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Price Elasticity and Pass-Through in the Nash Bargaining Solution in a Distribution Channel*

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Abstract

This paper analyzes the role of the industry's price elasticity and the determinants of pass-through in the Nash bargaining solution in a distribution channel. It is shown that the division of the upstream and downstream profits is characterized by the price elasticity and the distribution of bargaining powers. I also show that the demand curvature plays an important role in the determination of pass-through rates.

Keywords: Distribution Channels; Bargaining; Price Elasticity; Pass-Through; Demand Curvature.

JEL Classification: L13; L49; L66.

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1 Introduction

In vertical relationships, a retailer is usually a multi-product firm which sells competing brands (Choi, 1991; 1996). In this paper, I study the properties of the Nash bargaining solution (Nash, 1950) in such a distribution channel. First, I analyze the role of the *price elasticity* in the determination of the division of the upstream and downstream profits. It is shown that the common retailer's profit share becomes *lower* if the market demand becomes *less* elastic, holding the division of upstream and downstream bargaining powers is fixed. This is because through a negotiation process, an upstream firm can aggressively charge a higher wholesale price because it loses *less* from a sales reduction when the demand is not elastic.

Second, I show that an upstream cost increase is partly absorbed by the common retailer, and how much it is absorbed is determined by the *demand curvature*: if the market demand is "very convex," then the pass-through at the wholesale level is almost passed through to the final price, *irrespective of* the division of upstream and downstream bargaining powers. In this way, I argue the importance of the first- and the second-order demand characteristics as well as the division of bargaining powers in characterizing the Nash bargaining solution in a distribution channel.

In a similar vein, Aghadadashli, Dertwinkel-Kalt, and Wey (2016) study a model of *one upstream firm and N downstream firms* which bargain over the input price, produce outputs, and quantity-compete, and show that if the downstream firm's demand becomes *less* elastic, then *ceteris paribus*, the upstream monopolist earns a *higher* profit margin from the input price bargaining. Despite the differences in vertical structure and the mode of downstream competition, Aghadadashli, Dertwinkel-Kalt, and Wey (2016) and this paper's model of *one downstream firm and N upstream firms* share a similar intuition.

2 Model

Suppose that there are $N \geq 1$ symmetric upstream firms (manufacturers) whose marginal cost of production is constant, $c^U \geq 0$. They transact with a downstream firm (a common retailer) who is a monopolist in the geographical market. Each manufacturer produces one type of product, and for each manufacturer's product, the retailer incurs a constant marginal cost of sales, $c^D \geq 0$. The common retailer sells the manufacturers' products to the final market by choosing the prices $\mathbf{p} = (p_1, p_2, ..., p_N)$. Then, the demand (in terms of a market share) for product $j \in \mathcal{N} \equiv \{1, 2, .., N\}$ is written as $s_j = s_j(\mathbf{p})$. Here, N upstream firms are horizontally differentiated. The common retailer pays the unit price w_j to manufacturer j. Thus, the common retailer's total profit is written as $\Pi^D \equiv \sum_{j \in \mathcal{N}} \Pi_j^D$, where $\Pi_j^D \equiv (p_j - w_j - c^D) s_j(\mathbf{p})$. The first-order condition for p_j is given by

$$s_j(\mathbf{p}) + (p_j - w_j - c^D)\frac{\partial s_j}{\partial p_j} + \sum_{l \neq j}(p_l - w_l - c^D)\frac{\partial s_l}{\partial p_j} = 0. \tag{1}$$

On the other hand, if each manufacturer can directly distribute its product to the final market, then manufacturer j's profit is $\widetilde{\pi}_j^U \equiv (p_j - c^U - c^D) s_j(\mathbf{p})$, and thus the first-order condition for p_j is given by

$$s_j(\mathbf{p}) + (p_j - c^U - c^D)\frac{\partial s_j}{\partial p_j} = 0.$$

Hence, if $w_j \geq c^U$, then $\sum_{l \neq j}(p_l - w_l - c^D)\frac{\partial s_l}{\partial p_j} > (w_j - c^U)\frac{\partial s_j}{\partial p_j}$ because $\partial s_l / \partial p_j > 0$ for $l \neq j$ and $\partial s_j / \partial p_j < 0$. Thus, the final equilibrium in a distribution channel, $p^* = p(w)$, is *higher* than the equilibrium price without such a distribution channel, p^0. This is because the monopolistic retailer internalizes the effects of changing p_j on not only its own demand s_j but also on the demands for the other products, s_l's.

Following Aghadadashli, Dertwinkel-Kalt, and Wey (2016) and many others, I employ the simplifying assumption that each bargaining is played by an upstream firm and one of the N delegates from the common retailer, and each bargaining is unobservable from other upstream firms and other delegates. Additionally, it is also assumed that players hold "passive beliefs" in the sense that even if a player in one bargaining process observes out-of-equilibrium price offer, the player still holds the belief that the equilibrium is played (in the bargaining and the pricing decisions) by the players outside of this bargaining process.

Under these assumptions, I focus on the bargaining process over w_j. Given the players' belief that the symmetric equilibrium $\{w, p\}$ is played, it is determined to maximize the Nash product, $[\Pi^D - \underline{\Pi}^D]^\lambda [\pi_j^U]^{1-\lambda}$, where $\lambda \in (0, 1)$ is the common retailer's bargaining power, $\Pi^D = (p - w_j - c^D)s(p) + (N-1)(p - w - c^D)s(p)$ is the common retailer's total profit, $\underline{\Pi}^D$ is its *disagreement profit* that it obtains when the bargaining with manufacturer j breaks down, and $\pi_j^U = (w_j - c^U)s(p)$ is manufacturer j's profit from the wholesale bargaining. Here, note that the bargaining game is played by manufacturer j and the delegate of the common retailer, and they have a passive belief that the equilibrium is still played, in particular, the symmetric retail price p will still be chosen by the common retailer. Accordingly, the common retailer's disagreement payoff is perceived as $\underline{\Pi}^D = (N-1)(p-w-c^D)s(p)$, which implies that $\Pi^D - \underline{\Pi}^D = (p - w_j - c^D)s(p)$.[1]

3 Analysis

Now, I define the demand for each product under symmetric pricing by $s(p) \equiv s_j(p, p, ..., p)$. Then, the relationship, $s'(p) = \frac{\partial s_j}{\partial p_j} + (N-1)\frac{\partial s_j}{\partial p_l}$ holds. I also define *the industry's price elasticity* of demand by $\epsilon(p) \equiv -ps'(p)/s(p) > 0$.

3.1 The Role of the Industry's Price Elasticity

Then, the following proposition shows how the bargaining relationship is related to the demand conditions in the final market.

Proposition 1. *The upstream firm's share of the total profits, measured in terms of the final price, is expressed as*

$$\frac{w - c^U}{p} = \left(\frac{1-\lambda}{\lambda}\right)\left[\frac{1}{\epsilon(p)}\right] \tag{2}$$

for $\lambda \in (0, 1)$.

Proof. The first-order condition for w_j is given by

$$\begin{aligned}&\lambda[(p - w_j - s^D)s(p)]^{\lambda-1}[-s(p)][(w_j - c^U)s(p)]^{1-\lambda}\\&+(1-\lambda)[(w_j - c^U)s(p)]^{-\lambda}[s(p)][(p - w_j - c^D)s(p)]^{\lambda} = 0,\end{aligned}$$

[1]If consumers realize that the products are lost for the firms with which the negotiation breaks down while the retail prices remain the same even if such a breakdown happens, then the resulting market share for each firm is written as $\widetilde{s} \equiv s(\widetilde{p})$. Then,

$$\begin{aligned}\Pi^D - \underline{\Pi}^D &= (p - w_j - c^D)s(p) + (N-1)(p - w - c^D)s(p)\\&\quad -(N-1)(p - w - c^D)\widetilde{s}\\&= (p - w_j - c^D)s(p)\\&\quad +(N-1)(p - w - c^D)\{s(p) - \widetilde{s}\},\end{aligned}$$

which would not admit a tractable analysis as below.

which determines the symmetric w:

$$\lambda(w - c^U) = (1 - \lambda)(p - w - c^D).$$

Accordingly, the first-order condition for pricing (1) can rewritten as

$$\begin{aligned} & s(p) + (p - w - c^D)\left[\frac{\partial s_j}{\partial p_j} + (N-1)\frac{\partial s_j}{\partial p_l}\right] = 0 \\ \Leftrightarrow \quad & s(p) + (p - w - c^D)s'(p) = 0, \end{aligned}$$

where the common retailer takes care of the "industry" as a whole: it takes into account not only its own price effect, $\partial s_j/\partial p_j$, but also the aggregate spillover effects on other products, $(N-1)(\partial s_j/\partial p_l)$. Equation (2) is then obtained by combining these two equations. □

Equation (2) shows that the upstream firm's bargaining power becomes larger (i.e., $(1-\lambda)$ becomes larger), then, with other things unchanged, the upstream firm's share of profits also becomes larger. More importantly, if the industry's price elasticity of demand becomes *less* elastic (i.e., ϵ becomes smaller), then the upstream profit share becomes, *ceteris paribus*, *larger*. Intuitively, this is because the upstream firm loses *less* from a sales reduction by aggressively charging a high wholesale price to common retailer through the bargaining process.

The result above is even clearer if Holmes' (1989) decomposition is used. As Holmes (1989) shows, under symmetric pricing, the industry's price elasticity is *the firm's own price elasticity*, subtracted by *the cross price elasticity*: $\epsilon(p) = \epsilon_F(p) - \epsilon_C(p)$, where $\epsilon_F(p) \equiv -(p/s(p))\partial s_j(\mathbf{p})/\partial p_j|_{\mathbf{p}=(p,\ldots,p)}$ and $\epsilon_C(p) \equiv (N-1)(p/s(p))\partial s_l(\mathbf{p})/\partial p_j|_{\mathbf{p}=(p,\ldots,p)}$ for any distinct pair of indices j and l. Thus, an increase in the degree of competition (due to less product differentiation and/or an increase in N) *raises* the upstream firm's profit share, holding the final price p fixed. This result is, again, in line with the intuition above: the upstream firm loses *less* from an aggressive attitude in the bargaining because it already faces a severe level of competition. The total effect is less unambiguous, though, because the final price p would also be lower by a higher level of competition. In contrast, the effect of ϵ_F is exactly the opposite: if the own price elasticity is very elastic, then the upstream firm has to be less aggressive because an increase in w, and thus the associated increase in p, significantly reduces the demand.

Now, from Equation (2) above, the wholesale price is obtained by

$$w = \frac{1-\lambda}{\lambda}\left(\frac{-s}{s'}\right) + c^U,$$

which leads to the following equation:

$$p = c^U + c^D + \frac{1}{\lambda}\left(\frac{s}{-s'}\right). \tag{3}$$

Here, as N increases, p goes down. How does the margin ratio moves as N increases? The answer is that it is *independent* of N simply because

$$\frac{p - w - c^D}{w - c^U} = \frac{\lambda}{1 - \lambda}.$$

3.2 Pass-Through

Next, I define three different types of *cost pass-through*: the *wholesale* and the *final price pass-throughs of the upstream cost*, $\partial w/\partial c^U$ and $\rho^U \equiv \partial p/\partial c^U$, respectively, and *price pass-through of the downstream cost*, $\rho^D \equiv \partial p/\partial c^D$. I also define the *demand curvature* by $\sigma(p) \equiv ss''/[s']^2$. Among others, Adachi and Ebina (2014), Chen and Schwartz (2015), and Gaudin (2016) show that for the optimum it is necessary that $2 > \sigma$, and furthermore, $s(p)$ should not be "too convex," that is, s'' is sufficiently small that $1 > \sigma$. As Chen and Schwartz (2015) argue, many classes of demand functions satisfy this condition. Therefore, I also assume this restriction. Then, the following proposition is obtained.

Proposition 2. *The wholesale pass-through is larger than the upstream cost pass-through and the downstream cost pass-through, which are equal:*

$$\frac{\partial w}{\partial c^U} = (2 - \sigma)\rho^U > \rho^U = \rho^D.$$

Proof. Let $F(p, w, c^D) \equiv s(p) + (p - w - c^D)s'(p)$ and $G(p, w, c^U) \equiv \lambda(w - c^U)\epsilon(p) - (1 - \lambda)p$. Essentially, our simplifying assumptions make it unnecessary to consider the dependence of p on w: the retail prices and the wholesale prices are simultaneously determined. Then, these two equilibrium conditions, $F(p, w, c^D) = 0$ and $G(p, w, c^U) = 0$, can be utilized to develop implications for the three types of pass-through. First, the *wholesale* and the *final price pass-throughs of the upstream cost*, $\partial w/\partial c^U$ and $\rho^U \equiv \partial p/\partial c^U$, satisfy

$$\begin{bmatrix} \frac{\partial F}{\partial p} & \frac{\partial F}{\partial w} \\ \\ \frac{\partial G}{\partial p} & \frac{\partial G}{\partial w} \end{bmatrix} \begin{bmatrix} \rho^U \\ \\ \frac{\partial w}{\partial c^U} \end{bmatrix} = - \begin{bmatrix} \frac{\partial F}{\partial c^U} \\ \\ \frac{\partial G}{\partial c^U} \end{bmatrix}.$$

Now, let the determinant be defined by $D \equiv \left(\frac{\partial F}{\partial p}\right)\left(\frac{\partial G}{\partial w}\right) - \left(\frac{\partial F}{\partial w}\right)\left(\frac{\partial G}{\partial p}\right)$. Then

$$\begin{aligned} D &= s'\left\{\lambda\left(2 - \frac{ss''}{[s']^2}\right) + (1-\lambda)\left(\frac{p\epsilon'}{\epsilon} - 1\right)\right\} \\ &= s'\left\{\lambda\,(2-\sigma) + (1-\lambda)(1-\sigma)\epsilon\right\} < 0 \end{aligned}$$

for all $\lambda \in (0,1)$, because $\epsilon' = -\{s's + p\,[s''s - (s')^2]\}/s^2$ so that

$$\begin{aligned} \frac{p\epsilon'}{\epsilon} - 1 &= \left(-\frac{ps'}{s}\right)\left(1 - \frac{ss''}{[s']^2}\right) \\ &= \epsilon(1-\sigma). \end{aligned}$$

Now, it is proceeded as

$$\begin{bmatrix} \rho^U \\ \\ \dfrac{\partial w}{\partial c^U} \end{bmatrix} = \frac{-1}{D}\begin{bmatrix} \dfrac{\partial G}{\partial w}\dfrac{\partial F}{\partial c^U} - \dfrac{\partial F}{\partial w}\dfrac{\partial G}{\partial c^U} \\ \\ -\dfrac{\partial G}{\partial p}\dfrac{\partial F}{\partial c^U} + \dfrac{\partial F}{\partial p}\dfrac{\partial G}{\partial c^U} \end{bmatrix},$$

which implies that

$$\begin{aligned} \rho^U &= \left(\frac{s'}{D}\right)\lambda\epsilon \\ &= \frac{\lambda\epsilon}{\lambda\,(2-\sigma) + (1-\lambda)(1-\sigma)\epsilon} > 0, \end{aligned}$$

and

$$\begin{aligned} \frac{\partial w}{\partial c^U} &= \left(\frac{s'}{D}\right)(2-\sigma)\,\alpha\epsilon \\ &= \frac{\lambda\,(2-\sigma)\,\epsilon}{\lambda\,(2-\sigma) + (1-\lambda)(1-\sigma)\epsilon} > 0. \end{aligned}$$

Similarly, the *price pass-through of the downstream cost*, $\rho^D \equiv \partial p/\partial c^D$, is obtained by

$$\begin{aligned} \rho^D &= \frac{-1}{D}\left\{\frac{\partial G}{\partial w}\frac{\partial F}{\partial c^D} - \frac{\partial F}{\partial w}\frac{\partial G}{\partial c^D}\right\} \\ &= \left(\frac{s'}{D}\right)\alpha\epsilon \\ &= \frac{\lambda\epsilon}{\lambda\,(2-\sigma) + (1-\lambda)(1-\sigma)\epsilon} = \rho^U. \end{aligned}$$

Since $2 - \sigma > 1$, $\partial w/\partial c^U > \rho^U = \rho^D$. $\square$

This proposition shows that the common retailer absorbs the upstream cost shocks by $100 \times [(1-\sigma)/(2-\sigma)]\%$. Here, the demand curvature plays an important role: if the industry's demand becomes "very convex" (σ becomes close to one), then the wholesale price increase is almost passed through to the final price, irrespective of the common retailer's bargaining power, λ.[2] However, as λ increases, both ρ^U and $\partial w/\partial c^U$ also increase because

$$\frac{\partial \rho^U}{\partial \lambda} = \frac{\partial \rho^D}{\partial \lambda} = \frac{(1-\sigma)\epsilon^2}{[\lambda(2-\sigma)+(1-\lambda)(1-\sigma)\epsilon]^2} > 0,$$

and

$$\frac{\partial\left(\frac{\partial w}{\partial c^U}\right)}{\partial \lambda} = \frac{(2-\sigma)(1-\sigma)\epsilon^2}{[\lambda(2-\sigma)+(1-\lambda)(1-\sigma)\epsilon]^2} > 0.$$

Again, the latter is larger than the former.

Next, the direct effects of λ on the retail and the wholesale prices are obtained by

$$\begin{bmatrix} \dfrac{\partial p}{\partial \lambda} \\[2ex] \dfrac{\partial w}{\partial \lambda} \end{bmatrix} = \frac{-1}{D} \begin{bmatrix} -\dfrac{\partial F}{\partial w}\dfrac{\partial G}{\partial \lambda} \\[2ex] \dfrac{\partial F}{\partial p}\dfrac{\partial G}{\partial \lambda} \end{bmatrix},$$

which indicates that

$$\frac{\partial p}{\partial \lambda} = -\left(\frac{s'}{D}\right)\left(\frac{p}{\lambda}\right) < 0$$

and

$$\frac{\partial w}{\partial \lambda} = -\left(\frac{s'}{D}\right)(2-\sigma)\left(\frac{p}{\lambda}\right) < 0.$$

Interestingly, as the common retailer's bargaining power increases for all upstream manufacturers, the final price *decreases*. This is because the common retailer can lower the wholesale price though the bargaining, still maintaining product competition. If the common retailer loses its bargaining power, the loss from less product competition becomes larger than the gain from elimination of double marginalization.

Finally, imposing symmetry on Equation (3) yields:

$$p = c^U + c^D + \frac{p}{\lambda\epsilon(p)}$$

[2]On the other hand, an increase in the retailer's marginal cost c^D *lowers* the wholesale price:

$$\frac{\partial w}{\partial c^D} = -\frac{(1-\lambda)(1-\sigma)\epsilon}{\lambda(2-\sigma)+(1-\lambda)(1-\sigma)\epsilon} < 0.$$

$$\Leftrightarrow \quad p = \frac{c^U + c^D}{1 - \dfrac{1}{\lambda\epsilon(p)}},$$

which implies that

$$\rho^U = \rho^D = \frac{1}{1 - \dfrac{1}{\lambda\epsilon}}.$$

As a summary, a higher bargaining power of the common retailer (λ) *raises* the upstream cost and the downstream cost pass-throughs (ρ^U and ρ^D), and *lowers* the final price (p), suppressing the degree of double marginalization.

4 Concluding Remarks

In this paper, I characterize the Nash bargaining division of the profits between upstream firms and a common retailer in terms of the industry's price elasticity (Proposition 1) and the different types of cost pass-through (Proposition 2). How do these results change if downstream competition is introduced? This is an important issue for future research.

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