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Bargaining in Two-Sided Markets*

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Abstract

Negotiations regularly take place on the business-to-business side of two-sided markets. However, little is known about the consequences of these negotiations on participation, prices, and welfare when indirect network externalities exist between the business side and the consumer side. To fill this gap, we propose a tractable model of bargaining in two-sided markets where participating firms pay an entry fee and concession fee that is determined through negotiations with the platform. First, we show that as a platform has greater bargaining power, the concession fee and consumer entry fee rise, but the entry fee to firms goes down. In this case, more firms participate on the platform so that consumers are better off (even though they pay a higher entry fee), and firms with large network externalites are worse off (due to greater exploitation by the platform) while firms with small network externalites are better off. Altogether, greater platform bargaining power increases total welfare. We also show that these results are largely robust when considering platform competition using a Hotelling framework with bargaining.

Keywords: Indirect network externalities, platforms, two-sided markets, Nash cooperative bargaining
JEL Classifications: D42, D43, D62, L12, L13, L22

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1 Introduction

Transactions between oligopolistic firms are often determined through negotiations. *Two-sided markets* are no exception: in many cases, platforms post a final price for end consumers on one side, a standard business-to-consumer or (B2C) relationship, whereas the other side is often characterized as a business-to-business (B2B) relationship where price negotiations may take place between the platform and its firms who benefit from the consumer side through *indirect network externalities*. For example, it is well known that Apple Music and Spotify, two major music streaming platforms, negotiate with music labels.[1] Similarly, Netflix, a major video streaming platform, also negotiates with film producers and TV networks. Another example can be found in the cable television industry where programming distributors and content providers bargain over affiliate fees (Crawford and Yurukoglu (2012), Boik (2016), Crawford et al. (2018), and Goetz (2019)).[2] Furthermore, large retailers are attempting to negotiate with credit card companies over merchant fees.[3]

In spite of the prevalence of bargaining on the B2B side of two-sided markets, little is known about how these negotiations impact platform participation, prices, and welfare. To fill the gap, we study this issue by proposing a model of bargaining similar to that of Horn and Wolinsky (1988), but in the context of two-sided markets.[4] First, we study the case of a monopoly platform and then we consider platform competition using a Hotelling framework that includes heterogeneous network effects and bargaining between platforms

[1]See https://www.wsj.com/articles/spotify-seeks-to-fine-tune-music-rights-as-it-gears-up-for-ipo-1471983753 (dated August 23, 2016) and https://www.bloomberg.com/news/articles/2017-06-21/apple-said-to-seek-lower-rate-in-new-deals-with-record-labels (dated June 22, 2017). As a side note, Apple Music is currently acquiring Spotify's rival firm, Shazam.

[2]The market for video games provides another example of a two-sided market where the platform negotiates with the content side (video game publishers) over pricing and royalties.

[3]See http://nymag.com/intelligencer/2018/10/are-other-peoples-credit-card-rewards-costing-you-money.html (dated October 16, 2018).

[4]Note that indirect network externalities are not directly modeled in the aforementioned studies by Crawford and Yurukoglu (2012), Boik (2016), Crawford et al. (2018), and Goetz (2019). Although it is well recognized that it is difficult to estimate their effects: see Rysman (2019).

and firms. Across the two market structures, we find that the impact of bargaining is largely consistent.

We find that as a platform has greater bargaining power, the platform extracts more surplus from firms through a higher concession fee. However, this entails its own cost due to the network externalities: a lower size of participating firms will make the platform less attractive for end consumers. To combat this effect, the platform reduces its entry fee to firms so that firm participation actually increases when the platform has greater bargaining power. With more firms, consumers find the platform more attractive so that the consumer entry fee also increases when platform bargaining power is larger.

In terms of welfare, consumers benefit from greater platform bargaining power; that is, the additional network benefits from a greater number of firms is more than enough to compensate consumers for the higher entry fee. For firms, the results are mixed. Greater platform bargaining power results in a lower entry fee; however, the concession fee determined through bargaining becomes higher. Since firms with larger network benefits are more exploited in negotiations (they have more to lose), greater platform bargaining power dichotomizes firm welfare effects so that firms with larger network effects are harmed, but firms with smaller network effects actually benefit (the lower entry fee compensates for the higher losses in negotiations).

Another way to interpret our results is that greater bargaining power by firms disincentivizes the platform from promoting network growth which results in a platform network that is inefficiently small. As bargaining power shifts to the platform, the platform captures a greater share of the network surpluses that are generated and this creates an incentive for a larger network, benefiting the platform, consumers, and firms on the fringe of participation.

Since Rochet and Tirole (2003), Armstrong (2006), and Hagiu (2006), the literature on two-sided markets has emphasized how platforms are able to keep a balance between the two sides of the market by offering a price reduction on one side whenever they increase the price on the other side. In contrast, we find that when a platform is better able to exploit one side (which might seem *anti*-competitive), the platform is *better able to internalize the network externalities*. As a result, the two sides are better coordinated by this seemingly anti-competitive conduct, and thus social welfare is enhanced.

One paper that is similar to ours is Hagiu and Lee (2011). They consider an environment where two platforms bargain with content providers through a bidding game where the platforms simultaneously submit two bids to all content providers: a bid to be exclusive on one platform and a bid that allows content providers to multi-home. They find that the content providers decision depends on whether or not the content provider maintains control over the price of their content. Our paper differs from Hagiu and Lee (2011) in several respects; instead of considering the decision of control, we develop a model where bargaining power is explicitly included to consider how negotiations impact platform pricing, participation, and welfare for both platform competition as well as for a monopoly platform.[5]

Another paper that overlaps with ours is Chao and Derdenger (2013). We both show that seemingly anti-competitive behaviors may be welfare-enhancing in the context of two-sided markets. They study mixed bundling (a type of quantity discount for purchasing multiple products, such as purchasing a video game console bundled with a video game) in two-sided markets, and show that allowing for mixed bundling results in a lower price for the stand-alone platform (the stand-alone console price). This occurs because while consumers that already own a console are exploited, facing a higher price for a video game,

[5]There is also a well developed literature on bilateral bargaining within large networks in agents: Segal (1999), Segal and Whinston (2003), and De Fontenay and Gans (2014). These papers analyze bargaining when bilateral or multilateral negotiations generate externalities across agents. Thus, it is difficult to argue how the main results can be applied to two-sided markets with indirect network externalities.

mixed bundling enables more consumers to join the platform and this greater consumer participation benefits firms on the other side (i.e., game producers). This result is similar to ours where, through multiple tariffs (entry and negotiating fees on the B2B side instead of mixed bundling on the B2C side), the platform is able to exploit firms (instead of consumers) in a manner that maintains high levels of firm participation.

In this sense, our paper complements that of Chao and Derdenger (2013) by showing that caution should be taken when considering the seemingly anti-competitive effects of platform conduct on either side of the market. Furthermore, while Chao and Derdenger (2013) focus solely on a monopoly platform, we show that the overlapping results are robust to platform competition. In addition, our setting of bargaining allows for additional results relating to how changes to bargaining power impact platform pricing, participation, and welfare.[6]

The rest of the paper is organized as follows. Section 2 introduces a model of bargaining with a monopoly platform, characterizes the equilibrium, and analyzes the effects of bargaining power. Then, in Section 3, we extend the monopoly model to a setting of platform competition and show that our main results are largely robust across the two market structures. Finally, Section 4 concludes the paper.

[6]Ishihara and Oki (2017) also incorporate bargaining aspects in the context of two-sided platforms, and analyze how a single monopoly content provider determines the amount of exclusive content to provide for each duopolistic platform in a setting where this content provider can multi-home. The key factor that Ishihara and Oki (2017) discover is how the content provider's bargaining power is affected by the pros and cons of exclusive content provision. In this paper, we consider a richer setting of bargaining where platform bargaining power impacts platform fees and the participation of both consumers and firms.

2 A Monopoly Platform

2.1 The Model

We start with the situation where a two-sided market is served by a monopoly platform. There are two distinct sides (consumers on Side 1 and firms on Side 2), and both of them are connected via mediated agents known as platforms. On Side 1, there are heterogeneous consumers who have types $\theta_1 \in [0,1]$ distributed according to $F_1(\cdot)$. Consumers outside option is valued at 0. The utility that a consumer receives from joining the platform is given by

$$u_1(\theta_1) = \alpha_1(\theta_1)n_2 - p_1, \tag{1}$$

where $n_2 \in [0,1]$ is the number of firms on Side 2 and p_1 is the price or entry fee paid by consumers for participating on or joining the platform. The $\alpha_1(\theta_1)$ term expresses the intensity of the *indirect network externalities* which can be positive or negative.[7] We assume that $\alpha_1(\cdot)$ is decreasing and $\mathcal{C}^2$. A decreasing $\alpha_1(\cdot)$ naturally orders consumers so that consumers of type θ_1 close to zero are those consumers with greater network effects compared to consumers of type θ_1 far from zero who have smaller network effects.

The opposite side consists of heterogeneous firms whose outside options are valued at zero. A firm with type $\theta_2 \in [0,1]$, distributed according to $F_2(\cdot)$, has utility given by

$$u_2(\theta_2) = \alpha_2(\theta_2)n_1 - p_2 - w(\theta_2), \tag{2}$$

where $n_1 \in [0,1]$ is the number of consumers and p_2 is the entry fee that firms pay to the platform. Firms always benefit from more consumers on the platform so that $\alpha_2(\cdot) > 0$.

[7]For example, consumers clearly benefit from greater video games which implies $\alpha_1(\cdot) > 0$. However, some consumers might dislike advertisements ($\alpha_1(\cdot) > 0$) or some consumers might enjoy advertisements while others do not, in which case $\alpha_1(\cdot) \gtreqless 0$.

Similar to consumers, $\alpha_2(\cdot)$ is decreasing and $\mathcal{C}^2$, which implies that firms are equipped with the same order as consumers: a type θ_2 close to zero represents a firm with greater network benefits compared to a firm whose type is far from zero. Unlike consumers, firms pay an additional concession fee, $w(\theta_2)$, that is negotiated between each firm and the platform (we model the bargaining process explicitly below).

The platform earns profit which is given by

$$\Pi = n_1 \cdot (p_1 - c_1) + n_2 \cdot (p_2 - c_2) + \int_0^{n_2} w(\theta_2) dF_2(\theta_2),$$

where c_1 (c_2) is the marginal cost to the platform for an additional consumer (firm).[8]

The timing of the game is as follows. First the platform sets prices p_1 and p_2. Firms observe these prices and make their entry decision to pay p_2 and initiate negotiations with the platform; if a firm pays the entry fee, then the platform and the firm negotiate $w(\theta_2)$ and the firm makes their participation decision. Finally, consumers observe prices and the number of firms that join the platform and make participation decisions. The motivation for both the game's timing and for the firms incurring two prices (a bargained fee and a fixed fee) stems from the fact that endogenous firm entry is critical for platform markets as it effects consumer decisions which impacts the consumer price. Thus, to disentangle the bargaining game from endogenous firm entry, we incorporate a firm entry fee, p_2, which endogenously determines firm participation but is sunk to both the firms and the platform in the bargaining game.[9] This enables us to consider bargaining with endogenous entry of consumers and firms which is a critical feature of two-sided markets.[10]

[8] For simplicity, fixed costs are assumed to be zero.

[9] For example, in the video game market the p_2 denotes the entry price paid to the platform for a game developer to obtain the platforms software development kit; then, after game development, the game developer and platform negotiate the royalty represented here by $w(\cdot)$. For TV networks or magazines, the p_2 is an initial payment for a certain amount of advertising, and $w(\cdot)$ is the negotiated payment that occurs later to ensure the advertisements are paired with certain shows or articles.

[10] Without such a setup, bargain between firms and the platform would require exogenously given firm entry, which would eliminate the interaction between platform participation and platform prices. One could

2.2 Equilibrium Analysis

Our solution concept is the subgame perfect Nash equilibrium (SPNE). We solve the game using backward induction. Starting with consumers, for given values of n_2 and p_1, we see that setting Equation (1) equal to zero gives consumer inverse demand for the platform:

$$p_1(n_1) = \alpha_1(\theta_1')n_2, \tag{3}$$

where $\theta_1' = F_1^{-1}(n_1)$.

Now consider the bargaining subgame between the platform and a firm of type θ_2 that joins the platform. We model this bargaining process as a Nash cooperative game (Nash (1950); Horn and Wolinsky (1988); Stole and Zwiebel (1996a); Stole and Zwiebel (1996b)). The negotiation between the platform and firm θ_2 begins after p_2 was already paid as an entry fee (i.e., p_2 is sunk). At the negotiation stage, the participation of a θ_2 type firm on the platform (i) impacts the platform's revenues from consumers and (ii) generates $w(\theta_2)$ revenue. Thus, the marginal gain to the platform from a firm of type θ_2 is given by: $\frac{\partial(p_1 n_1)}{\partial n_2} + w(\theta_2) = \frac{\partial(\alpha_1(n_1)n_2 n_1)}{\partial n_2} + w(\theta_2) = \alpha_1(n_1)n_1 + w(\theta_2)$.[11] In contrast, the marginal benefit to a firm of type θ_2 from participating with the platform is given by $\alpha_2(\theta_2)n_1 - w(\theta_2)$.

In a cooperative model of bilateral bargaining, a player's bargaining position is determined by (i) bargaining weights assigned to each player, and (ii) the marginal benefits to each player from an agreement. More specifically, the Nash bargaining solution implies that the concession fee is determined by maximizing the Nash product of the marginal

think that this modeling is a concise way of capturing periodical negotiations in long-term relationships: firms pay an initial fee to start a relationship with a platform, and then they bargain over transaction fees on spot.

[11]Here, it is assumed that p_1 adjusts to the (off-the-equilibrium) breakdown according to $p_1 = \alpha_1(n_1)n_2$. If p_1 is treated as fixed, the expressions for n_1^* and n_2^* would be very complicated. It is also assumed that when the platform negotiates with content provider θ_2, each party assumes that other negotiations settle successfully (McAfee and Schwartz (1994)). Collard-Wexler et al. (2019) provide a non-cooperative foundation for this modeling.

gains:

$$\max_{w(\theta_2)} [\alpha_1(\theta_1')n_1 + w(\theta_2)]^{\lambda} [\alpha_2(\theta_2)n_1 - w(\theta_2)]^{1-\lambda},$$

where $\lambda \in [0,1]$ captures the Nash bargaining weight for the platform. Thus, the concession fee is for a firm of type θ_2 is given by

$$w(\theta_2) = [\lambda\alpha_2(\theta_2) - (1-\lambda)\alpha_1(\theta_1')]n_1. \tag{4}$$

If the platform has full bargaining power (i.e., $\lambda = 1$), then it exploits full surplus from the firm: $w(\theta_2) = \alpha_2(\theta_2)n_1$. Note that if $\lambda = 1$ so that $w(\theta_2) = \alpha_2(\theta_2)n_1$, then no firm will pay the concession fee, p_2, and enter into negotiations with the platform unless $p_2 \leq 0$. On the other hand, if firms have full bargaining power (i.e., $\lambda = 0$), then the net payment from the firm to the platform is negative, i.e, the platform refunds $\alpha_1(\theta_1')n_1$ to the firm. This can be understood as a firm specific discount on the entry fee p_2.[12]

Given n_1, p_2, and the negotiated $w(\theta_2)$, firms make their initial entry decision by setting Equation (2) equal to zero which generates the firm inverse demand for the platform:

$$p_2(n_2) = (1-\lambda)[\alpha_1(\theta_1') + \alpha_2(\theta_2')]n_1, \tag{5}$$

where $\theta_2' = F_2^{-1}(n_2)$. Finally, taking consumer and firm demands as given, the platform chooses prices p_1 and p_2 to maximize profit.[13]

To determine closed form solutions to the platform's problem, the distribution of agents and the structure of the network effects must be specified. For simplicity, let network effects be linear with respect to agent types and let agents be distributed uniformly on $[0,1]$ so that $\alpha_i(\theta_i) = (a_i - \theta_i)$ for $i = 1,2$. In addition, we have $a_2 \geq 1$ so that all firms

[12]In this way, p_2 captures the "going rate," while $w(\theta)$ entails the discount or markup from negotiations.

[13]Equivalently, the platform maximizes profit with respect to n_1 and n_2, taking the inverse demand functions as given.

earn positive profits from consumers. Note that on the consumer side we still allow for a variety of network benefit structures. For example, $a_1 \geq 1$ implies all consumers benefit from firms (the video game market), $a_1 \leq 0$ implies all consumers are harmed by firms (advertisements are a nascence), or $a_1 \in (0, 1)$ implies some consumers benefit while others are harmed by firms (some advertising markets). In this way, the network externalities are simply captured by two parameters: a_1 and a_2. For simplicity, let the platform marginal costs be zero ($c_1 = c_2 = 0$). While this assumption might not be realistic for certain platform markets, this simplification implies that a price below zero represents a platform subsidy while a price greater than zero corresponds to a markup.

Solving the platform's problem we have the following result:

Proposition 1. *There exists a unique platform equilibrium:*

$$n_1^* = \frac{1}{3}(a_1 + a_2),\ n_2^* = \frac{2(a_1 + a_2)}{3(2 - \lambda)},$$

$$p_1^* = \frac{2(a_1 + a_2)(2a_1 - a_2)}{9(2 - \lambda)},\ p_2^* = \frac{2(a_1 + a_2)^2(1 - \lambda)^2}{9(2 - \lambda)}, \tag{6}$$

$$w^*(\theta_2) = \left[\lambda(a_2 - \theta_2) - (1 - \lambda)\left(\frac{2a_1 - a_2}{3}\right)\right]\left(\frac{a_1 + a_2}{3}\right). \tag{7}$$

All proofs are in the appendix. Note that existence requires that $a_1 + a_2 \geq 0$.[14] In addition, we focus on the interior solution ($n_1^*, n_2^* < 1$) so that comparative statics can be considered. This requires that $a_1 + a_2 + \frac{3}{2} \cdot \lambda < 3$.

In terms of prices, note that consumers are subsidized (i.e., $p_1^* < 0$) when $a_2 > 2a_1$. This is guaranteed when $a_1 < 0$, or for media markets where advertisements are harmful to all consumers. Also notice that when $\lambda = 1$ so that the platform obtains full bargaining

[14] That is, the net network effects must be positive; otherwise, there is insufficient gains to make interactions worthwhile. For example, if advertisements are too harmful relative to the successfulness of the ads ($-a_1 > a_2$), then the platform cannot profitably support interaction between consumers and advertisers.

power, we have that $p_2 = 0$ so that all firms join the platform but the platform earns all firm surpluses from $w^*(\theta_2)$; that is, perfect price discrimination (in the form of a two-part tariff) occurs on the firm side when the platform has all bargaining power. This also motivates the platform to allow for all firms to enter the market as it extracts all firm surplus; in addition, more firms generates greater consumer surplus that the platform can partial extract through the consumer price.

Now consider how changes in bargaining power impact the platform equilibrium. In terms of participation, bargaining power does not affect equilibrium consumer participation, $\frac{\partial n_1^*}{\partial \lambda} = 0$, and more platform bargaining power results in greater firm participation, $\frac{\partial n_2^*}{\partial \lambda} > 0$. This implies that welfare increases with platform bargaining power. We will consider welfare shortly, but first consider how platform bargaining power impacts the equilibrium prices:

Corollary 1. *Greater platform bargaining power over firms implies that:*

1. *the consumer price increases,* $\frac{\partial p_1^*}{\partial \lambda} > 0$, *if and only if* $a_1 > \frac{1}{2}a_2$*;*
2. *the firm entry fee decreases,* $\frac{\partial p_2^*}{\partial \lambda} < 0$*; and*
3. *the concession fee increases,* $\frac{\partial w^*(\theta_2)}{\partial \lambda} > 0$.

Consider first the impact that changes in platform bargaining power have on the firm side of the market with respect to p_2^* and $w^*(\cdot)$. With greater platform bargaining power, the platform is able to better extract rents from the firms that join the platform through $w(\theta)$; however, the platform also preserves the size of the network by decreasing the entry fee to firms. This highlights how network effects generate a tradeoff for the platform between using its greater bargaining power to extract rents verses maintaining sufficient platform size to extract rents from consumers.

Now consider the impact of greater platform bargaining power on the price to consumers p_1^*, and notice that the effect is indeterminant. In particular, for greater platform bargaining power to increase the consumer price (i.e., $\frac{\partial p_1^*}{\partial \lambda} > 0$), it must be that consumer network benefits are greater than half of firm network benefits ($a_1 > \frac{a_2}{2}$). This is most likely true for the video game market where consumers receive significant benefit from games. In this case, greater bargain power by a gaming platform over its game developers will result in a higher consumer price on gaming consoles.

Alternatively, for the case of a media platform with advertising so that $a_1 < \frac{a_2}{2}$, greater bargaining power by the platform results in lower magazine or network subscription fees. The reason for this consumer price decrease is that when a_2 relatively large (e.g., $a_2 > 2a_1$), there is greater surplus (rent to be extracted) on the firm side so that the platform prefers to offer the firms more consumers (induced by a lower consumer price) while extracting greater firm concession fees.

2.3 Welfare

Now consider the welfare that is generated in this two-sided market. Welfare is given by:

$$W = \int_0^{n_1^*} \alpha_1(\theta_1) n_2^* \, dF_1(\theta_1) + \int_0^{n_2^*} \alpha_2(\theta_2) n_1^* \, dF_2(\theta_2). \tag{8}$$

The impact on welfare from greater platform bargaining power is characterized by the following proposition:

Proposition 2. *Greater platform bargaining power over firms implies that total welfare increases.*

The intuition for why greater platform bargaining power increases welfare is that, instead, if firms earn greater bargaining power over the platform, then it is more difficult for the platform to invest in the network size which reduces the welfare generated by the platform.

While understanding the impact of platform bargaining power on total welfare is important, measuring the impact of bargaining power on agent (consumers, firms, and the platform) surplus provides more informative welfare results.

Proposition 3. *Greater platform bargaining power increases platform profits and the surplus for every participating consumer:* $\frac{\partial \Pi}{\partial \lambda} > 0$ *and* $\frac{\partial u_1(\theta_1)}{\partial \lambda} > 0$ *for all* $\theta_1 \in [0, n_1^*]$. *However, greater platform bargaining power reduces the surplus for firms with large network benefits but increases surplus for firms with small network benefits. That is, there exists a* $\theta_2^* < n_2^*$ *so that* $\frac{\partial u_2(\theta_2)}{\partial \lambda} < 0$ *for* $\theta_2 < \theta_2^*$ *and* $\frac{\partial u_2(\theta_2)}{\partial \lambda} > 0$ *for* $\theta_2 > \theta_2^*$.

Not surprisingly, the platform benefits from greater bargaining power. In addition, given that greater platform bargaining power results in increased firm participation, it is also not surprising that all participating consumers benefit from greater platform bargaining power. On the firm side, greater platform bargaining power harms the firms with strong network benefits but actually benefits the firms with weak network benefits. This implies that only the firms with larger network benefits gain from greater firm bargaining power, whereas not only the platform, but also all consumers and the participating firms with smaller network benefits are harmed. Thus, greater firm bargaining power results in a kind of hold-up problem that only benefits some firms but is detrimental to the platform, consumers, and some of the other firms.

From a policy perspective, this result suggests that policy makers should implement policies that improve platform bargaining power instead of firm bargaining power. By doing so, a policy maker promotes the platform's greater incentive to invest in platform size on both sides of the market, instead of promoting firms who only care about the growth on the consumer side of the market.

3 Platform Competition

To make comparisons between the monopoly platform case and platform competition, we consider an environment where the original monopoly platform (which we refer to as Platform A in this section) faces competition from another platform. More specifically, we set up our model so that Platform A provides the same utilities to consumers and firms as the monopoly platform had in Section 2; however, now another platform exists (which we call Platform B) and offers agents a differentiated experience.

A natural modeling candidate for platform competition is the Hotelling framework for two-sided markets developed by Armstrong (2006). However, in our context, the bargaining game utilizes heterogenous network effects. Thus, we augment the two-sided market Hotelling model so that platforms are differentiated through the network effects instead of through stand-alone values. This maintains consistencies between Platform A and the monopoly platform and such a setting maps into many real world applications. For example, in the market for video games, there are consumers (game developers) that prefer to interact with Nintendo specific game developers (consumers) over Playstation game developers (consumers). Similarly, Vogue magazine and the Economist appeal to different consumers and advertisers so that an Economist consumer (advertiser) benefits more from an interaction with a Economist advertisement (consumer) than from an interaction with a Vogue advertisement (consumer).

3.1 The Model

Suppose that two horizontally differentiated platforms (Platform A and Platform B) compete over consumers and firms. As in the monopoly platform case, a platform charges consumers a single price, p_1^A and p_1^B, and firms pay an initial entry fee, p_2^A and p_2^B, and then bargain over a firm specific concession fee, $w^A(\theta_2)$ and $w^B(\theta_2)$. We follow an agent homing structure that is similar to the Choi (2010) and Belleflamme and Peitz (2019) who each extend the model by Armstrong (2006) so that consumers single-home while firms can either single-home or multi-home.

Consider first the consumer problem. A consumer $\theta_1 \in [0,1]$ can either join Platform A or Platform B and the utilities from doing so are given by:

$$u_1^A(\theta_1) = (a_1^A - \theta_1)n_2^A - p_1^A, \tag{9}$$

$$u_1^B(\theta_1) = [a_1^B - (1-\theta_1)]n_2^B - p_1^B, \tag{10}$$

where $n_2^X \in [0,1]$ denotes the number of firms on Side 2 for Platform $X = A, B$. For simplicity, we assume that platforms are symmetric so that $a_1^A = a_1^B \equiv a_1 > \frac{1}{2}$ (where $a_2 > \frac{1}{2}$ ensures that the market is covered).

The consumer utility structure implies that a consumer whose type θ_1 is close to zero has a preference for Platform A in the form of stronger network benefits on Platform A while a consumer whose type θ_1 is close to one has a preference for Platform B in the form of stronger network benefits on Platform B. Note that the marginal consumer that is indifferent between the two platforms identifies the consumers that join Platform A and those that join Platform B. Furthermore, notice that the consumer utility function for Platform A (Equation (9)) mirrors the consumer utility function for the monopoly platform (Equation (1)).[15] This is in an attempt to provide a clear comparison between the monopoly and duopoly frameworks.

On the other hand, we allow multi-homing on the firm side. Specifically, a firm of type θ_2 has utilities given by:

$$u_2^A(\theta_2) = (a_2^A - \theta_2)n_1^A - p_2^A - w^A(\theta_2), \tag{11}$$

[15] The same connection will be true on the firm side.

$$u_2^B(\theta_2) = [a_2^B - (1-\theta_2)]n_1^B - p_2^B - w^B(\theta_2), \tag{12}$$

$$u_2^M(\theta_2) = (a_2^A - \theta_2)n_1^A + [a_2^B - (1-\theta_2)]n_1^B - p_2^A - w^A(\theta_2) - p_2^B - w^B(\theta_2), \tag{13}$$

where $n_1^X \in [0,1]$ denotes the number of consumers on Side 1 for Platform $X = A, B$. For simplicity, we assume that platforms are symmetric so that $a_2^A = a_2^B \equiv a_2 > \frac{1}{2}$.

Given the initial entry decision made by firms, negotiation between the platform and firm θ_2 begins so that the entry fee, p_2, is sunk. And, similar to the monopoly platform case, the participation of a θ_2 type firm on Platform X for $X = A, B$ will (i) impact Platform X's revenues from consumers and (ii) generates $w^X(\theta_2)$ revenue. In contrast, the marginal benefit to a firm of type θ_2 from participating on Platform X is given by $\alpha_2(\theta_2)n_1^X - w^X(\theta_2)$. Thus, the Nash bargaining process is the same as in the monopoly platform case.[16]

3.2 Equilibrium

We have the following competitive equilibrium:

Proposition 4. *There exists a unique competitive bottleneck equilibrium:*

$$n_1^X = n_1^Y = n_1^c = \frac{1}{2},\ n_2^X = n_2^Y = n_2^c = \frac{2(a_1 + a_2) - 1}{2(2-\lambda)} \tag{14}$$

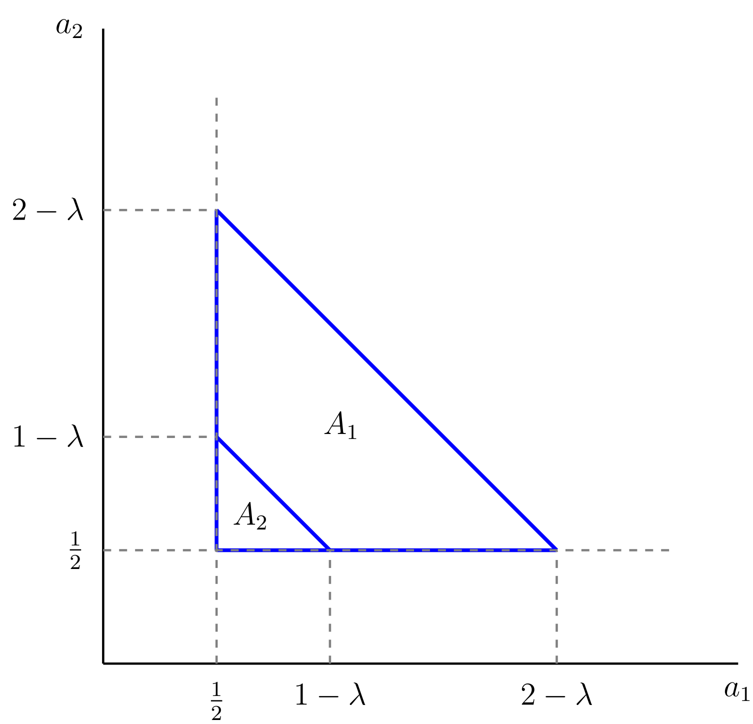
$$p_1^c = \frac{[2(a_1 + a_2) - 1][3 + 2(a_1 - a_2)]}{8(2-\lambda)},\ p_2^c = \frac{1-\lambda}{4}\left[2(a_2 - n_2^c) + (2a_1 - 1)\frac{p_1^c}{n_2^c}\right], \tag{15}$$

$$\begin{aligned} w^A(\theta_2) &= \tfrac{1}{4}\left[2\lambda(a_2 - \theta_2) - (1-\lambda)(2a_1 - 1)\tfrac{p_1^c}{n_2^c}\right], \\ w^B(\theta_2) &= \tfrac{1}{4}\left[2\lambda[a_2 - (1-\theta_2)] - (1-\lambda)(2a_1 - 1)\tfrac{p_1^c}{n_2^c}\right]. \end{aligned} \tag{16}$$

[16] That is, the Nash bargaining game between firms and platforms results in Platform X having a marginal gain from firm θ_2 given by $\frac{\partial(p_1^X n_1^X)}{\partial n_2^X} + w^X(\theta_2)$. As a result, the Nash bargaining problem for Platform X is formulated as $\max_{w^X(\theta_2)} \left[\frac{\partial(p_1^X n_1^X)}{\partial n_2^X} + w^X(\theta_2)\right]^{\lambda} \cdot \left[(a_2 - \theta_2)n_1^X - w^X(\theta_2)\right]^{1-\lambda}$.

To ensure that the equilibrium exists, we must have that $n_2^c < 1$. This requires that $a_1 + a_2 + \lambda \leq \frac{5}{2}$. Furthermore, notice that $n_2^c < \frac{1}{2}$ when $a_1 + a_2 + \lambda \leq \frac{3}{2}$. In this case, the platforms do not serve all firms and the firms single-home to one of the two platforms. Figure 1 shows the region where (a_1, a_2) is located, depending on the value of λ. If (a_1, a_2) is located in subspace A_1, then all firms are served. However, if (a_1, a_2) is in subspace A_2, then some firms are excluded and all participating firms are single-homing. Note also that subspace A_2 vanishes if λ is equal to or greater than $\frac{1}{2}$. Figure 2 depicts how the region of (a_1, a_2) changes as λ increases.

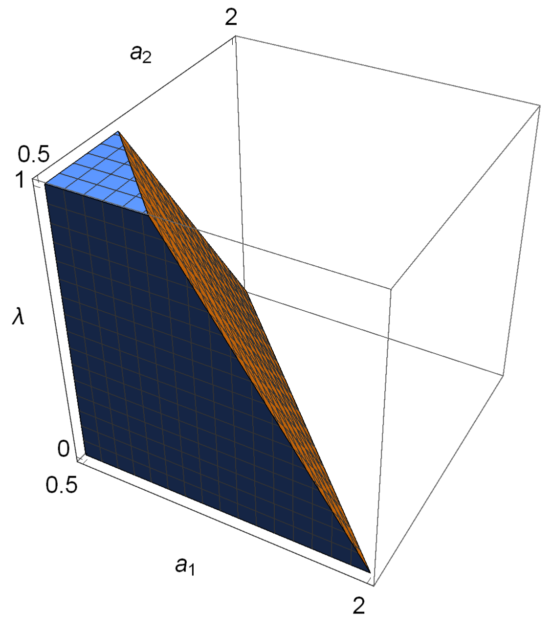
Figure 1: Firm Homing Constellations



Note: The set of all possible (a_1, a_2) is broken into two segments. If (a_1, a_2) is located in subregion A_1, then all firms transact with either or both of the platforms. Instead, if (a_1, a_2) is in subregion A_2, then some firms are excluded and all participating firms are single-homing.

In terms of prices, note that consumers are only subsidized (i.e., $p_1^c < 0$) when a_1 is

Figure 2: The Possible Parameter Region



Note: The parameter value (a_1, a_2, λ) is located within the colored region.

sufficiently small (i.e., $a_1 < a_2 - \frac{1}{2}$), which is similar to the case of a monopoly platform. Also, as in the case of a monopoly platform, when $\lambda = 1$ so that the platform obtains full bargaining power, we have that $p_2^c = 0$ so that all firms multi-home but the platforms earn all the firm surplus that they generate on their respective platform through the concession fees. That is, perfect price discrimination (in the form of a two-part tariff) occurs on the firm side when the platform has all bargaining power.

If we compare Propositions 1 and 4, then we can determine how platform competition impacts platform prices and concession fees. In particular, we see that the competition structure impacts how platforms extract surplus through bargaining:

Proposition 5. *Comparing monopoly platform fees to the fees with platform competition implies that:*

1. *The equilibrium consumer price is higher with platform competition than with a*

monopoly platform, $p_1^c > p_1^*$.

2. *The equilibrium firm entry fee is lower with platform competition than with a monopoly platform,* $p_2^c < p_2^*$*, for all* $\lambda \in [0,1]$ *if* $a_1 > 0.7$ *and* $a_2 > 1$.

3. *The equilibrium concession fee is higher with platform competition than with monopoly,* $w^A(n_2^c) > w^*(n_2^*)$*, for all* $\lambda \in [0,1]$ *if* $a_1 + a_2 < \frac{3}{2}$*. Naturally, this implies that* $w^A(n_2^c) > w^*(n_2^*)$ *when* $a_1 + a_2 + \lambda \leq \frac{3}{2}$*, that is, when firms single-home.*

First notice that for all $a_1 + a_2 + \lambda \leq \frac{3}{2}$ we have that $p_1^c > p_1^*$, $p_2^c < p_2^*$, and $w^A(n_2^c) > w^*(n_2^*)$. While these inequalities do not entirely hold across all parameters, they capture the main intuition behind the price differences between a monopoly platform and competing platforms: competing platforms focus on gaining firm participation. As a result, consumers face higher prices while firms typically face lower fees (especially entry fees).[17]

The reason why platforms compete fiercely over firm participation is that firm participation benefits the platform in two ways. First, greater firm participation benefits consumers allowing the platforms to charge a higher price to consumers. Second, firm participation allows the platforms to extract more concession fees. Instead, consumer participation provides the first benefit but not the second. In addition, obtaining an additional consumer is more difficult in the duopoly case so that duopoly platforms have an incentive to increase their consumer price.[18] Altogether this implies that platform competition is concentrated on the firm side of the market and greater markups are extracted from consumers.

The results in Proposition 5 capture the comparison between the monopoly platform case and the case of platform competition. Next, we consider how platform bargaining

[17]Unless the parameters satisfy those in Proposition 5 (2.), which requires a large λ and small as, then at least one of the fees to firms is lower with competition. This implies that the entry fee to firms is usually lower with platform competition. For a more extensive description of the firm fee comparison, see the proof of Proposition 5 which characterizes these relationships across parameters.

[18]Under monopoly, the consumer side is never covered (some consumers never purchase the product), whereas the entire market is covered under duopoly. Effectively, this brakes the downward pressure on the prices under duopoly.

power impacts the competitive equilibrium. In terms of participation, bargaining power does not affect equilibrium consumer participation, $\frac{\partial n_1^c}{\partial \lambda} = 0$, and greater platform bargaining power results in greater firm participation, $\frac{\partial n_2^c}{\partial \lambda} > 0$. Both of these effects are consistent with the monopoly platform case and they imply that total welfare increases with greater platform bargaining power. We will consider individual welfare shortly, but first consider how platform bargaining power impacts equilibrium pricing:

Corollary 2. *Greater platform bargaining power over firms implies that:*

1. *the consumer price increases,* $\frac{\partial p_1^c}{\partial \lambda} > 0$*, if and only if* $a_1 > \frac{2a_2-3}{2}$*;*
2. *there exists* $\hat{\lambda}(a_1, a_2) > 0.8$ *such that firm entry fee decreases,* $\frac{\partial p_2^c}{\partial \lambda} < 0$*, if and only if* $\lambda < \hat{\lambda}(a_1, a_2)$*;*
3. *the concession fee increases,* $\frac{\partial w^X(\theta_2)}{\partial \lambda} > 0$*.*

These results are almost entirely consistent with those in Corollary 1 for the monopoly platform case. Namely, greater platform bargaining power results in an increase in consumer price for a_1 sufficiently large and an increase in the concession fee. However, now with platform competition, we see that the firm entry fee does not always decreasing for greater platform bargain power. In fact, the firm entry fee increases with greater platform bargaining power when platform bargaining power is sufficiently high. Combining this finding with the results in Proposition 5 suggests that platform competition is stifled for higher levels of platform bargaining power, and in this case, the competing platforms are able to successfully charge higher fees.

3.3 Welfare

We now consider welfare with platform competition. Total welfare is given by:

$$W^c \equiv 2\int_0^{n_1^c}(a_1-\theta_1)n_2^c d\theta_1 + 2\int_0^{n_2^c}(a_2-\theta_2)n_1^c d\theta_2, \tag{17}$$

As in the monopoly platform market, greater platform bargaining power increases welfare.

Proposition 6. *Greater platform bargaining power over firms implies that total welfare rises.*

Like the monopoly platform case, if the platforms earn greater bargaining power over the firms, then the platforms have a greater incentive to invest in the size of their networks which increases welfare.

Due to the variety of pricing and participation effects from changes in platform bargaining power, we must consider agent utilities explicitly to determine welfare effects with platform competition. Turning to the impact that bargaining power has on individual agents (consumers, firms, and the platform), we see that our results do not necessarily mirror the monopoly platform case.

Proposition 7. *(a) Greater platform bargaining power increases platform profit:* $\frac{\partial \Pi^X}{\partial \lambda} > 0$ *for* $X = A, B$*. It also increases surplus for the consumers with large network benefits but decreases surplus for consumers with small network benefits: there exists an* $\theta_1^c < n_1^c$ *such that* $\frac{\partial u_1^A(\theta_1)}{\partial \lambda} > 0$ *for* $\theta_1 < \theta_1^c$ *and* $\frac{\partial u_1^A(\theta_1)}{\partial \lambda} < 0$ *for* $\theta_1 > \theta_1^c$*.*

(b) If $2(a_1+a_2) < 1 + \frac{2(1-\lambda)^2}{3-\lambda}$*, then there exists* $\theta_2^c < 1 - n_2^c$ *such that an increase in platform bargaining power raises single-homing firm utility for* $\theta_2 \in [\theta_2^c, 1-n_2^c]$ *and lowers other single-homing firms' utility. Otherwise, all single-homing utility decreases for greater bargaining power. Lastly, for each* $\widetilde{a}_2 \in (\frac{1}{2}, 1)$*, there exists a pair of* $(\widetilde{a}_1, \widetilde{\lambda}) \in (\frac{1}{2}, 2) \times [0,1]$ *so that multi-homing firms benefit from greater platform bargaining power if* $a_1 < \widetilde{a}_1$*,* $a_2 < \widetilde{a}_2$*,*

and $\lambda < \widetilde{\lambda}$.

Like the monopoly platform welfare effects described in Proposition 3, greater platform bargaining power increases platform profits. Unlike the monopoly case, not all consumers benefit from greater platform bargaining power. Recall that consumers face a higher price but also earn greater interaction with firms when platform bargaining power increase. With platform competition, we find that the increase in the number of firms is not enough to compensate all consumers for the increase in the consumer price so that the consumers with low network effects are worse off.

On the firm side, the welfare effects largely coincide between the two market structures. In the monopoly case, greater platform bargaining power reduces the surplus for firms with large network benefits but increases surplus for firms with small network benefits. Similarly with platform competition, single-homing firms (with relatively large network benefits) are usually better off from greater platform competition, and multi-homing firms (with relatively small network benefits) usually benefit from greater bargaining power.

To conclude this section on platform competition, it is important to note that the main results between the two market structures typically coincide. In terms of policy, this means that promoting greater platform bargaining power is generally welfare improving. However, when platform bargaining power strongly favors platforms, then greater bargaining power has a dampening effect on platform competition even though welfare increases.

4 Concluding Remarks

In this paper, we introduce bargaining, an important characteristic of B2B relationships, to a model of two-sided markets and we study its impact on platform participation, prices and welfare. We consider both a monopoly platform as well as two competing platforms and find that our main results are largely consistent between the two structures.

By introducing bargaining between platforms and firms, we find that a platform with stronger bargaining power will offer a reduced entry fee to firms in order to increase firm participation. Greater firm participation allows the platform to charge consumers a higher price. At the same time, the platform extracts additional firm surplus through higher concession fees that are earned through bargaining. While this typically harms firms, consumers benefit and total welfare increases. Instead, if firms have greater bargaining power over a platform, then it becomes more difficult for the platform to invest in network size, reducing the total welfare generated by the platform.

Appendix of Proofs

Proof of Proposition 1: Given that agents are distributed uniformly on $[0,1]$, $\alpha_i(\theta_i) = (a_i - \theta_i)$ for $i = 1, 2$, and $c_1 = c_2 = 0$, the inverse demands for consumers and firms implies that profits are given by:

$$\begin{aligned}
\Pi &= n_1 \cdot p_1 + n_2 \cdot p_2 + \int_0^{n_2} w(\theta_2) dF_2(\theta_2) \\
&= \alpha_1(n_1) n_1 n_2 + (1-\lambda)[\alpha_1(n_1) + \alpha_2(n_2)] n_1 n_2 + \int_0^{n_2} w(\theta_2) d\theta_2 \\
&= [\alpha_1(n_1) + (1-\lambda)\alpha_2(n_2)]\, n_1 n_2 + \lambda n_1 \int_0^{n_2} \alpha_2(\theta_2) d\theta_2 \\
&= \left[a_1 + a_2 - n_1 - \left(1 - \frac{\lambda}{2}\right) n_2\right] n_1 n_2,
\end{aligned}$$

where the second equality is given by Equations (3) and (5) as well as agents being uniformly distributed, the third equality is given by Equation (4), and the fourth is given by linear network effects. Taking first-order conditions with respect to n_1 and n_2 implies that

$$\frac{\partial \Pi}{\partial n_1} = \left[a_1 + a_2 - n_1 - \left(1 - \frac{\lambda}{2}\right) n_2\right] n_2 - n_1 n_2,$$

$$\frac{\partial \Pi}{\partial n_2} = \left[a_1 + a_2 - n_1 - \left(1 - \frac{\lambda}{2}\right) n_2\right] n_1 - \left(1 - \frac{\lambda}{2}\right) n_1 n_2 = 0.$$

Solving for n_1 and n_2 yields:

$$n_1^* = \frac{1}{3}(a_1 + a_2),$$

$$n_2^* = \frac{2(a_1 + a_2)}{3(2 - \lambda)}.$$

We have solution when $(n_1, n_2) \geq 0$ which requires that $a_1 + a_2 \geq 0$. Plugging n_1^* and n_2^* into Equations (3), (4), and (5) yields the prescribed equilibrium fees p_1^*, $w^*(\theta_2)$, and p_2^*. □

Proof of Corollary 1: Taking derivatives of Equations (6) and (7) implies

$$\frac{\partial p_1^*}{\partial \lambda} = \frac{p_1^*}{(2 - \lambda)} > 0 \text{ if and only if } a_1 > \frac{a_2}{2},$$

$$\frac{\partial p_2^*}{\partial \lambda} = -\frac{2(a_1 + a_2)^2(1 - \lambda)(3 - \lambda)}{9(2 - \lambda)^2} < 0,$$

$$\frac{\partial w^*(\theta_2)}{\partial \lambda} = \frac{(a_1 + a_2)(2a_1 + 2a_2 - 3\theta_2)}{9} > 0,$$

where the last inequality is greater than zero since $\theta_2 \in [0, n_2^*]$. □

Proof of Proposition 2: Welfare is given by:

$$\begin{aligned}
W &\equiv \int_0^{n_1^*} \alpha_1(\theta_1) n_2^* \, dF_1(\theta_1) + \int_0^{n_2^*} \alpha_2(\theta_2) n_1^* dF_2(\theta_2) \\
&= \int_0^{n_1^*} (a_1 - \theta_1) n_2^* d\theta_1 + \int_0^{n_2^*} (a_2 - \theta_2) n_1^* d\theta_2 \\
&= n_1^* n_2^* \left(a_1 + a_2 - \frac{n_1^* + n_2^*}{2}\right) \\
&= \frac{(a_1 + a_2)^3}{27(2 - \lambda)^2} \cdot (8 - 5\lambda),
\end{aligned}$$

which implies that

$$\frac{\partial W}{\partial \lambda} = \frac{(a_1 + a_2)^3}{27(2-\lambda)^3} \cdot (6 - 5\lambda) > 0.$$

$\square$

Proof of Proposition 3: Equilibrium platform profit is given by:

$$\Pi = \frac{2(a_1 + a_2)^3}{27(2-\lambda)},$$

which implies that

$$\frac{\partial \Pi}{\partial \lambda} = \frac{2(a_1 + a_2)^3}{27(2-\lambda)^2} > 0.$$

Equilibrium consumer surplus for a consumer of type $\theta_1 \in [0, n_1^*]$ is given by:

$$u_1(\theta_1) = \frac{2(a_1 + a_2)}{9(2-\lambda)} \cdot [(a_1 + a_2) - 3\theta_1],$$

which implies that

$$\frac{\partial u_1(\theta_1)}{\partial \lambda} = \frac{2(a_1 + a_2)}{9(2-\lambda)^2} \cdot [(a_1 + a_2) - 3\theta_1],$$

which is greater than zero for all consumers that join the platform (i.e. for all $\theta_1 \leq n_1^*$). Lastly, equilibrium firm surplus for a firm of type $\theta_2 \in [0, n_2^*]$ is given by:

$$u_2(\theta_2) = \frac{a_1 + a_2}{9} \cdot \frac{1-\lambda}{2-\lambda} \cdot [2\lambda a_1 - (1-\lambda)a_2 - (2-\lambda)\theta_2],$$

which implies that

$$\frac{\partial u_2(\theta_1)}{\partial \lambda} = \frac{a_1 + a_2}{9(2-\lambda)^2} \cdot [(2 - 4\lambda + \lambda^2)(2a_1 + a_2 + \theta_2) + 2\theta_2 + a_1],$$

which is less than zero only for firms of type θ_2 such that

$$\theta_2 < \frac{2(a_1 + a_2)\lambda}{3(1 + \lambda - \lambda^2)} \equiv \theta_2^* < n_2^*.$$

$\square$

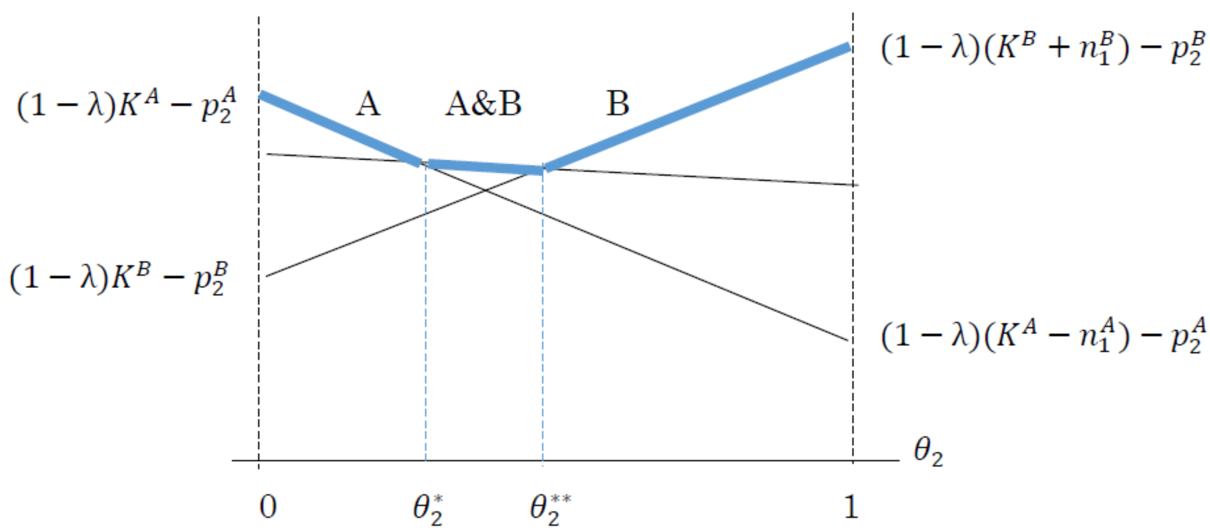
Proof of Proposition 4: Equations (9) and (10) imply that the marginal consumer that is indifferent between the two platforms is given by: $\frac{(n_2^A - n_2^B)a_1 + n_2^B - p_1^A + p_1^B}{n_2^A + n_2^B}$. Given that consumers are distributed uniformly between zero and one, the consumer demand for Platform A is $n_1^A = \frac{(n_2^A - n_2^B)a_1 + n_2^B - p_1^A + p_1^B}{n_2^A + n_2^B} \equiv n_1^A(p_1^A, p_1^B; n_2^A)$ and the consumer demand for Platform B is $n_1^B = 1 - n_1^A = \frac{(n_2^B - n_2^A)a_1 + n_2^A - p_1^B + p_1^A}{n_2^A + n_2^B} \equiv n_1^B(p_1^A, p_1^B; n_2^B)$.

Note that $w^A(\theta_2)$ and $w^B(\theta_2)$ in Equations (11) and (12) are endogenous. To fully identify the firm demand functions we must solve the bargaining subgame to determine the $w^A(\cdot)$ and $w^B(\cdot)$. Note that the marginal gain to Platform $X = A, B$ from a firm θ_2 is given by $\frac{\partial(p_1^X n_1^X)}{\partial n_2^X} + w^X(\theta_2) = \frac{\partial\left(p_1^X \frac{(n_2^X - n_2^Y)a_1 + n_2^Y - p_1^X + p_1^Y}{n_2^A + n_2^B}\right)}{\partial n_2^X} + w^X(\theta_2) = p_1^X \cdot \frac{a_1 - n_1^X}{n_2^A + n_2^B} + w^X(\theta_2)$, where the derivative derivation simplifies to the last equality because $n_1^X(n_2^A + n_2^B) = (n_2^X - n_2^Y)a_1 + n_2^Y - p_1^X + p_1^Y$. Therefore, the Nash bargaining problem for Platform A is formulated as $\max_{w^A(\theta_2)} \left[p_1^A \cdot \frac{a_1 - n_1^A}{n_2^A + n_2^B} + w^A(\theta_2)\right]^\lambda \left[(a_2 - \theta_2)n_1^A - w^A(\theta_2)\right]^{1-\lambda}$ which leads to Platform A's concession fee $w^A(\theta_2) = \lambda(a_2 - \theta_2)n_1^A - (1-\lambda)\left(\frac{a_1 - n_1^A}{n_2^A + n_2^B}\right)p_1^A$, whereas Platform B's concession fee is similarly given by $w^B(\theta_2) = \lambda[a_2 - (1-\theta_2)]n_1^B - (1-\lambda)\left(\frac{a_1 - n_1^B}{n_2^A + n_2^B}\right)p_1^B$. Therefore, firm θ_2 obtains utilities:

$$u_2^A(\theta_2) = (1-\lambda)\underbrace{\left(\frac{a_1 - n_1^A}{n_2^A + n_2^B}p_1^A + a_2 n_1^A\right)}_{\equiv K^A} - p_2^A - (1-\lambda)n_1^A\theta_2,$$

$$u_2^B(\theta_2) = (1-\lambda)\underbrace{\left(\frac{a_1 - n_1^B}{n_2^A + n_2^B}p_1^B - (1-a_2)n_1^B\right)}_{\equiv K^B} - p_2^B + (1-\lambda)n_1^B\theta_2,$$

Figure 3: Firm θ_2's utility when multi-homing is allowed



$$u_2^M(\theta_2) = (1-\lambda)\left(K^A + K^B\right) - p_2^A - p_2^B - (1-\lambda)(n_1^A - n_1^B)\theta_2.$$

Since $u_2(\theta_2)$ is linear in θ_2, firm θ_2's utility for each choice is depicted in Figure 3, where θ_2^* and θ_2^{**} are the thresholds. Given that firms are distributed uniformly between zero and one, the inverse demands are obtained from $n_2^A = \theta_2^{**} = \frac{(1-\lambda)\left(\frac{a_1-n_1^A}{n_2^A+n_2^B}p_1^A + a_2 n_1^A\right) - p_2^A}{(1-\lambda)n_1^A}$ and $n_2^B = 1 - \theta_2^* = \frac{(1-\lambda)\left(\frac{a_1-n_1^B}{n_2^A+n_2^B}p_1^B + a_2 n_1^B\right) - p_2^B}{(1-\lambda)n_1^B}$ so that

$$p_2^A = (1-\lambda)\left[(a_2 - n_2^A)n_1^A + \left(\frac{a_1 - n_1^A}{n_2^A + n_2^B}\right)p_1^A\right] \equiv p_2^A(n_1^A, n_2^A),$$

$$p_2^B = (1-\lambda)\left[(a_2 - n_2^B)n_1^B + \left(\frac{a_1 - n_1^B}{n_2^A + n_2^B}\right)p_1^B\right] \equiv p_2^B(n_1^B, n_2^B).$$

Now consider Platform A's problem. Given p_1^B and n_2^B, Platform A chooses p_1^A and n_2^A to maximize its profit:

$$\Pi^A = p_1^A \cdot n_1^A(p_1^A, p_1^B; n_2^A) + p_2^A(n_1^A(p_1^A, p_1^B), n_2^A) \cdot n_2^A + \int_0^{n_2^A} w^A(\theta_2)dF_2(\theta_2)$$

$$
\begin{aligned}
&= p_1^A \cdot \frac{(n_2^A - n_2^B)a_1 + n_2^B - p_1^A + p_1^B}{n_2^A + n_2^B} \\
&+ (1-\lambda)\left[(a_2 - n_2^A)n_1^A + \left(\frac{a_1 - n_1^A}{n_2^A + n_2^B}\right)p_1^A\right]n_2^A - \frac{\lambda n_1^A}{2}(n_2^A)^2 \\
&+ \left[\lambda a_2 n_1^A - (1-\lambda)\left(\frac{a_1 - n_1^A}{n_2^A + n_2^B}\right)p_1^A\right]n_2^A \\
&= \frac{(n_2^A - n_2^B)a_1 + n_2^B - p_1^A + p_1^B}{n_2^A + n_2^B}\left[p_1^A + a_2 n_2^A - \left(1 - \frac{\lambda}{2}\right)(n_2^A)^2\right].
\end{aligned}
$$

Therefore, the first-order conditions are

$$
\frac{\partial \Pi^A}{\partial p_1^A} = -\frac{1}{n_2^A + n_2^B}\left[p_1^A + a_2 n_2^A - \left(1 - \frac{\lambda}{2}\right)(n_2^A)^2\right] + \frac{(n_2^A - n_2^B)a_1 - n_2^B - p_1^A + p_1^B}{n_2^A + n_2^B} = 0,
$$

and

$$
\begin{aligned}
\frac{\partial \Pi^A}{\partial n_2^A} &= \frac{a_1 - n_1^A}{n_2^A + n_2^B}\left[p_1^A + a_2 n_2^A - \left(1 - \frac{\lambda}{2}\right)(n_2^A)^2\right] \\
&+ \frac{(n_2^A - n_2^B)a_1 + n_2^B - p_1^A + p_1^B}{n_2^A + n_2^B}\left[a_2 - 2\left(1 - \frac{\lambda}{2}\right)n_2^A\right] = 0,
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{\partial n_1^A}{\partial n_2^A} &= \frac{a_1(n_2^A + n_2^B) - [(n_2^A - n_2^B)a_1 + n_2^B - p_1^A + p_1^B]}{(n_2^A + n_2^B)^2} \\
&= \frac{a_1(n_2^A + n_2^B) - n_1^A(n_2^A + n_2^B)}{(n_2^A + n_2^B)^2} \\
&= \frac{a_1 - n_1^A}{n_2^A + n_2^B}
\end{aligned}
$$

is used (recall again that $(n_2^A - n_2^B)a_1 + n_2^B - p_1^A + p_1^B = n_1^A(n_2^A + n_2^B)$).

By imposing platform symmetry we have that $n_1^A = n_1^B = \frac{1}{2}$, $n_2^A = n_2^B = n_2^c$, $p_1^A = p_1^B = p_1^c$, and $p_2^A = p_2^B = p_2^c$. Applying this to the first-order conditions implies that $p_1^c = (1 - a_2)n_2^c + \left(1 - \frac{\lambda}{2}\right)(n_2^c)^2$ and $\left(a_1 - \frac{1}{2}\right)\left[p_1^c + a_2 n_2^c - \left(1 - \frac{\lambda}{2}\right)(n_2^c)^2\right] + n_2^c\left[a_2 - 2\left(1 - \frac{\lambda}{2}\right)n_2^c\right] = 0$

which imply that

$$n_2^c = \frac{2(a_1 + a_2) - 1}{2(2 - \lambda)},$$

$$p_1^c = \frac{[2(a_1 + a_2) - 1][3 + 2(a_1 - a_2)]}{8(2 - \lambda)}.$$

Lastly, by substitution we have that:

$$p_2^c = \frac{1 - \lambda}{4} \left[2(a_2 - n_2^c) + (2a_1 - 1)\frac{p_1^c}{n_2^c} \right],$$

$$w^A(\theta_2) = \frac{1}{4} \left[2\lambda(a_2 - \theta_2) - (1 - \lambda)(2a_1 - 1)\frac{p_1^c}{n_2^c} \right],$$

$$w^B(\theta_2) = \frac{1}{4} \left[2\lambda[a_2 - (1 - \theta_2)] - (1 - \lambda)(2a_1 - 1)\frac{p_1^c}{n_2^c} \right].$$

Formally, the closed form solutions are:

$$p_2^c = \frac{(1 - \lambda)\{3\lambda - 2 + 4a_1[(2 - \lambda)a_1 - \lambda] + 2[6 - 5\lambda - 2(2 - \lambda)a_1]a_2\}}{16(2 - \lambda)},$$

$$w^A(\theta_2) = \left(\frac{1}{2}\right) \left[\lambda(a_2 - \theta_2) - (1 - \lambda)\frac{(2a_1 - 1)[3 + 2(a_1 - a_2)]}{8} \right],$$

$$w^B(\theta_2) = \left(\frac{1}{2}\right) \left[\lambda[a_2 - (1 - \theta_2)] - (1 - \lambda)\frac{(2a_1 - 1)[3 + 2(a_1 - a_2)]}{8} \right].$$

□

Proof of Proposition 5: Comparing Equations (6) and (15) we see that $p_1^c > p_1^*$ occurs if and only if $\frac{9}{8}\left(a_1 + a_2 - \frac{1}{2}\right)(3 + 2a_1 - 2a_2) > (a_1 + a_2)(2a_1 - a_2)$. This holds for all a_1, a_2, and λ such that $a_1 + a_2 + \lambda \leq \frac{5}{2}$.[19]

[19]A graph of this space is available upon request.

Next, Equations (6) and (15) also imply that $p_2^* \gtrless p_2^c$ if and only if

$$
\begin{aligned}
&\frac{2(a_1+a_2)^2(1-\lambda)}{9} \gtrless \frac{3\lambda - 2 + 4a_1[(2-\lambda)a_1 - \lambda] + 2[6-5\lambda-2(2-\lambda)a_1]a_2}{16} \\
&\iff \left(4a_1^2 - 32a_2^2 - 100a_1a_2 + 36a_1 + 90a_2 - 27\right)\lambda \\
&\qquad \gtrless 40a_1^2 - 32a_2^2 - 136a_1a_2 + 108a_2 - 18,
\end{aligned} \tag{18}
$$

where the sign of $4a_1^2 - 32a_2^2 - 100a_1a_2 + 36a_1 + 90a_2 - 27$ can be positive or negative in the parameter region of (a_1, a_2) with a fixed value of λ in Figure 2. Figure 4 captures the (a_1, a_2, λ) where $p_2^c > p_2^*$ and $p_2^c < p_2^*$. Now, Inequality (18) is rearranged as $4(\lambda-10)a_1^2 - 32(\lambda-1)a_2^2 - 2(50\lambda-68)a_1a_2 + 36\lambda a_1 + 2(45\lambda-59)a_2 - 27\lambda + 18 \gtrless 0$. If $a_1 = 0.5$, then the left-hand side being equal to zero has two solutions, $a_2 = 0.25, 1$, when seen as a equality of a_2. Similarly, if $a_2 = 0.5$, then $a_1 = 1, \frac{7-2.5\lambda}{10-\lambda)}$. the latter of which is decreasing in λ and attains the maximum value, 0.7, when $\lambda = 0$. Thus, if $a_1 > 0.7$ and $a_2 > 1$, then $p_2^c < p_2^*$ for any $\lambda \in [0, 1]$.

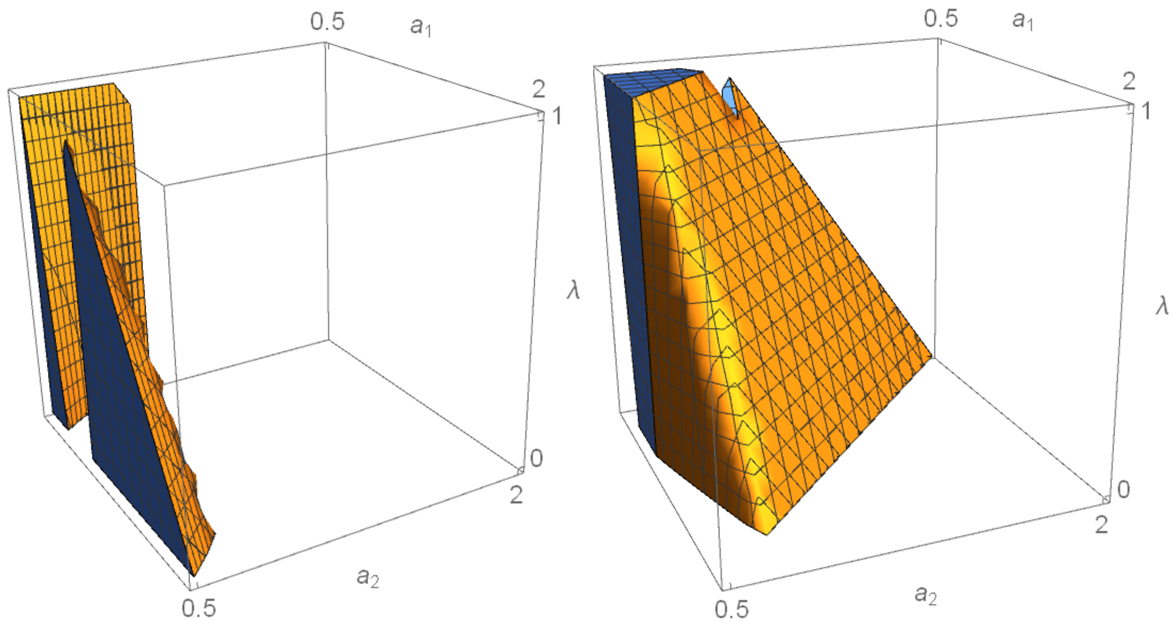
Third, Equations (7) and (16) imply that $w^A(n_2^c) \gtrless w^*(n_2^*)$ if and only if

$$
\begin{aligned}
&\left(\frac{1}{2}\right)\left[\lambda\left(a_2 - \frac{2(a_1+a_2)-1}{2(2-\lambda)}\right) - (1-\lambda)\frac{(2a_1-1)(3+2a_1-2a_2)}{8}\right] \\
&\qquad \gtrless \left(\frac{a_1+a_2}{3}\right)\left[\lambda\left(a_2 - \frac{2(a_1+a_2)}{3(2-\lambda)}\right) - (1-\lambda)\left(\frac{2a_1-a_2}{3}\right)\right].
\end{aligned} \tag{19}
$$

Figure 5 captures the (a_1, a_2, λ) where $w^A(n_2^c) > w^*(n_2^*)$ and $w^*(n_2^*) > w^A(n_2^c)$. If $a_2 = \frac{3}{2} - a_1$ is substituted, Inequality (19) is equivalent to $(2-\lambda)(1-\lambda)(1-a_1)(a_1 - \frac{1}{2}) > 0$. Thus, $w^A(n_2^c) > w^*(n_2^*)$ occurs for all λ when $a_1 + a_2 < \frac{3}{2}$.

Altogether we have that (i) $w^A(n_2^c) > w^*(n_2^*)$ with $p_2^* > p_2^c$, (ii) $w^*(n_2^*) > w^A(n_2^c)$ with $p_2^* > p_2^c$, and (iii) $w^A(n_2^c) > w^*(n_2^*)$ with $p_2^c > p_2^*$ are possible; however, $w^*(n_2^*) > w^A(n_2^c)$ with $p_2^c > p_2^*$ is not possible. Note that (iii) only occurs when λ large and a_1, a_2 small, the small parameter space in Figure 4 Panel (a). $\square$

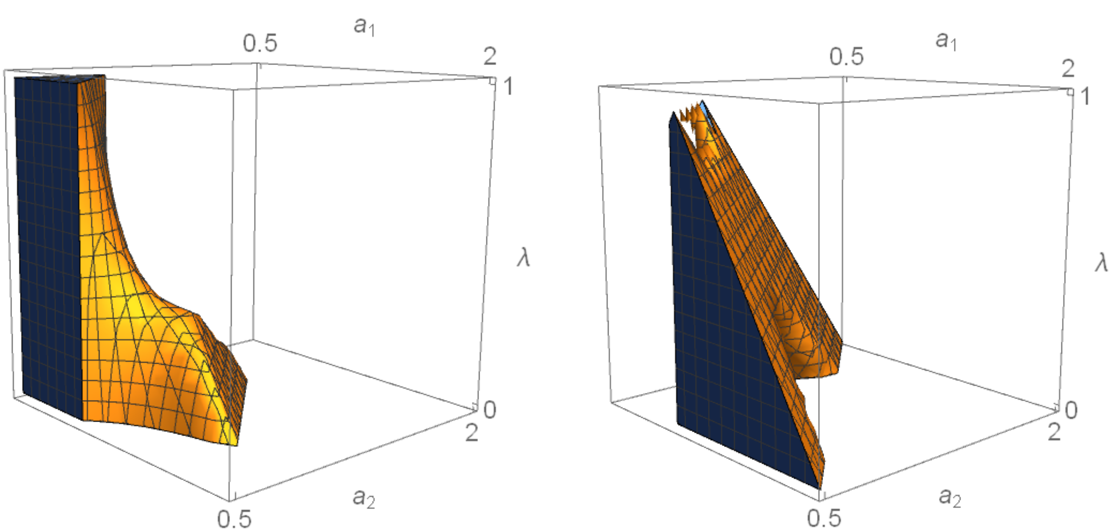
Figure 4: The Firm Entry Fee Comparison



(a) The (a_1, a_2, λ) where $p_2^c > p_2^*$.

(b) The (a_1, a_2, λ) where $p_2^c < p_2^*$.

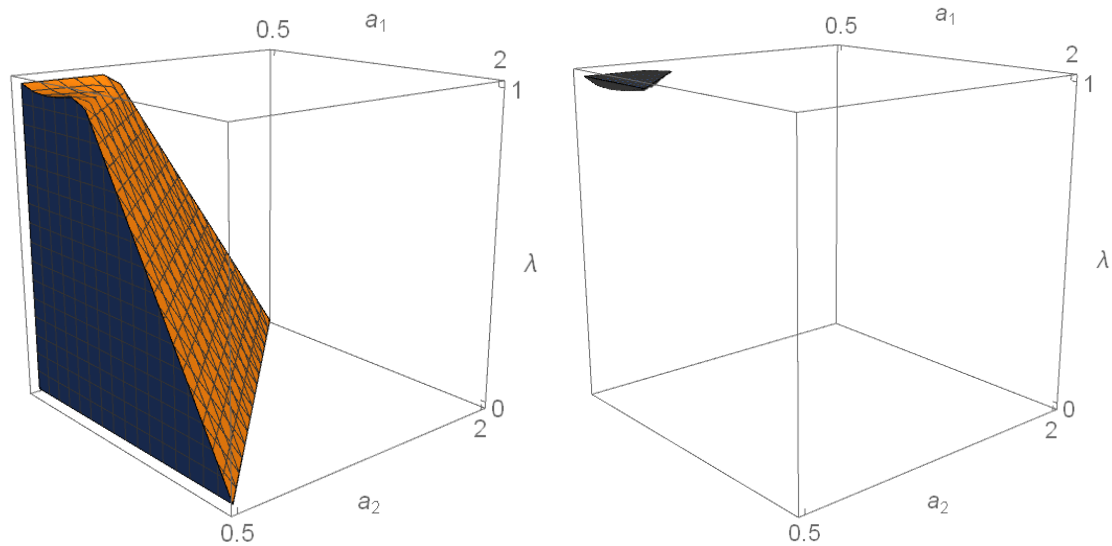
Figure 5: The Firm Concession Fee Comparison



(a) The (a_1, a_2, λ) where $w^A(n_2^c) > w^*(n_2^*)$.

(b) The (a_1, a_2, λ) where $w^A(n_2^c) < w^*(n_2^*)$.

Figure 6: The Impact of Bargaining Power on the Firm Entry Fee



(a) The (a_1, a_2, λ) where $\frac{\partial p_2^c}{\partial \lambda} < 0$. (b) The (a_1, a_2, λ) where $\frac{\partial p_2^c}{\partial \lambda} > 0$.

Proof of Corollary 2: Differentiating Equations (15) and (16) yields:

$$\frac{\partial p_1^c}{\partial \lambda} = \frac{p_1^c}{(2-\lambda)} > 0 \text{ if and only if } a_2 < \frac{3+2a_1}{2},$$

$$\frac{\partial p_2^c}{\partial \lambda} = 8 - 3(4-\lambda)\lambda - 2[16-5(4-\lambda)\lambda]a_2 - 4[2-(4-\lambda)\lambda-(2-\lambda)^2 a_2]a_1 - 4(2-\lambda)^2 a_1^2$$

$$\frac{\partial w^A(\theta_2)}{\partial \lambda} = \left(\frac{1}{2}\right)\left[a_2 - \theta_2 + \frac{(2a_1-1)[3+2(a_1-a_2)]}{8}\right] > 0,$$

$$\frac{\partial w^B(\theta_2)}{\partial \lambda} = \left(\frac{1}{2}\right)\left[a_2 - (1-\theta_2) + \frac{(2a_1-1)[3+2(a_1-a_2)]}{8}\right] > 0,$$

Figure 6 captures the (a_1, a_2, λ) where $\frac{\partial p_2^c}{\partial \lambda} < 0$ and $\frac{\partial p_2^c}{\partial \lambda} > 0$. Clearly, there exists there exists $\hat{\lambda}(a_1, a_2) > 0.8$ such that $\frac{\partial p_2^c}{\partial \lambda} < 0$ when $\lambda < \hat{\lambda}(a_1, a_2)$. □

Proof of Proposition 6: Substituting the results from Proposition 4 into Equation (17) we have that

$$W^c = \frac{[2(a_1+a_2)-1][(6-4\lambda)(a_1+a_2)-(1-\lambda)]}{8(2-\lambda)^2}.$$

Differentiating implies that

$$\frac{\partial W^c}{\partial \lambda} = \frac{[2(a_1+a_2)-1][4(1-\lambda)(a_1+a_2)+\lambda]}{8(2-\lambda)^3} > 0.$$

$\square$

Proof of Proposition 7: (a) Equilibrium profit for Platform A is given by:

$$\Pi^A = \frac{2(a_1+a_2)-1}{4(2-\lambda)},$$

which implies that

$$\frac{\partial \Pi^A}{\partial \lambda} = \frac{2(a_1+a_2)-1}{4(2-\lambda)^2} > 0.$$

Symmetry implies that $\frac{\partial \Pi^B}{\partial \lambda} > 0$.

Equilibrium consumer surplus for a consumer that joins Platform A (i.e. of type $\theta_1 \in [0, \frac{1}{2}]$) is given by:

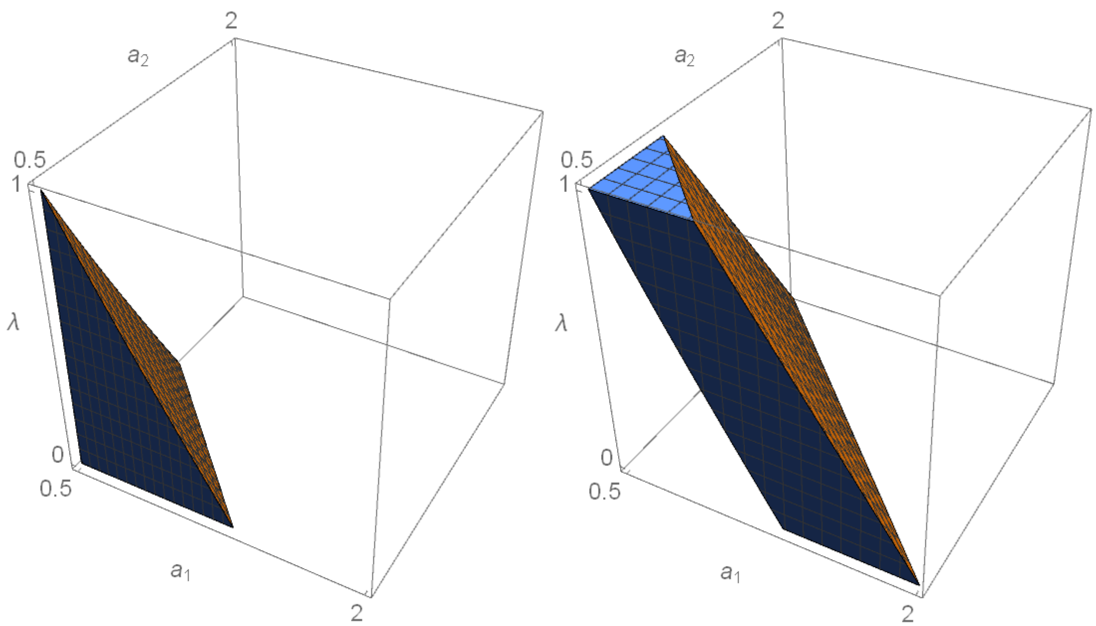
$$u_1^A(\theta_1) = \frac{2(a_1+a_2)-1}{8(2-\lambda)} \cdot [2(a_1+a_2)-3-4\theta_1],$$

which implies that

$$\frac{\partial u_1^A(\theta_1)}{\partial \lambda} = \frac{2(a_1+a_2)-1}{8(2-\lambda)^2} \cdot [2(a_1+a_2)-3-4\theta_1].$$

This is greater than zero if and only if $\theta_1 < \frac{2(a_1+a_2)-3}{4} \equiv \theta_1^c$. Note that $\theta_1^c = \frac{2(a_1+a_2)-3}{4} \leq \frac{5-3}{4} = \frac{1}{2} = n_1^c$ since $a_1 + a_2 + \lambda \leq \frac{5}{2}$. This implies that the Platform A consumers such that $\theta_1 \in [0, \theta_1^c]$ are better off while Platform A consumers such that $\theta_1 \in [\theta_1^c, n_1^A]$ are worse off

Figure 7: How the threshold for the Single-Homing Firm, θ_2^c, is determined



(a) The (a_1, a_2, λ) where $\theta_2^c < 1 - n_2^c$.

(b) The (a_1, a_2, λ) where $\theta_2^c > 1 - n_2^c$.

with greater platform bargaining power.

(b) Equilibrium firm surplus for a firm of type $\theta_2 \in [0, 1 - n_2^c]$ which *single-homes* on Platform A is given by:

$$u_2^A(\theta_2) = \frac{1-\lambda}{4}\left(\frac{2(a_1+a_2)-1}{2-\lambda} - 2\theta_2\right),$$

which implies that

$$\frac{\partial u_2^A(\theta_2)}{\partial \lambda} = \frac{1}{2}\left(\theta_2 - \frac{2(a_1+a_2)-1}{2(2-\lambda)^2}\right).$$

This is greater than zero if and only if $\theta_2 > \frac{2(a_1+a_2)-1}{2(2-\lambda)^2} \equiv \theta_2^c$. Figure 7 captures the (a_1, a_2, λ) where $\theta_2^c < 1 - n_2^c$ and $\theta_2^c > 1 - n_2^c$. Thus, if $\theta_2^c \geq 1 - n_2^c \Leftrightarrow 2(a_1 + a_2) \geq 1 + \frac{2(1-\lambda)^2}{3-\lambda}$, then all single-homing firms are worse off. However, if $\theta_2^c < 1 - n_2^c$, then single-homing firm of $\theta_2 \in [\theta_2^c, 1 - n_2^c]$ is better off, whereas all other firms with $\theta_2 < \theta_2^c$ are worse off.

Similarly, equilibrium firm surplus for a firm of type $\theta_2 \in [1-n_2^c, n_2^c]$ which *multi-homes* on Platforms A and B is given by:

$$\begin{aligned} u_2^M(\theta_2) = \frac{1-\lambda}{4(2-\lambda)}[&(2a_1^2+2a_2^2-4a_1a_2+3a_1-3a_2+2)\lambda \\ &-2(2a_1^2+2a_2^2-4a_1a_2+a_1-5a_2+3)], \end{aligned}$$

where the dependence of θ_2 can be suppressed. This implies that

$$\begin{aligned} \frac{\partial u_2^M}{\partial \lambda} \;=\; \frac{1}{4(2-\lambda)^2}[&\left(2a_1^2+2a_2^2-4a_1a_2+3a_1-3a_2+2\right)\lambda^2 \\ &-4\left(2a_1^2+2a_2^2-4a_1a_2+3a_1-3a_2+2\right)\lambda \\ &+4(2a_1^2+2a_2^2-4a_1a_2+2a_1-4a_2+2)+2]. \end{aligned}$$

Figure 8 captures the (a_1, a_2, λ) where $\frac{\partial u_2^M}{\partial \lambda} < 0$ and $\frac{\partial u_2^M}{\partial \lambda} > 0$, and Figure 9 shows the region for (a_1, a_2) with $\frac{\partial u_2^M}{\partial \lambda} > 0$ when $\lambda = 0$. Figures 8 and 9 together suggest that this region (a_1, a_2) shrinks as λ increases. Now, choose an arbtrary $\widetilde{a}_2 \in (\frac{1}{2}, 1)$, and define $\widetilde{a}_1 \equiv \frac{5}{2} - \widetilde{\lambda} - \widetilde{a}_2$ for an arbitrary $\widetilde{\lambda} \in [0,1]$, but $\widetilde{\lambda}$ must be chosen so that $\frac{1}{2} < \widetilde{a}_1 < 2$. Then, in the region (a_1, a_2, λ) with $a_1 \in (\frac{1}{2}, \widetilde{a}_1)$, $a_2 \in (\frac{1}{2}, \widetilde{a}_2)$, and $\lambda < \widetilde{\lambda}$, $\frac{\partial u_2^M}{\partial \lambda} > 0$.

□

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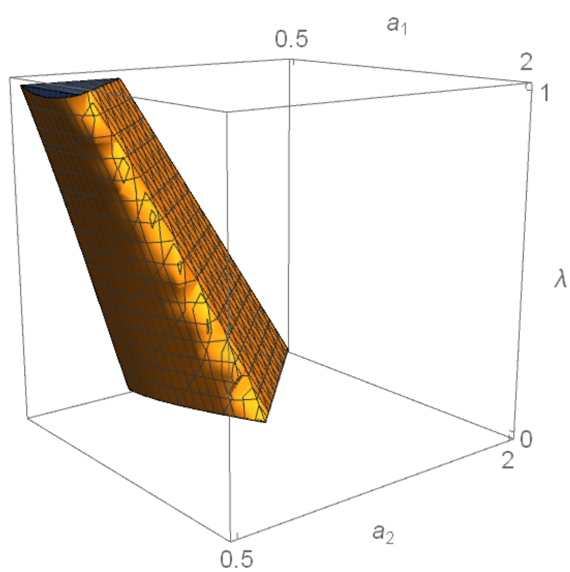
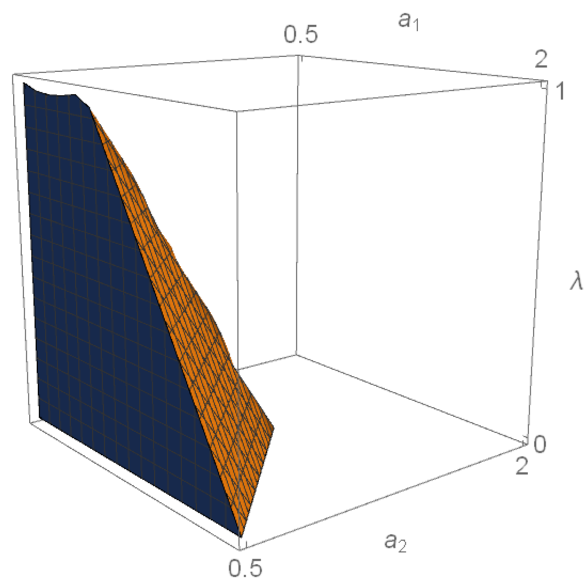
Figure 8: The Impact of Bargaining Power on Multi-Homing Firm Utility

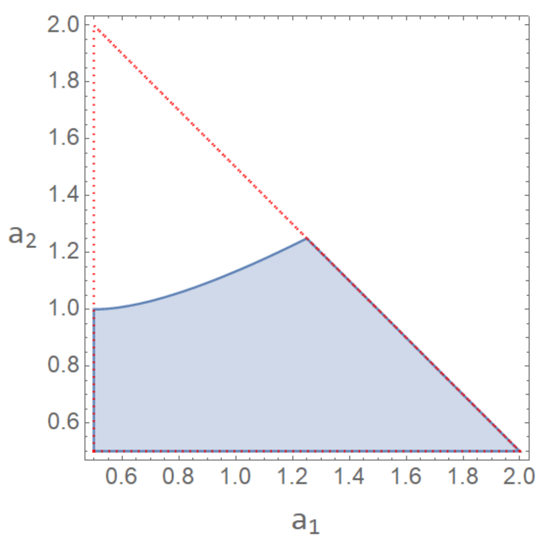


(a) The (a_1, a_2, λ) where $\frac{\partial u_2^M}{\partial \lambda} < 0$.



(b) The (a_1, a_2, λ) where $\frac{\partial u_2^M}{\partial \lambda} > 0$.

Figure 9: The Impact of Bargaining Power on Multi-Homing Firm Utility (continued)



Note: The (a_1, a_2) where $\frac{\partial u_2^M}{\partial \lambda} > 0$ when $\lambda = 0$.

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