equilibrium policies are possibly chosen. However, the federal government leadership excludes such extreme choices. Only the policy to attain the efficient supply of the public good is a unique choice for the federal government.

[12]See the interpretation of Proposition 1 and 5 for the case where $f_{kl} \geq 0$. Federal government has no alternative to $\tau = \alpha/\overline{w}l$.

5 Conclusion

This paper has investigated the interaction of tax policies, intergovernmental transfers, and strategic timing in the presence of a labor market imperfection. The equilibrium policies are determined by the cost of the public good relative to the maximum revenue attainable from the labor income tax, and the balance of the fiscal externality and employment externality of the capital tax. The fiscal externality is always positive, but the sign of the employment externality depends on the substitutability or complementarity between capital and labor in the production function.

First, consider the case where the cost of supplying the public good can be covered by using only the labor income tax. The equilibrium tax policies under simultaneous choice are Pareto efficient if capital and labor are complementary. However, when capital and labor are substitutes, the equilibrium policy may not be the efficient policy even if the labor income tax finances the cost of the public good.

Second, assume the labor income tax alone cannot finance the cost of providing the public good. Regardless of the substitutability/complementarity between capital and labor, the equilibrium taxes will not be at the efficient levels. However, there remains a possibility that the public good is efficiently provided when the aggregate externality (the sum of the fiscal and employment externalities) is negative. On the other hand, this outcome is one of multiple equilibria and may not be realized - in which case one of the inefficient equilibria will be established.

The timing of policy choices by the governments is an important determinant of the efficiency of public good provision and the comparison of timing structures developed in the paper has some significant policy implications: First, the equilibrium policies under simultaneous policy choice are identical to those under federal government leadership if capital and labor are complements: the federal government has no need to take a leadership role to improve social welfare. Second, federal government leadership improves the efficiency of providing public goods if capital and labor are substitutes. In this case, the federal government must act as a leader to achieve an efficient outcome.

Finally, we should note some possible future extensions of this research. First, it is insightful to incorporate asymmetry of regions into our basic model. Even though symmetric regions lead to analytically clear implications, the differences in wage levels, the capital and the labor endowments are important determinants of intergovernmental transfer. Second, public inputs should be considered because employment and pecuniary externalities are associated with public input provision. Then, the state governments tend to overprovide public input exceed its optimal level.[13]

[13] See Gillet and Pauser (2018) for the detail. Ogawa et al (2006b) examined the small open economy (i.e., there is no pecuniary externality).

Appendix

A. Derivation of equations (9)–(11)

Total differentiation of (2), (3), and (8) gives

$$\begin{pmatrix} f^1_{kk} & 0 & f^1_{kl} & 0 & -1 \\ 0 & f^2_{kk} & 0 & f^2_{kl} & -1 \\ f^1_{lk} & 0 & f^1_{ll} & 0 & 0 \\ 0 & f^2_{lk} & 0 & f^2_{ll} & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} dk_1 \\ dk_2 \\ dl_1 \\ dl_2 \\ dr \end{pmatrix} = \begin{pmatrix} dt_1 \\ dt_2 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

The determinant of the coefficient matrix is

$$\Delta \equiv \begin{vmatrix} f^1_{kk} & 0 & f^1_{kl} & 0 & -1 \\ 0 & f^2_{kk} & 0 & f^2_{kl} & -1 \\ f^1_{lk} & 0 & f^1_{ll} & 0 & 0 \\ 0 & f^2_{lk} & 0 & f^2_{ll} & 0 \\ 1 & 1 & 0 & 0 & 0 \end{vmatrix} = f^2_{ll}\left[f^1_{kk}f^1_{ll} - \left(f^1_{lk}\right)^2\right] + f^1_{ll}\left[f^2_{kk}f^2_{ll} - \left(f^2_{lk}\right)^2\right] < 0.$$

Applying Cramer's rule, we obtain equations (9)–(11).

B. Derivation of equation (16)

The condition for Pareto efficiency can be obtain by solving

$$\begin{aligned} \max_{\boldsymbol{s},\boldsymbol{t},\tau} \mathcal{L} \quad &= \quad r\sigma_1\overline{k} + \pi_1 + (1-\tau)\,\overline{w}l_1 + v(t_1k_1 + s_1) \\ &\quad + \lambda\left[r\sigma_2\overline{k} + \pi_2 + (1-\tau)\,\overline{w}l_2 + v(t_2k_2 + s_2)\right] + \mu\left[\tau\overline{w}\sum_{i=1}^{2} l_i - \sum_{i=1}^{2} s_i\right]. \end{aligned}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial t_i} = \frac{\partial V_1}{\partial t_i} + \lambda\frac{\partial V_2}{\partial t_i} + \mu\tau\overline{w}\left[\frac{dl_1}{dt_i} + \frac{dl_2}{dt_i}\right] = 0 \tag{34}$$

$$\frac{\partial \mathcal{L}}{\partial s_i} = 0 \Rightarrow v'(g_1) = \mu, \lambda v'(g_2) = \mu, \tag{35}$$

$$\frac{\partial \mathcal{L}}{\partial \tau} = 0 \Rightarrow l_1 + \lambda l_2 = (l_1 + l_2)\mu, \tag{36}$$

Equation (36) yields

$$\mu = \frac{l_1 + \lambda l_2}{l_1 + l_2}. \tag{37}$$

Furthermore, equation (34) can be reduced to

$$\frac{dr}{dt_1}\left[\left(\sigma_1\overline{k} - k_1\right) + \lambda\left(\sigma_2\overline{k} - k_2\right)\right] - k_1 + (1-\lambda)(1-\tau)\,\overline{w}\frac{dl_1}{dt_1} + \mu\left[k_1 + (t_1 - t_2)\,\frac{dk_1}{dt_1}\right] = 0, \tag{38}$$

$$\frac{dr}{dt_2}\left[\left(\sigma_1\overline{k} - k_1\right) + \lambda\left(\sigma_2\overline{k} - k_2\right)\right] - \lambda k_2 - (1-\lambda)(1-\tau)\,\overline{w}\frac{dl_2}{dt_2} + \mu\left[k_2 + (t_2 - t_1)\,\frac{dk_2}{dt_2}\right] = 0. \tag{39}$$

Due to symmetry, we have $t_1 = t_2$. Then, equations (38) and (39) lead to $\lambda = 1$. Inserting $\lambda = 1$ into equation (37), we obtain $\mu = 1$. These results and equation (35) indicate $v'(g_i) = 1$.

C. Proof of Propositions 1 and 2

C.1. Proof of Proposition 1

Interior solutions for federal government's optimization problem (23): $s > 0$ and $0 < \tau < 1$ ($\zeta = \xi = 0$). Equations (24)–(26) yield $\mu = 1$ and $v'(g) = 1$. If $t > 0$, equation (21) and $v'(g) = 1$ lead to

$$(1-\tau)\,\gamma = \epsilon \Leftrightarrow t = -\,(1-\tau)\,\overline{w}\,\frac{\partial l}{\partial t}\Big/\frac{\partial k}{\partial t} = -\frac{(1-\tau)\gamma}{\beta}. \tag{40}$$

(i) $f_{kl} \geq 0$. Let be $f_{kl} > 0$. Then, $t < 0$ is derived from equation (40). This contradicts $t > 0$. Let be $t = 0$. Equations (14), (15), $t = 0$, and $v'(g) = 1$ yield

$$g^* = \alpha = \tau \overline{w} l. \tag{41}$$

From equation (41), we obtain

$$\tau^N = \frac{\alpha}{\overline{w} l}. \tag{42}$$

When $\alpha < \overline{w}l$, equation (22) does not contradict $0 < \tau^N < 1$ and $v'(g) = 1$. Thus, there is a Nash equilibrium with $t^N = 0$, equation (42), and g^*. We now assume $f_{kl} = 0$. Equation (40) leads to $t = 0$. If $\alpha < \overline{w}l$, equations (41) and (42) hold. These equations derive the same result shown above.

(ii) $f_{kl} < 0$. From equation (40), $t > 0$ is derived. Equations (14), (15), (16), and (40) yield

$$\tau^N = \frac{\alpha\beta + \gamma k}{\beta\overline{w}l + \gamma k}. \tag{43}$$

From equations (40) and (43), we obtain

$$t^N = \frac{(\alpha - \overline{w}l)\,\gamma}{\beta\overline{w}l + \gamma k}. \tag{44}$$

When $\alpha < \overline{w}l$, equation (44) shows that $\beta\overline{w}l > |\gamma k|$ is required to ensure $t^N > 0$. Since $\alpha < \overline{w}l$, $\beta\overline{w}l > \alpha\beta > |\gamma k|$ holds. Equation (43) yields $0 < \tau^N < 1$ if $\alpha\beta > |\gamma k|$. There exists a Nash equilibrium which satisfies equations (41), (43), and (44).

Corner solutions for federal government's optimization problem (23): $s = \overline{w}l$ and $\tau = 1$ ($\zeta = 0$, $\xi > 0$). Equation (25) becomes $(\mu - 1)\sum_{i=1}^{2} l_i = \xi > 0$; $\mu > 1$ holds. By equation (24), $v'(g) = \mu > 1$. For $t > 0$, equations (24) and (21) provide

$$v'(g) = \frac{1}{1+\epsilon} = \mu > 1. \tag{45}$$

Equations (15), (45), symmetry of regions, $\tau = 1$, and $v'(g) = \alpha/g$ give

$$t^N = \frac{\alpha - \overline{w}l}{\alpha\beta + k}, \tag{46}$$

$$\mu = \frac{\alpha\beta + k}{\beta\overline{w}l + k}. \tag{47}$$

Note that these equations hold for $f_{kl} \in \mathbb{R}$. When $\alpha < \overline{w}l$, $t^N < 0$ and $\mu > 1$ are derived from equations (46) and (47). These results contradict $t^N > 0$ and equation (45). There is no Nash equilibrium with $\tau = 1$ if $\alpha < \overline{w}l$.

Corner solutions for federal government's optimization problem (23): $s = 0$ and $\tau = 0$ ($\zeta > 0$, $\xi = 0$). $v'(g) = 1 - \zeta < 1$ is derived from from equations (24) and (25).

(i) $f_{kl} > 0$. For $f_{kl} > 0$, equation (21) indicates $v'(g) > 1$ for $t > 0$. These results contradict each other. For $t = 0$, $g = 0$ holds because of $\tau = s = 0$. This is also inconsistent with $v'(g) = 1 - \zeta < 1$.

For $f_{kl} = 0$, equation (21) is $v'(g) > 1$ for $t > 0$. When $t = 0$, we obtain $v'(0) = \infty$. These results contradict $v'(g) < 1$.

(ii) $f_{kl} < 0$. For $t > 0$, equation (21) is

$$v'(g) = \frac{1 + (1 - \tau)\gamma}{1 + \epsilon} = \frac{1 + \gamma}{1 - t\beta} \gtreqless 1 \Leftrightarrow |\gamma| \lesseqgtr t\beta.$$

When $\gamma > 0$ or $\gamma < 0$ and $|\gamma| < t\beta$, the equation mentioned above contradicts $v'(g) > 1$. If $\gamma < 0$ and $t\beta < |\gamma|$, $0 < v'(g) < 1$ holds. Equations (14), (15), and $\tau = 0$ provide

$$t^N = \frac{\alpha}{\alpha\beta + (1 + \gamma) k} = t^O > 0. \tag{48}$$

Equations (14), (15), (48), Lemma 2 and $\tau = 0$ give

$$g^N = g^O > g^* \text{ for } \alpha\beta < |\gamma k|. \tag{49}$$

C.2. Proof of Proposition 2

If $\alpha > \overline{w}l$, equations (46) and (47) lead to $t^N > 0$ and $\mu > 1$, which are consistent with $t^N > 0$ and equation (45). Equations (14), (15), (46), and $\tau^N = 1$ provide

$$g^N = t^N k + \overline{w}l = \frac{(\beta\overline{w}l + k)\alpha}{\alpha\beta + k}. \tag{50}$$

Therefore, the triplet of equations (46), (50), and $\tau^N = 1$ is a Nash equilibrium for $f_{kl} \in \mathbb{R}$.

In equation (44), $\beta\overline{w}l < |\gamma k|$ is necessary to be $t^N > 0$ for $\alpha > \overline{w}l$. Note that $\alpha\beta > \beta\overline{w}l$ holds if $\alpha > \overline{w}l$. Equations (43) and (44) lead to $t^N > 0$ and $0 < \tau^N < 1$ if $|\gamma k| > \alpha\beta$. Note that (41) holds. Therefore, a Nash equilibrium with $t^N > 0$, $0 < \tau^N < 1$, and g^* exists if $\gamma < 0$ and $|\gamma k| > \alpha\beta$. Furthermore, equations (48), (49), and $\tau^N = 0$ hold if $\gamma < 0$ and $|\gamma k| > \alpha\beta$. These equations also give a Nash equilibrium. Therefore, three Nash equilibria exist when $|\gamma k| > \alpha\beta$.

D. Proof of Propositions 3 and 4

D.1. Proof of Proposition 3

Case (i). From lemma 1, Lemma 2, and Proposition 1, we obtain $t^N = 0 < t^O$, $\tau^N = \tau^* > 0$, and $g^* = \alpha > g^O$.

Case (ii). $|\gamma k| < \alpha\beta$: Lemma 2 and Proposition 1 lead to $\tau^N > 0$ and $g^* = \alpha > g^O$. Since $t^N > 0 = t^*$, $\tau^N < \tau^*$ holds. Using Lemma 1 and Proposition 1, we have

$$\begin{aligned} t^N - t^O &= \frac{(\alpha - \overline{w}l)\gamma}{\beta\overline{w}l + \gamma k} - \frac{\alpha}{\alpha\beta + (1 + \gamma) k} = \frac{[\alpha\beta + (1 + \gamma) k](\alpha - \overline{w}l)\gamma - (\beta\overline{w}l + \gamma k)\alpha}{(\beta\overline{w}l + \gamma k)[\alpha\beta + (1 + \gamma) k]} \\ &= \frac{(\alpha\beta + \gamma k)\alpha\gamma - [\alpha\beta + (1 + \gamma) k]\overline{w}l\gamma - \alpha\beta\overline{w}l}{(\beta\overline{w}l + \gamma k)[\alpha\beta + (1 + \gamma) k]} \\ &= \frac{(\alpha\beta + \gamma k)[\alpha\gamma - (1 + \gamma)\overline{w}l]}{(\beta\overline{w}l + \gamma k)[\alpha\beta + (1 + \gamma) k]} < 0 \Leftrightarrow t^N < t^O. \end{aligned}$$

Furthermore, equation (17) and Proposition 1 yield

$$\tau^N - \tau^* = \frac{\alpha\beta + \gamma k}{\beta\overline{w}l + \gamma k} - \frac{\alpha}{\overline{w}l} = -\frac{(\alpha - \overline{w}l)\gamma k}{(\beta\overline{w}l + \gamma k)\overline{w}l} < 0 \Leftrightarrow \tau^N < \tau^*.$$

$|\gamma k| > \alpha\beta$: Since we have $\tau^N = 0$, $t^N = t^O > 0$, and $g^N = g^O$.

D.2. Proof of Proposition 4

Case (i). From Lemma 1 and Proposition 2, we obtain

$$\begin{aligned} t^N - t^O &= \frac{\alpha - \overline{w}l}{\alpha\beta + k} - \frac{\alpha}{\alpha\beta + (1+\gamma)k} = \frac{[\alpha\beta + (1+\gamma)k](\alpha - \overline{w}l) - (\alpha\beta + k)\alpha}{(\alpha\beta + k)[\alpha\beta + (1+\gamma)k]} = \frac{\alpha\gamma k - [\alpha\beta + (1+\gamma)k]\overline{w}l}{(\alpha\beta + k)[\alpha\beta + (1+\gamma)k]} \\ &= \frac{(\alpha - \overline{w}l)\gamma k - (\alpha\beta + k)\overline{w}l}{(\alpha\beta + k)[\alpha\beta + (1+\gamma)k]} \gtreqless 0. \end{aligned} \tag{51}$$

Equation (51) shows that t^N can be either larger or smaller than t^O. Lemma 2 and Proposition 2 provide $g^N < g^*$ and

$$\begin{aligned} g^N - g^O &= \frac{(\beta\overline{w}l + k)\alpha}{\alpha\beta + k} - \frac{\alpha k}{\alpha\beta + (1+\gamma)k} = \frac{[\alpha\beta + (1+\gamma)k](\beta\overline{w}l + k)\alpha - (\alpha\beta + k)\alpha k}{(\alpha\beta + k)[\alpha\beta + (1+\gamma)k]} \\ &= \frac{\left\{[\alpha\beta + (1+\gamma)k]\beta\overline{w}l + \gamma k^2\right\}\alpha}{(\alpha\beta + k)[\alpha\beta + (1+\gamma)k]} = \frac{\{(\alpha\beta + \gamma k)\beta\overline{w}l + (\beta\overline{w}l + \gamma k)k\}\alpha}{(\alpha\beta + k)[\alpha\beta + (1+\gamma)k]}. \end{aligned} \tag{52}$$

Equation (52) is positive for $\gamma > 0$. Therefore, we have $g^N > g^O$.

Case (ii). $|\gamma k| < \alpha\beta$: Equations (51) and (52) are derived from Lemma 1, Lemma 2, and Proposition 2. Equation (51) gives $t^N < t^O$ for $\gamma < 0$. From equation (52), the magnitude relation between g^N and g^O is ambiguous. $|\gamma k| > \alpha\beta$: There are three equilibria. First, we consider the equilibrium where $\tau^N = \tau^* = 1$. From equation (51), $t^N < t^O$ holds. Equation (52) shows $g^N < g^O$. Next, we examine the equilibrium with $\tau^N \in (0,1)$. $\tau^N < 1 = \tau^*$ holds. Lemma 1 and Proposition 2 lead to

$$\begin{aligned} t^N - t^O &= \frac{(\alpha - \overline{w}l)\gamma}{\beta\overline{w}l + \gamma k} - \frac{\alpha}{\alpha\beta + (1+\gamma)k} = \frac{[\alpha\beta + (1+\gamma)k](\alpha - \overline{w}l)\gamma - (\beta\overline{w}l + \gamma k)\alpha}{(\beta\overline{w}l + \gamma k)[\alpha\beta + (1+\gamma)k]} \\ &= \frac{(\alpha\beta + \gamma k)[(\alpha - \overline{w}l)\gamma - \overline{w}l]}{(\beta\overline{w}l + \gamma k)[\alpha\beta + (1+\gamma)k]} < 0 \Leftrightarrow t^N < t^O. \end{aligned}$$

From Lemma 2 and Proposition 2, we have $g^N = g^* < g^O$. The last case is the equilibrium with $\tau^N = 0$. Then, the result is same as the one in Proposition 3.

E. Derivation of reaction functions

Equation (21) yields

$$\begin{aligned} D_1 dt_1 &= -(1+\epsilon_1)k_1 v''(g_1)ds_1 + \overline{w}\frac{\partial l_1}{\partial t_1}d\tau, \\ D_2 dt_2 &= -(1+\epsilon_2)k_2 v''(g_2)ds_2 + \overline{w}\frac{\partial l_2}{\partial t_2}d\tau. \end{aligned}$$

These equations provide equation (28).

F. Proof of Propositions 5 and 6

F.1. Proof of Proposition 5

Interior solution for state government's optimization problem (19): Using Lemma 1, we can verify the capital tax rate as

$$t = \frac{\alpha - [1 + (1-\tau)\gamma]\tau\overline{w}l}{\alpha\beta + [1 + (1-\tau)\gamma]k}.$$

From Assumption 1, $t > 0 \Leftrightarrow \alpha - [1 + (1-\tau)\gamma]\tau\overline{w}l > 0$. Combined equation (32) with equation (31), we obtain

$$\left[(\mu - 1)l - \frac{\xi}{2}\right]\overline{w}\frac{\partial t}{\partial s} = [v'(g) - \mu]\frac{\partial t}{\partial \tau}. \tag{53}$$

Interior solution for federal government's optimization problem (30): $\tau < 1$ $(\xi = 0)$. For the interior solution $(t > 0)$ of (19), equations (21), (31), and (32) with $\xi = 0$ yield simultaneous (non-linear) equations system composed of equation (21),

$$\begin{aligned} [(1-\tau)\,\gamma + \beta t]\, k\frac{\partial t}{\partial s} + 1 + (1-\tau)\,\gamma - (1-\beta t)\,\mu &= 0, & (54) \\ \sum_{i=1}^{2} [(1-\tau)\,\gamma + \beta t]\, k\frac{\partial t}{\partial \tau} + (1-\beta t)\,(\mu - 1)\,\overline{w}\sum_{i=1}^{2} l &= 0. & (55) \end{aligned}$$

One of the solution for the system is $(1-\tau)\,\gamma + \beta t = 0$.

(i) $f_{kl} \geq 0$. First, we consider the case where $f_{kl} > 0$. Equation (53) provides $v'(g) \gtreqless \mu \Leftrightarrow \mu \lesseqgtr 1$. From $t > 0$, $\tau < 1$, and $\gamma > 0$, $(1-\tau)\,\gamma + \beta t > 0$ holds. From equation (21), we obtain $v'(g) > 1 > \mu$. This result cannot be an equilibrium because they create a contradiction: equations (28), (54), and (55) produce

$$\begin{aligned} 0 &= k\frac{\partial t}{\partial s} + 1 + \frac{k}{\overline{w}l}\frac{\partial t}{\partial \tau} \\ &= \frac{(1+\epsilon_i)\,k^2 v''(g_i)}{D_i} + 1 + \frac{k}{l}\frac{1}{D_i}\frac{\partial l_i}{\partial t_i} > 0. \end{aligned}$$

We now consider the case where $f_{kl} = 0$. Equation (31) and (32) leads to $v'(g) = \mu = 1$. Using equation (31), $f_{kl} = 0$, and $\mu = 1$, we have

$$v'(g) = \frac{1}{1 - \epsilon k \frac{\partial t}{\partial s}}.$$

When $f_{kl} = 0$, $t = 0$ is the interior solution of (19). From the above equation and equation (21) with $t = 0$, we have $v'(g) = 1$. Equations (14), (15), (16), $t^F = 0$, and $v'(g) = 1$ yield

$$\tau^F = \frac{\alpha}{\overline{w}l}. \tag{56}$$

From equation (56), we obtain $\tau^F < 1$ for $\alpha < \overline{w}l$. This result shows (i) in Proposition 5 for $f_{kl} = 0$.

(ii) $f_{kl} < 0$. From equation (53), we have $v'(g) \gtreqless \mu \Leftrightarrow \mu \gtreqless 1$. Note that equation (40) holds. Hence, we have

$$t^F = \frac{(\alpha - \overline{w}l)\,\gamma}{\beta\overline{w}l + \gamma k}, \tau^F = \frac{\alpha\beta + \gamma k}{\beta\overline{w}l + \gamma k}. \tag{57}$$

Equation (57) and $|\gamma k| < \alpha\beta < \beta\overline{w}l$ lead to $t^F > 0$ and $\tau^F \in (0,1)$. If $\alpha\beta < |\gamma k| < \beta\overline{w}l$, equation (57) yields $t^F > 0$ and $\tau^F < 0$. For $\alpha\beta < \beta\overline{w}l < |\gamma k|$, $t^F < 0$ is obtained from equation (57). This makes a contradiction.

Corner solution for federal government's optimization problem (30): $\tau = 1$ $(\xi > 0)$. From Lemma 1, equations (31), and (32), we have $v'(g) > 1$, and

$$t^F = \frac{\alpha - \overline{w}l}{\alpha\beta + k}. \tag{58}$$

For $\alpha < \overline{w}l$, equation (58) is negative. Therefore, this contradicts the assumption of $t > 0$.

Corner solution for state government's optimization problem (19): $v'(g) \leq 1 + (1-\tau)\gamma$ holds and the optimization problem (30) can be reduced to (23) with $\zeta = 0$. Since $t = 0$, $\tau \geq 0$ is necessary to be $g \geq 0$.

Interior solution for federal government's optimization problem (30): $\tau < 1$ $(\xi = 0)$. From equations (24)–(26), we obtain $\mu = 1$ and $v'(g) = 1$. Equation (56) and $t^F = 0$ are derived from equations (14) and (15). *(i)* $f_{kl} > 0$. Then, we have $v'(g) \leq 1 \leq 1 + (1-\tau)\gamma$. This inequality is consistent with $v'(g) = 1$. Therefore, the result (i) in Proposition 1 for $f_{kl} > 0$ is alive if $\alpha < \overline{w}l$. *(ii)* $f_{kl} < 0$. $v'(g) \leq 1 + (1-\tau)\gamma < 1$ holds. $v'(g) = 1$ contradicts this inequality.

Corner solution for federal government's optimization problem (30): $\tau = 1$ ($\xi > 0$). For $t = 0$, $v'(g) \leq 1$ from (22). On the other hand, equations (31) and (32) lead to $v'(g) > 1$. This makes a contradiction.

F.2. Proof of Proposition 6

Interior solution for state government's optimization problem (19). *Interior solution for federal government's optimization problem (30):* $\tau < 1$ ($\xi = 0$).

(i) $f_{kl} \geq 0$. For $f_{kl} > 0$, the result is same as the one shown in the proof of Proposition 5. If $f_{kl} = 0$, equation (56) leads to $\tau > 1$. This makes a contradiction.

(ii) $f_{kl} < 0$. Equation (57) holds. If $\alpha\beta > |\gamma k| > \beta\overline{w}l$, we have $t^F > 0$ and $\tau^F < 0$. If $\alpha\beta > \beta\overline{w}l > |\gamma k|$, $t^F < 0$ from equation (57). This result contradicts the assumption of $t > 0$. For $|\gamma k| > \alpha\beta > \beta\overline{w}l$, equation (57) provides $t^F > 0$ and $\tau^F \in (0, 1)$.

Corner solution for federal government's optimization problem (30): $\tau = 1$ ($\xi > 0$). From Lemma 1, equations (31), and (32), we have $v'(g) > 1$, equation (58),

$$\begin{aligned} \epsilon v'(g)k\frac{\partial t}{\partial s} &= v'(g) - \mu, \\ \epsilon v'(g)k\frac{\partial t}{\partial \tau} &= (\mu - 1)\,\overline{w}l - \frac{\overline{w}\xi}{2}. \end{aligned}$$

These equations yield

$$v'(g) = \frac{\mu}{1 - \epsilon k\frac{\partial t}{\partial s}} = \frac{1}{1+\epsilon} \Leftrightarrow \mu = \frac{1 - \epsilon k\frac{\partial t}{\partial s}}{1+\epsilon}.$$

(i) $f_{kl} \geq 0$. For $f_{kl} > 0$, equation (21) provides

$$t^F = \frac{\alpha - \overline{w}l}{\alpha\beta + k} > 0 \text{ and } g^F = \left(\frac{\beta wl + k}{\alpha\beta + k}\right)\alpha < g^* \text{ for } \alpha > \overline{w}l.$$

Then, the values of μ and ξ are determined by equations (31) and (32). Furthermore, this result holds if $f_{kl} = 0$. *(ii)* $f_{kl} < 0$. All results of *(i)* holds.

Corner solution for state government's optimization problem (19). As shown in Proof of Proposition 5, there is no equilibrium outcome when $\alpha > \overline{w}l$.

G. Proof of Propositions 7 and 8

G.1. Proof of Proposition 7

Case (i). The outcome is same as that of Proposition 3. *Case (ii).* From Lemma 1 and Proposition 5, we have

$$t^F - t^O = \frac{(\alpha - \overline{w}l)\,\gamma}{\beta\overline{w}l + \gamma k} - \frac{\alpha}{\alpha\beta + (1+\gamma)\,k} = \frac{(\alpha\beta + \gamma k)\,[\alpha\gamma - (1+\gamma)\,\overline{w}l]}{(\beta\overline{w}l + \gamma k)\,[\alpha\beta + (1+\gamma)\,k]}. \tag{59}$$

When $|\gamma k| < \alpha\beta < \beta\overline{w}l$, equation (59) is negative: $0 < t^F < t^O$. If $\alpha\beta < |\gamma k| < \beta\overline{w}l$, equation (59) is positive $0 < t^O < t^F$. Furthermore, Lemma 2 and Proposition 5 lead to

$$g^F = g^* \gtreqless g^O \Leftrightarrow |\gamma k| \gtreqless \alpha\beta. \tag{60}$$

G.2. Proof of Proposition 8

Case (i). The results of Proposition 4 hold. *Case (ii)*. Lemma 1 and Proposition 6 provide equation (59) for $\beta\overline{w}l < |\gamma k|$ and

$$\begin{aligned} t^F - t^O &= \frac{\alpha - \overline{w}l}{\alpha\beta + k} - \frac{\alpha}{\alpha\beta + (1+\gamma)\,k} = \frac{[\alpha\beta + (1+\gamma)\,k]\,(\alpha - \overline{w}l) - (\alpha\beta + k)\,\alpha}{(\alpha\beta + k)\,[\alpha\beta + (1+\gamma)\,k]} = \frac{\alpha\gamma k - [\alpha\beta + (1+\gamma)\,k]\,\overline{w}l}{(\alpha\beta + k)\,[\alpha\beta + (1+\gamma)\,k]} \\ &= \frac{(\alpha - \overline{w}l)\,\gamma k - (\alpha\beta + k)\,\overline{w}l}{(\alpha\beta + k)\,[\alpha\beta + (1+\gamma)\,k]} \text{ for } \beta\overline{w}l > |\gamma k|\,. \end{aligned} \tag{61}$$

For $\beta\overline{w}l < |\gamma k| < \alpha\beta$, equation (59) is positive: $0 < t^O < t^F$. For $\beta\overline{w}l < \alpha\beta < |\gamma k|$, equation (59) is negative: $0 < t^F < t^O$. If $|\gamma k| < \beta\overline{w}l < \alpha\beta$, equation (61) yields $0 < t^F < t^O$. From Lemma 2 and Proposition 6, we obtain equation (59) for $\beta\overline{w}l < |\gamma k|$ and

$$\begin{aligned} g^F - g^O &= \frac{(\beta\overline{w}l + k)\,\alpha}{\alpha\beta + k} - \frac{\alpha k}{\alpha\beta + (1+\gamma)\,k} = \frac{\left\{[\alpha\beta + (1+\gamma)\,k]\,\frac{\beta\overline{w}l}{\gamma k} + k\right\}\alpha\gamma k}{(\alpha\beta + k)\,[\alpha\beta + (1+\gamma)\,k]} \\ &= \frac{\alpha\gamma k}{(\alpha\beta + k)\,[\alpha\beta + (1+\gamma)\,k]}\left[(\alpha\beta + \gamma k)\,\frac{\beta\overline{w}l}{\gamma k} + \left(1 + \frac{\beta\overline{w}l}{\gamma k}\right)k\right] \text{ for } \beta\overline{w}l > |\gamma k|\,. \end{aligned} \tag{62}$$

If $\gamma > 0$, equation (62) is positive: $g^F > g^O$. If $\gamma < 0$ and $\beta\overline{w}l > |\gamma k|$, equation (62) is positive: $g^F > g^O$.

References

[1] Akai, N. and M. Sato (2008), Too big or too small? A synthetic view of the commitment problem of interregional transfers, *Journal of Urban Economics* 64, 551–559.

[2] Altshuler R. and T.J. Goodspeed (2015), Follow the Leader? Evidence on European and US tax competition, *Public Finance Review*, 43 (4), 485–504.

[3] Aronsson, T. and S. Wehke (2008), Public goods, unemployment and policy coordination, *Regional Science and Urban Economics* 38, 285–298.

[4] Baskaran, T. and M. Lopes da Fonseca (2013), The economics and empirics of tax competition: a survey, Discussion Papers, Center for European Governance and Economic Development Research, No. 163.

[5] Bettendorf, L., A. Horst and R. De Mooij (2009), Corporate tax policy and unemployment in Europe: an applied general equilibrium analysis, *World Economy* 32, 1319–1347.

[6] Boadway, R. (2006), Intergovernmental redistribute transfers: efficiency and equity, In: Ahmad, E. and G. Brosio (eds.) *Handbook of Fiscal Federalism*, Edward Elgar, Northampton.

[7] Boadway, R. and M.J. Keen (1996), Efficiency and the optimal direction of federal-state transfers, *International Tax and Public Finance* 3, 137–155.

[8] Boadway, R., M. Marchand, and M. Vigneault (1998), The consequences of overlapping tax bases for redistribution and public spending in a federation, *Journal of Public Economics* 68, 453–478.

[9] Boadway, R. and A. Shah (eds.) (2007), *Intergovernmental Fiscal Transfers: Principles and Practices*, The World Bank, Washington DC.

[10] Breuillé, M.-L., T. Madiès, and E. Taugourdeau (2010), Gross versus net equalization scheme in a federation with decentralized leadership, *Journal of Urban Economics* 68, 205–214.

[11] Bucovetsky S. and M. Smart. (2006), The efficiency consequences of local revenue equalization: tax competition and tax distortions, *Journal of Public Economic Theory* 8, 119-144.

[12] Caputo, E. and D. Silva (2014), Selective decentralized leadership, *Journal of Urban Economics* 83, 1–5.

[13] Caputo, E. and D. Silva (2015), Efficient earmarking under decentralized fiscal commitments, *International Tax and Public Finance* 22, 683–701.

[14] Caputo, E. and D. Silva (2017), Tax competition and federal equalization schemes with decentralized leadership, *International Tax and Public Finance* 24, 164–178.

[15] Chirinko, R.S. and D.J. Wilson (2008) State investment tax incentives: a zero-sum game? *Journal of Public Economics* 92, 2362–2384.

[16] Dahlby, B. (1996), Fiscal externalities and the design of intergovernmental grants, *International Tax and Public Finance* 3, 397–412.

[17] Dahlby, B. and N.A. Warren. (2003), Fiscal incentive effects of the Australian equalization system, *Economic Record* 79, 434–445.

[18] DePeter, J. and G.M. Myers. (1994), Strategic capital tax competition: a pecuniary externality and a corrective device, *Journal of Urban Economics* 36, 66–78.

[19] Devereux, M.P., B. Lockwood, and M. Redoano (2008), Do countries compete over corporate tax rates? *Journal of Public Economics* 92, 1210–1235.

[20] Egger, P., M. Köethenbüerger, and M. Smart (2010), Do fiscal transfers alleviate business tax competition? Evidence from Germany, *Journal of Public Economics* 94, 235–246.

[21] Eichner, T. and T. Upmann (2012), Labor markets and capital tax competition, *International Tax and Public Finance* 19, 203–215.

[22] Exbrayat, N., C. Gaign, and S. Riou (2012), The effects of labour unions on international capital tax competition, *Canadian Journal of Economics* 45, 1480–1503.

[23] Feld, L. and G. Kirchgassner (2002), The impact of corporate and personal income taxes on the location of firms and on employment: some panel evidence for the Swiss cantons, *Journal of Public Economics* 87, 129–155.

[24] Feldmann, H. (2011), The unemployment puzzle of corporate taxation, *Public Finance Review* 39, 743–769.

[25] Felix, R.A. (2009), Do state corporate income taxes reduce wages? *Economic Review, Federal Reserve Bank of Kansas City* 94 (2), 77–102.

[26] Figuieres, C., J. Hindriks, and G.D. Myles (2004), Revenue sharing versus expenditure sharing in a federal system, *International Tax and Public Finance* 11, 155–174.

[27] Gillet, H. and J. Pauser (2018), Public input provision in asymmetric regions with labor market imperfections, *German Economic Review* 19, 466–492.

[28] Harden, J.W. and W.H. Hoyt (2003), Do states choose their mix of taxes to minimize employment losses? *National Tax Journal* 56, 7–26.

[29] Hayashi, M. and R. Boadway (2001), An empirical analysis of intergovernmental tax interaction: The case of business income taxes in Canada, *Canadian Journal of Economics* 34, 481–503.

[30] Keen, M. and K.A. Konrad (2013), The theory of international tax competition and coordination: In *Handbook of Public Economics* 5, Auerbach, A.J., Chetty, R., Feldstein, M. and E. Saez (ed.)

[31] Kikuchi, Y. and T. Tamai (2019), Tax competition, unemployment, and intergovernmental transfers, forthcoming in *International Tax and Public Finance*.

[32] Köethenbüerger, M. (2002), Tax competition and fiscal equalization, *International Tax and Public Finance* 9, 391–408.

[33] Köethenbüerger, M. (2004), Tax competition in a fiscal union with decentralized leadership, *Journal of Urban Economics* 55, 498–513.

[34] Köthenbüerger, M. (2008), Federal tax-transfer policy and intergovernmental pre-commitment, *Regional Science and Urban Economics* 38, 16–31.

[35] OECD (2017), *Tax Policy Reforms 2017: OECD and Selected Partner Economies*, OECD Publishing, Paris.

[36] Ogawa, H., Y. Sato, and T. Tamai (2006a), A note on unemployment and capital tax competition, *Journal of Urban Economics* 60, 350–356.

[37] Ogawa, H., Y. Sato, and T. Tamai (2006b), Tax competition and public input provision with imperfect labor markets, DEE Discussion Paper No. 06-2, Nagoya University.

[38] Sato, Y. (2009), Capital tax competition and search unemployment, *Papers in Regional Science* 88, 749–764.

[39] Wildasin, D.E. (1989), Interjurisdictional capital mobility: fiscal externality and a corrective subsidy, *Journal of Urban Economics* 25, 193–212.

[40] Wildasin D.E. and J.D. Wilson (2004), Capital tax competition: Bane or boon? *Journal of Public Economics* 88, 1065–1091.

[41] Wilson J.D. (1986), A theory of inter-regional tax competition, *Journal of Urban Economics* 19, 296–315.

[42] Zodrow G.R. (2010), Capital mobility and tax competition, *National Tax Journal* 63, 865–901.

[43] Zirgulis, A. and T. Šarapovas (2017), Impact of corporate taxation on unemployment, *Journal of Business Economics and Management* 18, 412–426.

[44] Zodrow G.R. and P. Mieszkowski (1986), Pigou, Tiebout, property taxation, and the underprovision of local public goods, *Journal of Urban Economics* 19, 356–370.