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Is a Big Entrant a Threat to Incumbents? The Role of Demand Substitutability in Competition among the Big and the Small*

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Abstract

We establish a model of market competition between large and small firms and investigate the way in which demand substitutability affects how the entry of big firms impacts incumbents. We focus on the relative strength of two opposing effects of entry on large incumbent firms' demand: the direct substitution effect among large firms (negative) and the indirect feedback effect through the change in small firms' aggregated behavior (positive). If the substitutability between large and

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small firms is sufficiently high, the indirect effect dominates the direct effect and large incumbents' equilibrium prices and profits increase. We show that welfare effects are ambiguous, which calls for careful assessment when regulating large firms' entry.

Keywords: big firms, small firms, substitutability, entry, market impacts

JEL classification: D21, D43, L11, L13

1 Introduction

The entry of a large firm has substantial effects on market competition and market structure, and evidence suggests that the impact on large and small firms differs across markets. Igami (2011) studies the supermarket industry in Japan after deregulation (that is, the relaxation of the Large-Scale Retail Law) and shows that the entry of large supermarkets has a negative impact on large (and medium) supermarkets and a neutral or positive impact on small supermarkets. However, IKEA's entry into South Korea in 2014 increased the profits of large furniture makers while substantially lowering the profits of small furniture makers.[1] Why do the impacts differ? What factors determine these differences? Does the entry of a large firm enhance overall efficiency?

To answer these questions, we consider the following model of competition among large and small firms and study the impact of large firms' entry. A large firm is modeled as a multi-product firm, and the number of products it offers is a choice variable.[2] A small firm is modeled as a single-product firm with free entry and exit. Following the model of monopolistic competition, we treat each variety symmetrically. While the number of large firms is finite and exogenous, the variety of each large firm is determined through oligopolistic competition and the measure of small firms is determined by the free entry condition. We consider a static game in which all decisions, including entry, variety choices, and production, are simultaneous. Our model has the following two key features. First, a large firm can exert market power through variety choice and coordinated pricing of its own varieties. Second, by adopting a quadratic quasi-linear utility function for the representative consumer, demand in our model displays rich configurations of product substitutability within and across large and small firms.[3] We derive the condition under which a unique mixed market equilibrium exists (i.e., the coexistence of active large and small firms) and investigate the impact of a large firm's entry on other firms' behavior and welfare.

Our main result shows how product substitutability affects the impact of a large firm's

[1] After IKEA's entry into South Korea in 2014, large national furniture makers such as HANSSEM and LIVART enjoyed an increase in revenue. After an IKEA store was established in Gwangmyeong, the revenue of LIVART's Gwangmyeong branch increased by 27%, while HANSSEM's Gwangmyeong store saw a 10% rise in sales relative to the same period in the previous year. Small furniture makers, however, suffered from a revenue decrease of more than 70% on average. Source: *Korea Bizwire*. March 27, 2015, "Korea's Large Furniture Makers Boost Revenues Thanks to IKEA," http://koreabizwire.com/koreas-large-furniture-makers-boost-revenues-thanks-to-ikea/32438

[2] Bernard et al. (2010) show that multi-product firms are nearly omnipresent in the U.S. manufacturing industry. According to data from the period between 1979 and 1992, multi-product firms account for 41% of all firms but supply 91% of total output. In addition, 89% of multi-product firms adjust their product range every five years.

[3] Our modeling approach draws closely on that of Parenti (2016), who also considers multi-product large firms and single-product small firms in a given market based on quasi-linear utility with quadratic subutility. More broadly, a quasi-linear quadratic form of the utility function is also adopted by Singh and Vives (1984), Ottaviano and Thisse (1999), and Ottaviano et al. (2002). Unlike our paper, none of these papers considers different degrees of substitutability across firms.

entry on incumbent large firms. Indeed, we find a necessary and sufficient condition under which the product range, the price of each product, and the profit of each incumbent all increase (Proposition 2). The key is to separate two effects of a large firm's entry on the demand for incumbent large firms' products. The first effect is the direct substitution effect: the new large firm's products are substitutes for the incumbent large firms' products. This negatively affects the demand for large firms' products. The second effect is the indirect feedback effect due to the change in the number of small firms.[4] A large firm's entry squeezes out some small firms if the products are substitutes across large and small firms, whereas such entry invites more small firms to participate if the products are complements. It is important to note that the resulting indirect feedback effect is non-negative on the demand for large firms' products, regardless of whether large firms' and small firms' products are substitutes or complements. If the degree of substitutability or complementarity between large firms' and small firms' products is relatively larger than the substitutabilities within large firms' products and within small firms' products, the indirect feedback effect outweighs the direct substitution effect, thereby resulting in a rise in demand for the large firms' products, which characterizes the condition for an increase in the product range, the price, and the profit of an incumbent large firm.

Our finding may explain the different impacts of a large firm's entry in the empirical evidence reported above. As Igami (2011) observes in his analysis of supermarkets in Japan, large and medium incumbents seem to compete directly with large entrants, while "small incumbents are insulated by product differentiation and even benefit from the positive demand externality (additional flow of shoppers)." Thus, the substitutability among large supermarkets is strong, while the substitutability between large and small supermarkets is weakly positive (or even slightly negative).[5] Therefore, large incumbents may suffer from intensified competition from new large rivals because the feedback effect may not be strong enough to offset the substitution effect. In contrast, small stores may benefit from the large entrants because the latter generate a positive demand externality (in addition to a negative but small substitution effect) on the former. In the South Korean furniture industry, large furniture makers have a superior ability to design furniture, which implies that the products of large furniture makers are more differentiated

[4]Shimomura and Thisse (2012) are the first to identify this effect but with a different term – the "market expansion effect" – which is analogous to our "indirect feedback effect". Note that the "market expansion effect" originates from the exit of some small firms in Shimomura and Thisse (2012), while the "indirect feedback effect" in this paper stems from the exit of some small firms when products are substitutes across large and small firms and from the entry of small firms when products are complementary between large and small firms.

[5]Large incumbents with a similar store size face direct competition with new large rivals. In contrast, small retailers differentiate themselves from large ones in terms of store size and serve different demands. Therefore, the substitution effect on small stores from large supermarkets is weak. Furthermore, new large retailers attract extra shopper traffic, which raises the demand in the neighborhood and hence creates profit opportunities for small stores. This process generates a positive demand externality for small supermarkets. If the positive demand externality dominates the substitution effect, the existence of large retailers would increase the demand for small ones, and thus, large and small stores are weakly complementary.

than those of small furniture makers and hence suggests that the substitutability among large firms is weaker than that between large and small firms. Therefore, the feedback effect could outweigh the substitution effect, resulting in a rise in demand for large firms' products, while the squeezing-out effect on small firms is strong.

The welfare effects of a large firm's entry are ambiguous. The change in social welfare depends on the replacement effect, the variety effect, and the quantity effect (on producer surplus). To fix ideas, consider the case in which large and small firms' products are substitutes. Observe that some small firms exit after the entry of a large firm, while large firms' (total) product range expands, and thus, a portion of small firms' product range is replaced by that of large firms. Furthermore, the total variety range after entry may expand or shrink. We examine the associated changes in consumer welfare. For each variety in the replacement range, the consumer welfare increases or decreases depending on the relative allocative inefficiency between large and small firms. Moreover, if the net variety range expands (shrinks), consumer welfare improves (worsens) because consumers prefer diversified consumption. For the associated change in producer surplus, the marginal change in large firms' aggregate profits is always positive due to the expansion of their total quantity.[6] This argument shows that the welfare effect depends on each effect's sign and/or the relative strength. Our model thus implies that policy makers should carefully assess the implications of a large firm's entry.

There are several strands of literature on markets with large and small firms and they differ in terms of how they capture large firms' market power. The first strand is the so-called dominant firm model (e.g., Markham, 1951, Chen, 2003, and Gowrisankaran and Holmes, 2004). The dominant firm is large since it is the leader and the price-maker, while the price-taking followers are small. Another strand employs the Stackelberg model (Etro, 2004, 2006, and Ino and Matsumura, 2012). In this model, the first mover is large due to the commitment power in the market.

The third strand models a mixture of oligopolistic and monopolistic competition (Shimomura and Thisse, 2012; and Parenti, 2016). With respect to the modeling approach, our paper is closely linked to these recent advances in studies of market structure in the presence of large and small firms. Shimomura and Thisse (2012) are the first to connect oligopolistic and monopolistic competition, where oligopolistic firms are large due to their ability to produce a large amount, while monopolistically competitive firms are so small that they can produce only a negligible amount in the market. These authors establish a general equilibrium model in which large firms account for (i) strategic interactions within their group, (ii) the aggregate behavior of small firms, and (iii) the endogenous income generated by the profit distribution. This model is an important starting point for the present paper, where we introduce asymmetry in product substitution across oligopolistic large firms and small monopolistic competitors. In addition, this paper is also related to Parenti (2016), who characterizes a similar mixed market structure in a representative consumer model with linear quadratic preferences and examines the impact of trade lib-

[6] Consumer welfare and producer surplus are also affected by the price change due to a large firm's entry. However, these effects on social welfare are neutral.

eralization on large and small firms. Our model also employs a linear quadratic utility function, but we go a step further by allowing for different substitutabilities of products within and across large and small firms. This approach is crucial for determining the interactions between large and small firms.[7]

This paper's focus on the impact of a large firm's entry is shared by Shimomura and Thisse (2012) and Ino and Matsumura (2012). Ino and Matsumura (2012) study a homogeneous-good Stackelberg game with multiple leaders and free-entry followers and find that the impact of adding another Stackelberg leader is beneficial to social welfare since it eliminates some excessive entry by followers while increasing the total quantity supplied. Shimomura and Thisse (2012) study a general equilibrium model featuring a mixture of oligopolistic and monopolistic competition. They find that the entry of a large firm increases the incumbent large firms' profits and raises consumer and social welfare. The present paper differs from Shimomura and Thisse (2012) in the following respects. First, Shimomura and Thisse (2012) employ Cobb-Douglas CES nested utility, while we employ a quasi-linear utility function with quadratic subutility.[8] Therefore, this paper does not account for the general equilibrium effect on the differentiated goods market due to the endogenous income from large firms' profits. Second – and more importantly – Shimomura and Thisse (2012) assume the same substitution among the products of large and small firms, but we consider richer substitutability among the products, which is the essential element that distinguishes our model from theirs. Incorporating different degrees of substitutability and complementarity, our study sheds new light on competition among large and small firms.[9] The absence of the general equilibrium effect in our model allows us to highlight the role of different degrees of substitutability across large and small firms with sharper results. Therefore, our framework provides a feasible way to generate flexible patterns of production behavior and welfare changes.[10]

The remainder of the paper is organized as follows. We construct the model in Section 2. In Section 3, we analyze the equilibrium of the model and explore the impact of a large firm's entry. Section 4 briefly discusses the robustness of the established results, while Section 5 concludes.

[7] Another strand of research differentiates between large and small firms from the perspective of firm heterogeneity in cost, represented by Lahiri and Ono (1988) and Matsumura and Matsushima (2010).

[8] For a detailed discussion of different classes of utility functions, see Parenti, Ushchev, and Thisse (2017).

[9] There is an additional difference between our model and Shimomura and Thisse's (2012): in their model, a large firm is a single-product firm supplying a non-negligbile quantity, while in our model, a large firm is a multi-product firm: it chooses a variety range and the quantities of each chosen variety. Since the equilibrium per-variety quantity level is the same, the variety range in our model effectively plays the same role as the large firm's quantity in Shimomura and Thisse (2012).

[10] For example, social welfare may improve or worsen in our model, depending on the relative degree of substitutability between large and small firms, while it always improves in Shimomura and Thisse (2012) and Ino and Matsumura (2012).

2 The Model

Consider a closed economy consisting of two sectors. Firms in sector 1 are perfectly competitive and produce a homogenous good under constant returns to scale. Sector 2 provides differentiated goods that are produced by two types of firms. The first type of firm is large in size, and the number of these firms is exogenous. The second type of firm is infinitesimal and freely enters or exits the market. All the varieties in Sector 2 are differentiated so that no two firms offer identical products.

The large and small firms differ in four respects. First, each large firm has a non-negligible impact on the market and competes in an oligopolistic manner, whereas each small firm is negligible in the market and behaves as a monopolistic competitor. Second, following the approach of Shimomura and Thisse (2012), we assume that the number of large firms is exogenous, while the number of small firms is endogenously determined by free entry and exit. Third, each large firm produces a range of varieties and strategically chooses both the product range and the quantity of each variety, while each small firm produces only one variety of product. Fourth, the varieties are equally substitutable within the group of large firms and that of small firms, but the level of substitution across these two types of firms can be different.

2.1 Preferences and Demand

The utility of the representative consumer U is described by a quasi-linear utility function with a quadratic subutility:

$$U = \alpha \left[\int_0^N q_S(i)di + \sum_{m=1}^{M} \int_0^{\overline{\omega}_m} q_L^m(\omega)d\omega \right] - \frac{\beta}{2} \int_0^N [q_S(i)]^2 di - \frac{\beta}{2} \sum_{m=1}^{M} \int_0^{\overline{\omega}_m} [q_L^m(\omega)]^2 d\omega$$

$$- \frac{\gamma_1}{2} \left[\int_0^N q_S(i)di \right]^2 - \frac{\gamma_2}{2} \left[\sum_{m=1}^{M} \int_0^{\overline{\omega}_m} q_L^m(\omega)d\omega \right]^2 - \gamma_3 \left[\int_0^N q_S(i)di \right] \left[\sum_{m=1}^{M} \int_0^{\overline{\omega}_m} q_L^m(\omega)d\omega \right] + q_0, \tag{1}$$

where $q_S(i)$ is the quantity of small firm i with $i \in [0, N]$. The output of each small firm is of zero measure, and the total mass of small firms is N, describing the competitive fringe. Large firm m ($m = 1, \ldots, M$) provides multiple varieties of products, with the product range represented by $\bar{\omega}_m \geq 0$, which is chosen by large firm m. The quantity of variety $\omega \in [0, \overline{\omega}_m]$ is $q_L^m(\omega)$. The total number of the incumbent large firms is $M \geq 2$, which is exogenously given. Here, we treat $\overline{\omega}_m$ continuously. The output of sector 1 is q_0, which is treated as the numeraire.

Consumer preferences are characterized by five parameters, which are α, β, and γ_j ($j = 1, 2, 3$). The intensity of preferences for the differentiated good is measured by $\alpha > 0$, which determines the size of the differentiated goods market, and $\beta > 0$ implies the consumer's preference for diversified product consumption. The substitutability among varieties is characterized by γ_j ($j = 1, 2, 3$). Specifically, the substitutability among the varieties produced by small firms and that among the varieties of large firms are expressed by γ_1 and γ_2, respectively, and the cross-substitutability between the varieties of large firms and those of small firms is expressed by γ_3[11]. The products are substitutes if $\gamma_j > 0$ and complements if $\gamma_j < 0$, and the products are closer substitutes (complements) when $|\gamma_j| > 0$ $(j = 1, 2, 3)$ is higher. Finally, to ensure the concavity of the quadratic subutility, we impose the following conditions:

(i) $\beta/N + \gamma_1 > 0$,

(ii) $\beta/(\sum_m \overline{\omega}_m) + \gamma_2 > 0$, and

(iii) $(\beta/N + \gamma_1)[\beta/(\sum_m \overline{\omega}_m) + \gamma_2] > \gamma_3^2$.[12]

It should be noted here that the measure of small firms, N, and the variety of each large firm, $\bar{\omega}_m$, are determined in equilibrium. For the main analysis, we need greater flexibility in the substitution parameters, γ_j. To ensure this, we assume that there are some large $\bar{N} < \infty$ and $\omega^{\max} < \infty$ such that γ_1, γ_2, and γ_3 satisfy (i) - (iii) for $N \leq \bar{N}$ and $\bar{\omega}_m \leq \omega^{\max}$ for all m.

The representative consumer's budget constraint is

$$\int_0^N p_S(i)q_S(i)di + \sum_{m=1}^{M} \int_0^{\overline{\omega}_m} p_L^m(\omega)q_L^m(\omega)d\omega + q_0 = I,$$

where $p_S(i)$ and $p_L^m(\omega)$ are the prices of the small firm i's and large firm m's variety ω, respectively. The price of the numeraire is normalized to 1. The representative consumer's income is I, which is exogenously given. The inverse demand functions facing small firms and large firms are determined by the maximization of the consumer's utility subject to the budget constraint:[13]

$$p_S(i) = \alpha - \beta q_S(i) - \gamma_1 Q_S - \gamma_3 Q_L, \tag{2}$$

$$p_L^m(\omega) = \alpha - \beta q_L^m(\omega) - \gamma_3 Q_S - \gamma_2 Q_L, \tag{3}$$

where $Q_S \equiv \int_0^N q_S(i)di$ and $Q_L \equiv \sum_{m=1}^{M} \int_0^{\overline{\omega}_m} q_L^m(\omega)d\omega$ are the total output of the small firms and that of the large firms, respectively.

[11] A few existing works on multiproduct firms allowed for substitutability between varieties within a firm to differ from the substitutability between varieties across firms (Allanson and Montagna, 2005; Hottman et al., 2011; Bernard et al., 2016). Different from these works, this paper focuses on asymmetric substitutabilities across large and small firms. We can generalize our model to allow for the asymmetric substitutability within and across multiproduct large firms, and our results still hold qualitatively.

[12] See Appendix **A** for details. Here, γ_i $(i = 1, 2, 3)$ can be positive or negative as long as the conditions for the concavity of the utility function hold. We will discuss the different signs of γ_i in Section 3.

[13] We establish the strict concavity of the utility function to derive the inverse demand function from the variety-wise first-order conditions almost everywhere. For the optimization in an infinite-dimensional vector space, see Luenberger (1969, Lemma 1 in Section 8.7).

2.2 Firms

Both large and small firms incur variable costs and fixed costs. All firms incur a common and constant marginal cost, which is normalized to zero, whereas the fixed cost may differ across the two types of firms.

Small Firms The profit of the small firm is expressed by

$$\Pi_S(i) = p_S(i)q_S(i) - f,$$

where $\Pi_S(i)$ is the profit of small firm i, and f is the fixed cost of the small firm.

Plugging $p_S(i)$ from equation (2) into the above profit function, $\Pi_S(i)$ can be rewritten as

$$\Pi_S(i) = \alpha q_S(i) - \beta[q_S(i)]^2 - [\gamma_1 Q_S + \gamma_3 Q_L]q_S(i) - f, \tag{4}$$

Each small firm maximizes its profit with respect to its quantity $q_S(i)$, taking as given the total output of small firms, Q_S, and that of large firms, Q_L.

The free entry and exit of small firms pins the equilibrium profit of the small firm to zero:

$$\Pi_S(i) = \alpha q_S(i) - \beta[q_S(i)]^2 - [\gamma_1 Q_S + \gamma_3 Q_L]q_S(i) - f = 0. \tag{5}$$

Large Firms The profit of the large firm is

$$\Pi_L^m(\overline{\omega}_m, q_L^m(\cdot)) = \int_0^{\overline{\omega}_m} (p_L^m(\omega)q_L^m(\omega) - F)d\omega,$$

where $\Pi_L^m(\overline{\omega}_m, q_L^m(\cdot))$ is the profit of large firm m, and F is the fixed production cost for the large firm to produce one variety.

Substituting $p_L^m(\omega)$ from equation (3) into the above profit function, $\Pi_L^m(\overline{\omega}_m, q_L^m(\cdot))$ can be rewritten as

$$\begin{aligned}\Pi_L^m(\overline{\omega}_m, q_L^m(\cdot)) = &\left\{\alpha - \gamma_3 Q_S - \gamma_2 \sum_{k\neq m} \int_0^{\overline{\omega}_k} q_L^k(\omega)d\omega\right\} \int_0^{\overline{\omega}_m} q_L^m(\omega)d\omega \\ &- \beta \int_0^{\overline{\omega}_m} [q_L^m(\omega)]^2 d\omega - \gamma_2 \left[\int_0^{\overline{\omega}_m} q_L^m(\omega)d\omega\right]^2 - F\overline{\omega}_m.\end{aligned} \tag{6}$$

The large firm maximizes its profit with respect to both its product range, $\overline{\omega}_m$, and the quantity of each variety, $q_L^m(\omega)$. Unlike a small firm, the large firm does not take the total output of large firms, Q_L, as given. Instead, each large firm takes as given the total output of other large firms, Q_L^{-m}, defined by $Q_L^{-m} \equiv \sum_{k\neq m} \int_0^{\overline{\omega}_k} q_L^k(\omega)d\omega$, in addition to Q_S.

2.3 Definition of Equilibrium

Since consumers are passive, an equilibrium state arises if no firm wishes to unilaterally deviate. Note that we consider market competition by the large firms and small firms, in which all firms behave simultaneously, including the entry decision of the small firms. Our solution concept is Nash equilibrium.

An equilibrium is characterized by the mass of small firms, N^*, the output of each small firm, $q_S^*(i)$, $\forall i \in N^*$, the product range of each large firm, $\overline{\omega}_m^*$, $\forall m = 1, \dots, M$, and the output of each variety for the large firm, $q_L^{m*}(\omega)$, $\forall \omega \in [0, \overline{\omega}_m^*]$, $\forall m = 1, \dots, M$, such that each firm maximizes the profits given other firms' behavior and no other small firms can earn positive profits due to free entry. An equilibrium is called a mixed market equilibrium if $Q_S^* > 0$ and $Q_L^* > 0$.

2.4 Welfare

The social welfare comprises consumer welfare and producer surplus. Consumer welfare is measured by

$$CW = U - I,$$

and hence, the change in consumer welfare is the same as that in consumer utility.

Since small firms earn zero profit, producer surplus is given by the sum of all large firms' profits:

$$PS = \sum_{m=1}^{M} \Pi_L^m.$$

Then, social welfare is the sum of consumer welfare and producer surplus:

$$SW = U - I + \sum_{m=1}^{M} \Pi_L^m. \tag{7}$$

3 Equilibrium Analysis

In this section, we derive the equilibrium results and conduct the comparative static analysis to investigate the impact of the entry of a large firm on the other firms' behavior and welfare.

3.1 Derivation of Mixed Market Equilibrium

Small Firms' Profit Maximization and Entry A small firm only accounts for the impact of the market's total production because its own impact on the market is negligible. Thus, it does not internalize its externality into its production. The small firm maximizes its profit given by equation (4) with respect to its output $q_S(i)$. The first-order condition of a small firm with respect to its output $q_S(i)$ is

$$\alpha - 2\beta q_S^*(i) - \gamma_1 Q_S - \gamma_3 Q_L = 0.$$

which implies that

$$q_S^*(i) = q_S^* = \frac{\alpha - \gamma_1 Q_S - \gamma_3 Q_L}{2\beta}. \tag{8}$$

All small firms choose the same level of output. Hence, $Q_S^* = N q_S^*$.

Using equation (2), the price of the small firm can be expressed by

$$p_S^*(i) = \beta \frac{\alpha - \gamma_3 Q_L}{2\beta + \gamma_1 N}. \tag{9}$$

Accordingly, the equilibrium price of the small firm decreases with the mass of small firms and the total output of large firms.

Entry and exit are free for small firms. Using equation (5) after plugging in (8) and (9), we can express the mass of small firms (N) as a function of the total output of large firms (Q_L):

$$N(Q_L) = \frac{1}{\gamma_1}\left[\sqrt{\frac{\beta}{f}}(\alpha - \gamma_3 Q_L) - 2\beta\right]. \tag{10}$$

As shown by the above expression, the mass of small firms decreases with the total output of large firms.

Substituting (10) into (8), the optimal quantity of each small firm is

$$q_S^* = \sqrt{\frac{f}{\beta}}.$$

Owing to free entry and exit, the quantity produced by a small firm is independent of the behavior of large firms. In other words, the aggregate behavior of small firms responds to the change in the market condition only by adjusting the competitive fringe. (See also Lemma 1.)

Plugging q_S^* into (9) yields the equilibrium price of small firms:

$$p_S^* = \sqrt{\beta f}.$$

Large Firms' Profit Maximization and Variety Choice Unlike small firms, large firms impose a non-negligible impact on the market. Large firm m maximizes its profit given by equation (6) with respect to its output. The first-order condition of large firm m with respect to the quantity of its ωth variety, $q_L^m(\omega)$, is

$$\alpha - 2\beta q_L^m(\omega) - 2\gamma_2 \int_0^{\overline{\omega}_m} q_L^m(v)dv - \gamma_3 Q_S - \gamma_2 Q_L^{-m} = 0. \tag{11}$$

Equation (11) implies that $q_L^m(\omega) = q_L^m$ for each $\omega \in [0, \overline{\omega}_m]$. Therefore, we have

$$\int_0^{\overline{\omega}_m} q_L^m(v)dv = \overline{\omega}_m q_L^m.$$

Substituting the above symmetry property into equation (11), we solve the optimal quantity of each variety (q_L^{m*}) as a function of the expected aggregate output of small firms (Q_S), the expected output of other large firms (Q_L^{-m}), and its own product range ($\overline{\omega}_m$):

$$q_L^{m*} = \frac{\alpha - \gamma_3 Q_S - \gamma_2 Q_L^{-m}}{2(\beta + \gamma_2 \overline{\omega}_m)}. \tag{12}$$

Everything else being equal, an increase in firm m's product range (larger $\overline{\omega}_m$) results in a reduction in the quantity of each variety (smaller q_L^m), implying the cannibalization effect.

Large firm m maximizes its profit with respect to its product range. After substituting (12), the first-order condition of large firm m with respect to its product range, $\overline{\omega}_m$, is

$$2(\beta + \gamma_2 \overline{\omega}_m) = \sqrt{\frac{\beta}{F}}(\alpha - \gamma_3 Q_S - \gamma_2 Q_L^{-m}). \tag{13}$$

From equations (12) and (13), we obtain the optimal output per variety for the large firm:

$$q_L^{m*} = \sqrt{\frac{F}{\beta}},$$

which is determined only by the fixed cost of large firms and the demand parameters but is independent of its product range and other firms' behavior. (See also the discussion after Lemma 1.) This equation also implies that $q_L^{m*} = q_L^*$ for each $m = 1, ..., M$. Thus, $Q_L^{-m} = \sum_{k \neq m}^{M} \overline{\omega}_k q_L^*$, and equation (13) can be expressed as

$$2(\beta + \gamma_2 \overline{\omega}_m) = \sqrt{\frac{\beta}{F}}(\alpha - \gamma_3 Q_S - \gamma_2 \sum_{k \neq m}^{M} \overline{\omega}_k q_L^*). \tag{14}$$

Substituting $q_L^* = \sqrt{F/\beta}$, the above equation can be rearranged into

$$\gamma_2 \overline{\omega}_m = \sqrt{\frac{\beta}{F}}(\alpha - \gamma_3 Q_S) - \gamma_2 \sum_{k=1}^{M} \overline{\omega}_k - 2\beta, \tag{15}$$

which implies that $\overline{\omega}_m = \overline{\omega}$ for each $m = 1, .., M$. Hence, we can express the product range $\overline{\omega}$ as a function of the expected aggregate output of small firms Q_S:

$$\overline{\omega}(Q_S) = \frac{\sqrt{\beta/F}(\alpha - \gamma_3 Q_S) - 2\beta}{\gamma_2(M+1)}. \tag{16}$$

Mixed Market Equilibrium The above analysis indicates that firms behave symmetrically within the group of large firms and that of small firms. That is, $q_S^*(i) = q_S^* = \sqrt{f/\beta}$ for each $i \in [0, N]$, $q_L^{m*} = q_L^* = \sqrt{F/\beta}$ for each $m = 1, .., M$, and $\overline{\omega}_m = \overline{\omega}$ for each $m = 1, .., M$. Then, the total output of large firms can be expressed by $Q_L = M\overline{\omega}q_L^{m*} = M\overline{\omega}\sqrt{F/\beta}$, and the aggregate output of small firms is $Q_S = Nq_S^* = N\sqrt{f/\beta}$. Plugging in these two expressions, (10) and (16) can be expressed as

$$\begin{aligned} N &= \frac{1}{\gamma_1}\left[\sqrt{\frac{\beta}{f}}(\alpha - \gamma_3 M\overline{\omega}\sqrt{\frac{F}{\beta}}) - 2\beta\right], \\ \overline{\omega} &= \frac{\sqrt{\beta/F}(\alpha - \gamma_3 N\sqrt{f/\beta}) - 2\beta}{\gamma_2(M+1)}. \end{aligned}$$

These two equations determine the equilibrium mass of small firms N^* and the equilibrium product range of a large firm $\overline{\omega}^*$. A mixed market equilibrium exists if and only if $N^* > 0$ and $\overline{\omega}^* > 0$. Assuming this, the mass of small firms and product range of large firms in equilibrium are

$$\begin{aligned} N^* &= \sqrt{\frac{\beta}{f}}\frac{\alpha[\gamma_2(M+1) - \gamma_3 M] - 2\sqrt{\beta}[\gamma_2(M+1)\sqrt{f} - \gamma_3 M\sqrt{F}]}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}, \\ \overline{\omega}^* &= \sqrt{\frac{\beta}{F}}\frac{\alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}. \end{aligned}$$

Plugging N^*, $\overline{\omega}^*$, q_S^* and q_L^{m*} into equation (3), the price of the large firm in equilibrium is

$$p_L^* = \sqrt{\beta F} + \frac{\gamma_2[\alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})]}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}.$$

Substituting the equilibrium range of varieties $\overline{\omega}^*$, the output of each variety q_L^{m*} and the equilibrium price of large firms p_L^* into equation (6), we obtain the equilibrium profit of the large firm:

$$\Pi_L^* = \gamma_2[\frac{\alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}]^2.$$

The total output is

$$\begin{aligned} Q^* = &\frac{1}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}\{\alpha M(\gamma_1 + \gamma_2 - 2\gamma_3) + \gamma_2(\alpha - 2\sqrt{\beta f}) \\ &- 2M\sqrt{\beta}[(\gamma_2 - \gamma_3)\sqrt{f} + (\gamma_1 - \gamma_3)\sqrt{F}]\}. \end{aligned}$$

3.2 Conditions for a Unique Mixed Market Equilibrium

We focus on a market in which large and small firms coexist. To ensure that the market is mixed and stable in equilibrium, we need to guarantee that

(i) large firms earn positive profits with a positive product range (that is, $\overline{\omega}^* > 0$);

(ii) the mass of small firms is positive (that is, $N^* > 0$); and

(iii) the equilibrium is (locally) stable (that is, with slight perturbation from the mixed market equilibrium, firms' strategies would converge back to the equilibrium values)[14].

We identify the conditions for the existence and stability of a mixed market equilibrium through the aggregate reactions between large and small firms. (A precise analysis is available in Appendix **B.1**.)

The aggregate reaction of large firms to the competitive fringe is

$$Q_L(Q_S) = \max\{0, \frac{M}{\gamma_2(M+1)}(\alpha - 2\sqrt{\beta F} - \gamma_3 Q_S)\}, \tag{17}$$

and that of the competitive fringe to large firms is

$$Q_S(Q_L) = \max\{0, \frac{1}{\gamma_1}(\alpha - 2\sqrt{\beta f} - \gamma_3 Q_L)\}. \tag{18}$$

Figure 1 depicts the aggregate reactions of large and small firms under different conditions. When goods are substitutes across large and small firms (that is, $\gamma_3 > 0$), large and small firms' aggregate reactions behave like strategic substitutes. Figure 1a depicts a stable mixed market equilibrium. First, the coexistence of large and small firms in equilibrium requires that the two aggregate reaction functions intersect. Second, the (local) stability of the intersection requires that the slope of $Q_L(Q_S)$ is flatter than the slope of $Q_S(Q_L)$ (that is, $\gamma_3 M/\gamma_2(M+1) < \gamma_1/\gamma_3$). Even when the two aggregate reaction functions intersect, if the stability condition does not hold, and upon a slight perturbation, the mixed market equilibrium is unstable, and firms will deviate from the intersection of $Q_L(Q_S)$ and $Q_S(Q_L)$ to the equilibrium with only small firms or the equilibrium with only large firms (see Figure 1b). If the two aggregate reaction functions do not intersect, there is no mixed market equilibrium even when the stability condition is satisfied. As shown in Figure 1c, for instance, there is only one equilibrium in which small firms operate. This is the case when the fixed production cost of a large firm F is too high, meaning that large firms are at a technological disadvantage relative to small firms. Similarly, there is only one market equilibrium in which large firms operate if the fixed cost of a small firm f is too high. Therefore, a unique mixed market equilibrium requires that (i) $Q_L(Q_S)$ and $Q_S(Q_L)$ intersect and (ii) the slope of $Q_L(Q_S)$ is flatter than the slope of $Q_S(Q_L)$. Given the second condition, the first condition implies that the intercept of $Q_S(Q_L)$ should be smaller than the intercept of $Q_L(Q_S)$ on the horizontal axis (that is, $(\alpha - 2\sqrt{\beta f})/\gamma_1 < (\alpha - 2\sqrt{\beta F})/\gamma_3$), and the intercept of $Q_S(Q_L)$ should be larger than the intercept of $Q_L(Q_S)$ on the vertical axis (that is, $(\alpha - 2\sqrt{\beta f})/\gamma_3 > (\alpha - 2\sqrt{\beta F})M/[\gamma_2(M+1)]$).

[14]The local stability here is the pseudo-stability often seen in textbook discussions of Cournot models. In the dynamic adjustment, we assume non-cooperative behavior among large firms and collective behavior among small firms. (See Appendix **B.1** for details.) We would like to thank the anonymous referees for their clarifying suggestions.

When $\gamma_3 < 0$, in aggregate, large firms and small firms behave like strategic complements (See Figure 1d). To ensure the existence of a unique market equilibrium in this case, the slope of $Q_L(Q_S)$ should be flatter than the slope of $Q_S(Q_L)$ (that is, $\gamma_3 M/\gamma_2(M+1) > \gamma_1/\gamma_3$).

[Figure 1 around here]

Summing up the above discussion, we formally establish the conditions in the following proposition.

Proposition 1 *A unique equilibrium exists, which is (locally) stable and features a mixed market, if the following three conditions hold:*
(i) $(\alpha - 2\sqrt{\beta F})\gamma_1 > (\alpha - 2\sqrt{\beta f})\gamma_3$*;*
(ii) $(\alpha - 2\sqrt{\beta f})(M+1)\gamma_2 > (\alpha - 2\sqrt{\beta F})M\gamma_3$*;*
(iii) $\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$*.*
These conditions require that (a) the intensity of preferences for the differentiated goods market, i.e., α*, is sufficiently high relative to the fixed costs of large and small firms, i.e.,* F *and* f*, and (b) the substitutability/complementarity across large and small firms, i.e.,* $|\gamma_3|$*, is not too strong relative to the substitutabilities among large firms, i.e.,* γ_1*, and among small firms, i.e.,* γ_2*.*

Proof. See Appendix **??**. ■

Conditions (i) and (iii) guarantee that large firms operate with a positive product range ($\overline{\omega}^* > 0$), conditions (ii) and (iii) guarantee that the mass of small firms is positive ($N^* > 0$), and condition (iii) is a necessary condition for the (local) stability of the mixed market equilibrium.

These three conditions impose constraints on the parameters. No complementarity is allowed among the varieties of small firms and among those of large firms, that is, $\gamma_1 > 0$ and $\gamma_2 > 0$.[15] In addition, the intensity of preferences for the differentiated goods market α should be sufficiently large (that is, $\alpha > 2\sqrt{\beta f}$ and $\alpha > 2\sqrt{\beta F}$).[16] Conditions (ii) and (iii) imply that the range of parameters narrows with an increase in the number of large firms M. In other words, the market condition for small firms becomes more severe when more large firms coexist.

We can express the three conditions parametrically by γ_1/γ_3 and γ_2/γ_3, which represent the relative substitutability for the small firm and that for the large firm, respectively.

[15] Notice that the concavity of a large firm's profit with respect to its product range requires $\gamma_2 > 0$. To be precise, large firm m's profit after substituting equation (12) can be expressed as $\Pi_L^m = \overline{\omega}_m[(\alpha - \gamma_3 Q_S - \gamma_2 Q_L^{-m})^2/4(\beta + \gamma_2\overline{\omega}_m)^2 - F]$. Thus, $\gamma_2 > 0$ guarantees that the second-order derivative of Π_L^m with respect to the product range $\overline{\omega}_m$ is negative, i.e., $-\gamma_2\beta(\alpha - \gamma_3 Q_S - \gamma_2 Q_L^{-m})^2/2(\beta + \gamma_2\overline{\omega}_m)^3 < 0$. Intuitively, $\gamma_2 > 0$ implies that the infra-marginal effect of adding a new variety is negative.

Given that $\gamma_2 > 0$, if $\gamma_1 < 0$, condition (iii) of Proposition 1 is violated. Thus, $\gamma_1 > 0$.

[16] Given that $\gamma_1 > 0$ and $\gamma_2 > 0$, if either $\alpha < 2\sqrt{\beta f}$ or $\alpha < 2\sqrt{\beta F}$, then at least one of the three conditions in Proposition 1 would be violated. For instance, when $\gamma_3 > 0$, if $2\sqrt{\beta f} < \alpha < 2\sqrt{\beta F}$, condition (i) is violated. The results are analogous in other cases.

Figure 2 shows the areas where a unique mixed market equilibrium exists. The horizontal axis and vertical axis are γ_1/γ_3 and γ_2/γ_3, respectively. Note that, according to condition (iii), $|\gamma_3|$ should not be too high relative to γ_1 and γ_2. If the varieties are highly substitutable across large and small firms, then the mixed market equilibrium is unstable, and market competition typically causes one group to survive.[17] If the varieties are highly complementary between large and small firms, then the total quantity of both groups of firms would always expand. However, it is unrealistic to observe quantities and varieties expand, exhausting all the income for products in a particular market.

In addition, if $\gamma_1 = \gamma_2 = \gamma_3 > 0$, condition (i) implies that $f > F$. That is, if the varieties are equally substitutable among all firms, the existence of large firms requires that the fixed cost of a small firm be larger than the large firm's per-variety fixed production cost. If the large and small firms share the same fixed production cost, this condition indicates that the small firm's entry cost should be positive to ensure that the large firm enjoys economies of scope (Parenti, 2016). Even when the small firm's entry cost is close to zero, the large firm may also exist if it incurs a lower fixed cost to produce a variety.

[Figure 2 around here]

3.3 The Impact of a Large Firm's Entry

Now, we investigate the impact of a large firm's entry, that is, an exogenous increase in the number of large firms from M to $M + 1$. We have some preliminary results that will be useful for subsequent analysis. Formally, based on q_S^* and q_L^*, we have the following lemma:

Lemma 1 *The entry of a large firm has no impact on (i) the output and price level of the small firm (that is, p_S^* and q_S^*) or (ii) the output of each variety of the large firm (that is, q_L^*).*

The first outcome is in line with the traditional monopolistic competition model. Here, small firms can be considered monopolistic competitors. The free entry and exit of small firms shifts the demand curve such that there is only one equilibrium quantity at which the marginal revenue is equal to the marginal cost and the price is equal to the average cost.

To see the second result, first note that profit maximization of a large firm is accompanied by the following two conditions. First, for each variety, the marginal revenue from

[17] If γ_3 is high, according to equation (2), an increase in Q_L would substantially shift the demand for a small firm downward. Thus, small firms would have much lower incentives to operate so that the competitive fringe significantly shrinks, which further stimulates the expansion of large firms. Eventually, all small firms would be driven out of the market, and only large firms would operate. Analogously, there is another equilibrium in which only small firms operate.

output (price plus infra-marginal effect) equals the marginal cost (MC), adjusted by the internalization effects on other varieties, that is,

$$p_L^m - \underbrace{\beta q_L^m}_{\text{Infra-marginal}} = MC + \underbrace{\gamma_2 \overline{\omega}_m q_L^m}_{\text{Internalization}} .$$

Second, the profit from adding one more variety is zero. The second condition implies that the price equals the average cost, adjusted by the (average) cannibalization effect, that is,

$$p_L^m = \underbrace{MC + F/q_L^m}_{\text{Average Cost}} + \underbrace{\gamma_2 \overline{\omega}_m q_L^m}_{\text{Average Cannibalization}} .$$

Note that the cannibalization effect can be regarded as an aggregation of internalization effects, as each output of the marginal variety imposes a negative externality on its own varieties, which implies that the average cannibalization must equal the internalization.

Combining the above two conditions, we obtain the following general condition: the infra-marginal effect equals the difference between the marginal cost and the average cost. Under linear demand and a constant marginal cost, we obtain a simpler condition: the infra-marginal effect (βq_L^m) equals the average fixed cost (F/q_L^m), which pins down the unique per-variety quantity.

3.3.1 Substitution between the Big and the Small ($\gamma_j > 0$, $j = 1, 2, 3$)

Based on the results in Lemma 1, we first investigate the impact of a large firm's entry on firms' behavior when products are substitutes across large and small firms. Proposition 2 establishes the results.

Proposition 2 *The entry of a large firm (that is, an increase in the number of large firms, M) will exert the following impact on firms' behavior:*

(i) The competitive fringe shrinks (that is, N^* decreases);

(ii) The price, product range, and profit of each large firm increase if the substitutability across large and small firms is sufficiently high relative to the substitutabilities within the groups of large and small firms, but otherwise, they all decline (formally, p_L^*, $\overline{\omega}^*$ and Π_L^* increase if $\gamma_1\gamma_2 < \gamma_3^2$, decrease if $\gamma_1\gamma_2 > \gamma_3^2$, and remain the same if $\gamma_1\gamma_2 = \gamma_3^2$); and

(iii) The total output rises if the substitutability among small firms is higher than the substitutability across large and small firms and declines otherwise (formally, Q^* increases if $\gamma_1 > \gamma_3$, decreases if $\gamma_1 < \gamma_3$, and remains the same if $\gamma_1 = \gamma_3$).

Proof. See Appendix **B.2**. ■

We explain the economic intuition of the above results as follows. First, the entry of a large firm raises the total output of large firms, which generates a negative impact on the demand faced by small firms by substituting some of the goods produced by small firms. Consequently, some small firms are squeezed out, and hence, the competitive fringe shrinks. This argument explains Proposition 2-(i).

Second, a large firm's entry may raise or reduce large firms' prices and profits. The impact of a large firm's entry on incumbent large firms depends primarily on two opposing effects. On one hand, the entry of a large firm intensifies the competition among large firms through a negative substitution effect. On the other hand, the shrinkage of the competitive fringe generates a positive impact on the demand faced by large firms. If the substitutability across large and small firms is sufficiently high relative to the substitutability among large firms and that among small firms, the positive squeezing-out effect outweighs the negative substitution effect, leading to an increase in the demand each large firm faces. Therefore, each large firm's price, product range, and profit increase. If the substitutability across large and small firms is relatively small, the negative substitution effect dominates, resulting in a reduction in each large firm's price, product range, and profit.

For a precise illustration of the mechanism, we examine how the inverse demand of each variety shifts after a large firm's entry. Plugging the equilibrium values into the inverse demand, we have the following relationships:

$$p_S^* = \alpha - \beta q_S^* - \gamma_1 Q_S^* - \gamma_3 Q_L^*, \tag{19}$$

$$p_L^* = \alpha - \beta q_L^* - \gamma_2 Q_L^* - \gamma_3 Q_S^*. \tag{20}$$

Recall that in Lemma 1, q_L^*, p_S^* and q_S^* are independent of the number of large or small firms, which implies that the small firms' inverse demand for each variety does not shift after entry and thus that $\gamma_1 Q_S^* + \gamma_3 Q_L^*$ remains constant. Hence, for an increase in large firms' aggregate quantity, ΔQ_L^*, small firms' aggregate quantity is substituted by $\Delta Q_S^* = -(\gamma_3/\gamma_1)\Delta Q_L^*$. Now, we can see how the demand for a variety offered by large firms shifts after entry. For an increase ΔQ_L^*, the substitution effect is $-\gamma_2 \Delta Q_L^*$, whereas the feedback effect through the change in small firms' aggregate quantity is $-\gamma_3 \Delta Q_S^* = (\gamma_3^2/\gamma_1)\Delta Q_L^*$. Thus, the total effect is $\Delta Q_L^*[\gamma_3^2 - \gamma_1\gamma_2]/\gamma_1$. If $\gamma_1\gamma_2 > \gamma_3^2$, i.e., the substitution level between large and small firms is relatively small, then the negative substitution effect dominates the positive feedback effect such that the large firm's price p_L^* falls. Since the revenue from each variety decreases, the equilibrium product range of the large firm $\overline{\omega}^*$ must shrink, and consequently, the profit of each large firm π_L^* decreases. The cases of $\gamma_1\gamma_2 < \gamma_3^2$ and $\gamma_1\gamma_2 = \gamma_3^2$ are analogous.[18]

Finally, the change in total output is determined by the comparison between the substitutability among small firms (γ_1) and the substitutability across large and small firms (γ_3). If the substitutability across large and small firms is higher than the substitutability among small firms, that is, $\gamma_3 > \gamma_1$, then the squeezing-out effect is so strong that the increase in the output of large firms is insufficient to offset the shrinkage of the competitive fringe. Thus, total output is reduced. Conversely, the squeezing-out effect is modest, meaning that the entry of a large firm increases total output. Precisely, by equation (19), it is straightforward that $\Delta Q_S^* = -(\gamma_3/\gamma_1)\Delta Q_L^*$, and thus, the change in total output

[18] When $\gamma_1\gamma_2 = \gamma_3^2$, our result is consistent with that of Parenti (2016), who assumes that the product substitution is the same across large and small firms, that is, $\gamma_1 = \gamma_2 = \gamma_3 = \gamma$.

is $\Delta Q^* = \Delta Q^*_S + \Delta Q^*_L = (\gamma_1 - \gamma_3)/\gamma_1 \Delta Q^*_L$, which is positive if $\gamma_1 > \gamma_3$ and negative otherwise.

Our established model and findings are related to two major strands of the literature. First, our model is closely linked to recent works on the coexistence of oligopolists and monopolistic competitors. Representative works include Shimomura and Thisse (2012) and Parenti (2016). Accounting for the endogenous income from the greater aggregate profits of large firms, Shimomura and Thisse (2012) demonstrate that entry always increases the large incumbents' quantities and profits. Our result is complementary to theirs in that we focus on the interaction of demand substitutabilities but abstract from the income effects on the differentiated market using quasi-linear utility. Another related paper is Parenti (2016), who analyzes the impact of trade liberalization when large and small firms coexist in the same market. Based on a quasi-linear utility model with the assumption of symmetric substitutability among all varieties, he finds that the entry of a large firm has no impact on large incumbents' behavior in a closed economy. While this result is consistent with our findings, we go a step further and introduce asymmetric substitutabilities across large and small firms, showing how demand substitutability affects the impact of a large firm's entry.

Second, our result also sheds light on the literature on price-increasing competition.[19] A key idea in this literature is that as the market becomes more competitive, the slope of each firm's demand curve can be steeper, especially in discrete choice models of product differentiation (Chen and Riordan (2008) call this the price sensitivity effect). Our model does not capture this effect due to the assumption of representative consumers. We find that large firms' demand function may shift upward after the entry of a large firm due to the interaction between large and small firms. The effect of the exit of small firms can be large enough to dominate the negative effects of a large firm's entry on the demand for large incumbents.

Now let us consider how the entry of a large firm influences consumer welfare, producer surplus and social welfare. We normalize the entry cost of the large entrant to zero. Proposition 3 establishes the results.

Proposition 3 *The entry of a large firm generates the following impact on welfare:*

(i) Consumer welfare rises if

$$\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f}) + E(\gamma_1\gamma_2 - \gamma_3^2)[\frac{M}{D(M)} + \frac{(M+1)}{D(M+1)}] > 0$$

and falls otherwise;

(ii) Producer surplus rises if

$$\frac{(M+1)}{D^2(M+1)} - \frac{M}{D^2(M)} > 0$$

[19] See Chen and Riordan (2007, 2008), Perloff and Salop (1985), Perloff, Suslow, and Seguin (2005), and Gabaix, Laibson, and Li (2005).

and falls otherwise; and

(iii) Social welfare rises if

$$\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f}) + \gamma_1\gamma_2 E(\frac{1}{D(M)} + \frac{1}{D(M+1)}) > 0$$

and falls otherwise,

where $D(M) = \gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$, $D(M+1) = \gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)(M+1) > 0$, *and* $E = \alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f}) > 0$ *according to the conditions in Proposition 1.*

Proof. See Appendix **B.3**. ∎

Proposition 3 shows that a large firm's entry will only conditionally raise consumer welfare, producer surplus and social welfare. The change in consumer welfare is determined by three effects. First, a portion of the small firms' product range is replaced by large firms' output in the representative consumer's consumption. This replacement effect affects consumer welfare because the quantity of one variety differs across large and small firms. Second, the entry of a large firm affects consumer welfare through the change in the total variety range. The consumer benefits from an increase in the variety range.[20] Third, the change in consumer welfare depends on the price charged by large firms, which is determined by the relative substitution levels. These three effects determine the sign of the change in consumer welfare.

Specifically, the impact of a large firm's entry on consumer welfare can be decomposed as

$$\Delta CW = \underbrace{(\frac{\beta q_S^{*2}}{2} - \frac{\beta q_L^{*2}}{2})\Delta V_S^*}_{\text{Replacement Effect}} + \underbrace{\frac{\beta q_L^{*2}}{2}\Delta V^*}_{\text{Variety Effect}} - \underbrace{\frac{1}{2}[Q_L^*(M) + Q_L^*(M+1)]\Delta p_L^*}_{\text{Price Effect}}. \tag{21}$$

Here, ΔV_S^* is the (negative) change in the variety range of small firms, and ΔV^* is the change in the total variety range. The first term characterizes the replacement effect. The utility obtained from each variety of the small firm and that of the large firm are $\beta q_S^{*2}/2$ and $\beta q_L^{2*}/2$, respectively. Therefore, when the ΔV_S^* of small firms' product range is replaced by that of large firms, the utility change is measured by the first term. The second term characterizes the variety effect. If the total variety range expands (ΔV^*), the representative consumer's utility gain is $(\beta q_L^{2*}/2)\Delta V^*$ because (s)he consumes more varieties from large firms. The third term characterizes the price effect arising from the substitutabilities among varieties. As shown in Proposition 2, the sign of this term is ambiguous, depending on the relative substitution level across large and small firms. If the substitutability among large and small firms is such that a large firm's entry raises the price charged by large firms (p_L^*), then the consumer suffers from a higher price.

[20]See Ottaviano, Tabuchi and Thisse (2002) for a detailed discussion of the consumer's "love for variety."

Conversely, consumer welfare improves if the price charged by large firms falls after a large firm's entry. Moreover, owing to free entry and exit, the price charged by small firms does not change with the number of large firms, and thus, there is no price effect of small firms on consumer welfare. Therefore, the total impact on consumer welfare depends on the comparison of the replacement effect, the variety effect, and the price effect.

The impact on producer surplus depends simply on the comparison between the profit change due to the increase in the total quantity of large firms (positive marginal effect) and the profit change due to the price change (infra-marginal effect). A sufficient condition for an increase in producer surplus is $\gamma_1\gamma_2 < \gamma_3^2$, which means that the marginal and infra-marginal effects are both positive. Producer surplus decreases only if the infra-marginal effect is negative and outweighs the marginal effect. Formally,

$$\Delta PS = \frac{1}{2}[p_L^*(M) + p_L^*(M+1) - 2\beta q_L^*]\Delta Q_L^* + \frac{1}{2}[Q_L^*(M) + Q_L^*(M+1)]\Delta p_L^*.$$

Summing up the change in consumer welfare and that in producer surplus, the impact on social welfare can be expressed by

$$\Delta SW = \underbrace{(\frac{\beta q_S^{*2}}{2} - \frac{\beta q_L^{*2}}{2})\Delta V_S^*}_{\text{Replacement Effect}} + \underbrace{\frac{\beta q_L^{*2}}{2}\Delta V^*}_{\text{Variety Effect}} + \underbrace{\frac{1}{2}[p_L^*(M) + p_L^*(M+1) - 2\beta q_L^*]\Delta Q_L^*}_{\text{Quantity Effect}}. \quad (22)$$

Similar to ΔCW, the first two terms here characterize the replacement effect and variety effect on consumer welfare. The third term, which is always positive, measures the quantity effect (on producer surplus) arising from the increase in large firms' total output. Note that the price effect on consumer welfare and the infra-marginal increase in producer surplus offset one another. In consequence, the change in social welfare depends on the replacement effect and variety effect on consumer welfare (the first two terms) and the marginal increase in producer surplus (the third term). Figure 3 provides a graphical depiction of these three effects.

[Figure 3 around here]

We also analyze a few specific cases. First, we consider the case in which $\gamma_1 = \gamma_2 = \gamma_3$, i.e., the products are equal substitutes across large and small firms. In this case, the large firm's price does not change, and thus, the price effect on consumer welfare is zero. The total output does not change, either. Moreover, by Proposition 1-(i), the condition for the coexistence of large and small firms implies that $f > F$. Therefore, we have $\Delta CW < 0$, $\Delta PS > 0$, and $\Delta SW > 0$.[21] Although the total consumption does not change, the proportion of large firms' products increases in the consumption bundle, and therefore, the representative consumer suffers from an increased average price. Producer surplus

[21] Assuming the same product substitution across large and small firms, Shimomura and Thisse (2012) also find that a large entrant raises producer surplus and social welfare.

increases because of the expansion of the total output of large firms. Social welfare improves because the deterioration of consumer welfare is dominated by the increase in producer surplus that results from the entry of a large firm.[22]

Now, we consider the case in which the fixed cost of a small firm is the same as the per-variety fixed cost of a large firm, i.e., $f = F$. In this case, the coexistence condition in Proposition 1 implies that $\gamma_1 > \gamma_3$, and thus, the total variety range (and total output) increases, while the replacement effect is zero because per-variety output is the same across large and small firms. With the marginal increase in producer surplus, social welfare improves. A sufficient condition for the improvement of consumer welfare is $\gamma_1\gamma_2 > \gamma_3^2$, which implies a decline in the large firm's price. However, a sufficient condition for an increase in producer surplus is $\gamma_1\gamma_2 < \gamma_3^2$, as shown above.

We summarize these findings in the following corollary:

Corollary 1 *Let the parameters satisfy the conditions of Proposition 1 so that the market is mixed in equilibrium. We consider the following special cases:*

(i) If the products are equally substitutable across large and small firms ($\gamma_1 = \gamma_2 = \gamma_3$), the entry of a large firm results in a deterioration of consumer welfare, an increase in producer surplus and improved social welfare;

(ii) If the fixed cost of a small firm is the same as the per-variety fixed cost of a large firm ($f = F$), the entry of a large firm improves social welfare. A sufficient condition for consumer welfare to increase is $\gamma_1\gamma_2 > \gamma_3^2$, whereas a sufficient condition for producer surplus to increase is $\gamma_1\gamma_2 < \gamma_3^2$.

In the following example, we show the case in which the entry of a large firm worsens consumer welfare and social welfare.

Example 1 *Consider a mixed market with 2 large firms and a host of small firms. Let the size of the differentiated goods market $\alpha = 1$, the preference for diversity $\beta = 1$, the substitutability among small firms' products $\gamma_1 = 0.4$, the substitutability among large firms' products $\gamma_2 = 0.625$, the substitutability across small and large firms' products $\gamma_3 = 0.5$, a small firm's fixed cost $f = 0.16$, and a large firm's fixed production cost of one variety $F = 0.1444$. In this case, $\gamma_1\gamma_2 = \gamma_3^2$, and the conditions for the coexistence of large and small firms are satisfied. The change in consumer welfare is -0.056, and the change in social welfare is -0.032. Here, the consumer suffers from a negative replacement effect and a reduction of the total variety range, and these two negative effects dominate the marginal increase in producer surplus.*

3.3.2 Complementarity between the Big and the Small ($\gamma_1 > 0$, $\gamma_2 > 0$, and $\gamma_3 < 0$)

Having examined the case in which all the products in the differentiated goods market are substitutes, we now consider the case in which the varieties produced by the large

[22] This case is also discussed in Parenti (2016).

firms complement those produced by the small firms; this complementarity between large and small firms implies $\gamma_3 < 0$. In this case, the results in the previous section are mainly robust, except for the change in the number of small firms and social welfare. We establish the results in the following proposition.

Proposition 4 *When large firms' goods are complementary to those of small firms (that is, $\gamma_3 < 0$), the entry of a large firm (that is, an increase in M) generates the following impact:*

(i) The impact on the behavior of large firms is the same as in Proposition 2, but the competitive fringe expands (that is, N^* increases).

(ii) The impact on consumer welfare and producer surplus is also ambiguous, based on the same conditions as in Proposition 3. Nevertheless, social welfare always increases.

First, the entry of a large firm raises the total output of large firms, which generates a positive impact on the demand faced by small firms, which provide goods that complement large firms' goods. Consequently, more small firms enter the market, and hence, the competitive fringe expands. The expansion of the competitive fringe thus generates a positive effect on the price charged by large firms, according to equation (20). Thus, the change in the price charged by the large firm is determined by the comparison between the negative substitution effect $-\gamma_2$ and the positive indirect effect from the expansion of the competitive fringe, which is represented by γ_3^2/γ_1. Therefore, the impact on the large firm is also determined by the same conditions as in Proposition 2.

Compared with the case in which all products are substitutes, it is more likely that consumer welfare improves when the products are complementary between large and small firms in that both the total variety range of large firms and that of small firms increase, i.e., the first two terms in expression (21) are positive. The only possible negative effect originates from the increases in the large firms' price when $\gamma_1\gamma_2 < \gamma_3^2$. In this case, the third term of expression (21) is negative.

Because the large firms' behavior is affected in the same way as in the case in which products are substitutes across large and small firms, the impact on producer surplus here follows the same intuition as in the previous section.

Finally, social welfare always increases here because there is no (negative) replacement effect and variety expansion occurs in both large and small firms.

3.4 Implications

From the perspective of demand substitution, our findings may explain the distinct impacts of a large firm's entry across industries featuring the coexistence of large and small firms. In the Japanese supermarket industry, large stores directly compete with one another, and thus, substitutability among large firms is high. However, small stores are insulated from such direct competition with large firms and benefit from the increase in consumer flows from the opening of large stores; hence, the substitutability across large and small firms is slightly negative. As implied by Proposition 4, in this case, the feedback effect may not be strong enough to offset the substitution effect for large incumbents, which are expected to shrink their production scale and experience a decline in profits. Moreover, because of the weak complementarity between large and small stores, the entry of a large firm increases the demand for small supermarkets. As a result, the competitive fringe may expand. Our theoretical analysis is consistent with the empirical evidence in Igami (2011), as discussed in the introduction.[23] In the South Korean furniture industry, large furniture makers differentiate themselves from others through a unique design, while small furniture makers' products are less differentiated. Thus, the substitutability between large firms is weaker than that between large and small firms. Therefore, according to Proposition 2, the feedback effect could outweigh the substitution effect, resulting in a rise in demand for the large firms' products and a significant squeezing-out effect on small firms.[24]

Our results also provide several policy implications for regulations on large firm's entry. Many countries, such as France and the United Kingdom, enforce laws and regulations to restrict the entry of large firms. Our results partially support such restrictions in terms of welfare because the entry of a large firm may reduce consumer welfare or social welfare in some cases; however, we also show that the entry of large firms may improve welfare under certain conditions. With different levels of substitution across large and small firms, incumbent firms react differently to the entry of a large firm. When products are substitutes across large and small firms ($\gamma_3 > 0$), if the substitutability between large and small firms is sufficiently weak ($\gamma_3^2 < \gamma_1\gamma_2$), a large firm's entry squeezes out a few small firms and intensifies the competition among large firms, and consumers benefit from reduced prices. Such entry behavior may be welcome in terms of the social benefits, with modest costs paid by small firms. However, if the cross-substitution effect is sufficiently strong ($\gamma_3^2 > \gamma_1\gamma_2$), more small firms will be squeezed out, and consumers are more likely to suffer due to the increased price. If the products of large and small firms are complementary ($\gamma_3 < 0$), such entry behavior actually expands small firms' business. Owing to the ambiguity of the impact of a large firm's entry, our results suggest that governments should implement more meticulous and flexible policies to address different entry types.

[23] Similar impacts on small local stores are observed after the entry of Wal-Mart in the United States. Basker (2005) finds that approximately four of two hundred small competitors closed within five years after Wal-Mart's entry, implying a minor effect on the number of small stores. However, the effect of Wal-Mart's entry on large competitors is more complex and subtle due to the difference in their strategies with respect to location choice and product mix, etc. (Basker, 2007). These asymmetric and endogenous choices by large supermarkets are not captured in the present paper and may require further study. We thank an anonymous referee for suggesting the connection between our model and the "Wal-Mart effect."

[24] The report on IKEA's entry did not mention how the entry affected the number of small furniture makers but showed how small makers' sales decreased. Given the sharp decline in their sales, some small firms are very likely to shut down in the medium to long run. In addition, when small firms are heterogenous in efficiency, the least efficient ones are predicted to exit first, while those remaining small firms with higher efficiency would experience revenue declines.

4 Discussion

In the basic model, we have assumed that (i) large firms are multiproduct firms, (ii) there is no vertical differentiation across large and small firms, and (iii) the marginal costs of large and small firms are zero. In this section, we relax each of these three assumptions and discuss the robustness of our results in the following extensions.

Single-product Large Firms In this subsection, we discuss the case when large firms are single-product firms, that is, $\overline{\omega} = 1$. our results are robust, and the change in each large firm's output is qualitatively the same as the change in the large firm's variety choice in our original model. Specifically, the impact of a large firm's entry on firms' behavior is the same as in Proposition 2. The welfare effects are also ambiguous, with slight changes in the conditions. The conditions for the unique mixed market equilibrium are also slightly modified. We summarize the results in the following proposition.

Proposition 5 *When both large and small firms are single-product firms,*

(i) There exists a unique mixed market equilibrium if the following three conditions hold:

(i-1) $\gamma_1(2\beta + \gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$;

(i-2) $\alpha[2\beta + \gamma_2(M+1) - \gamma_3 M] > 2\sqrt{\beta f}[2\beta + \gamma_2(M+1)]$; *and*

(i-3) $\alpha(\gamma_1 - \gamma_3) + 2\gamma_3\sqrt{\beta f} > [\gamma_1(2\beta + \gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M]\sqrt{F/(\beta + \gamma_2)}$.

(ii) The impact of a large firm's entry on firms' behavior is the same as in Propositions 2 and 4.

(iii) The entry of a large firm generates the following impact on social welfare:

(iii-1) Consumer welfare rises if $\alpha\beta(\gamma_1 - \gamma_3) - \gamma_2\gamma_3\sqrt{\beta f} + (\beta + \gamma_2)(\gamma_1\gamma_2 - \gamma_3^2)J[M/K(M) + (M+1)/K(M+1)] > 0$ *and falls otherwise;*

(iii-2) Producer surplus rises if $(M+1)/K^2(M+1) - M/K^2(M) > 0$ *and falls otherwise; and*

(iii-3) Social welfare rises if $\alpha\beta(\gamma_1 - \gamma_3) - \gamma_2\gamma_3\sqrt{\beta f} + \gamma_1(\beta + \gamma_2)(2\beta + \gamma_2)J[1/K(M) + 1/K(M+1)] > 2FK(M)K(M+1)/J$ *and falls otherwise,*

where $J = \alpha(\gamma_1 - \gamma_3) + 2\gamma_3\sqrt{\beta f} > 0$, $K(M) = \gamma_1(2\beta + \gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$, *and* $K(M+1) = \gamma_1(2\beta + \gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)(M+1) > 0$, *according to the conditions in (i).*

Proof. See Appendix **B.4.** ■

Vertical Differentiation across Large and Small Firms In many industries, large firms and small firms are vertically differentiated. In this case, consumer preference can be represented by

$$\begin{aligned}
U &= \alpha_S \int_0^N q_S(i)di + \alpha_L \sum_{m=1}^M \int_0^{\overline{\omega}_m} q_L^m(\omega)d\omega - \frac{\beta}{2}\int_0^N [q_S(i)]^2 di - \frac{\beta}{2}\sum_{m=1}^M \int_0^{\overline{\omega}_m} [q_L^m(\omega)]^2 d\omega \\
&\quad - \frac{\gamma_1}{2}\left[\int_0^N q_S(i)di\right]^2 - \frac{\gamma_2}{2}\left[\sum_{m=1}^M \int_0^{\overline{\omega}_m} q_L^m(\omega)d\omega\right]^2 - \gamma_3 \left[\int_0^N q_S(i)di\right]\left[\sum_{m=1}^M \int_0^{\overline{\omega}_m} q_L^m(\omega)d\omega\right] + q_0.
\end{aligned}$$

If $\alpha_L > (<)\alpha_S$, large firms' products have a higher (lower) quality than small firms'.

The (inverse) demand for small firm i and that for large firm m's ωth variety are

$$\begin{aligned}
p_S(i) &= \alpha_S - \beta q_S(i) - \gamma_1 \int_0^N q_S(i)di - \gamma_3 \sum_{m=1}^M \int_0^{\overline{\omega}_m} q_L^m(\omega)d\omega, \\
p_L(\omega) &= \alpha_L - \beta q_L^m(\omega) - \gamma_2 \sum_{m=1}^M \int_0^{\overline{\omega}_m} q_L^m(\omega)d\omega - \gamma_3 \int_0^N q_S(i)di.
\end{aligned}$$

In equilibrium, a small firm's quantity (q_S^*) and a large firm's per variety quantity (q_L^*) are the same as in the basic model. The competitive fringe and a large firm's product range are

$$\begin{aligned}
N^* &= \sqrt{\frac{\beta}{f}}\frac{(\alpha_S\gamma_2(M+1) - \alpha_L\gamma_3 M) - 2\sqrt{\beta}[\gamma_2(M+1)\sqrt{f} - \gamma_3 M\sqrt{F}]}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}, \\
\overline{\omega}^* &= \sqrt{\frac{\beta}{F}}\frac{(\alpha_L\gamma_1 - \alpha_S\gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}.
\end{aligned}$$

We can obtain the conditions for a unique mixed market equilibrium following the same approach that is adopted in the basic model. Moreover, it is readily verified that the change in firms' production behavior and welfare is almost the same as in the basic model. We summarize the results of this extension in the following proposition.

Proposition 6 *When large and small firms are vertically differentiated, that is,* $\alpha_S \neq \alpha_L$*,*

(i) There exists a unique mixed market equilibrium if the following three conditions hold:

(i-1) $(\alpha_L - 2\sqrt{\beta F})\gamma_1 > (\alpha_S - 2\sqrt{\beta f})\gamma_3$*;*

(i-2) $(\alpha_S - 2\sqrt{\beta f})(M+1)\gamma_2 > (\alpha_L - 2\sqrt{\beta F})M\gamma_3$*;*

(i-3) $\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$*.*

(ii) The impact of a large firm's entry on firms' behavior is the same as in Propositions 2 and 4.

(iii) The entry of a large firm generates an ambiguous impact on welfare, depending on the same conditions as those in Proposition 3, except that E represents $(\alpha_L\gamma_1 - \alpha_S\gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})$*, which is positive according to condition (i-1).*

Proof. See the above discussion. ■

Cost Asymmetry across Large and Small Firms In the basic model, we have assumed zero marginal cost for both large and small firms. In reality, however, marginal cost is generally positive and may differ across large and small firms. We consider two possible extensions. In the first case, firms incur a constant marginal cost. In the second case, firms incur an increasing variable cost.

In the first case, the variable costs of the large and small firms are represented by $c_L q_L$ and $c_S q_S$, respectively. Under this setting, the aggregate reactions of large and small firms are

$$\begin{aligned} Q_L(Q_S) &= \max\{0, \frac{M}{\gamma_2(M+1)}(\alpha - c_L - 2\sqrt{\beta F} - \gamma_3 Q_S)\}, \\ Q_S(Q_L) &= \max\{0, \frac{1}{\gamma_1}(\alpha - c_S - 2\sqrt{\beta F} - \gamma_3 Q_S)\}. \end{aligned}$$

In equilibrium, a small firm's quantity (q_S^*) and a large firm's per variety quantity (q_L^*) are the same as in the basic model. The competitive fringe and a large firm's product range are slightly modified:

$$\begin{aligned} N^* &= \sqrt{\frac{\beta}{f}} \frac{(\alpha - c_S)\gamma_2(M+1) - (\alpha - c_L)\gamma_3 M - 2\sqrt{\beta}[\gamma_2(M+1)\sqrt{f} - \gamma_3 M\sqrt{F}]}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}, \\ \overline{\omega}^* &= \sqrt{\frac{\beta}{F}} \frac{(\alpha - c_S)\gamma_1 - (\alpha - c_L)\gamma_3 - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}. \end{aligned}$$

The conditions for a unique mixed market equilibrium and comparative analysis are qualitatively the same as in the basic model. Specifically,

Proposition 7 *When large and small firms incur a constant marginal cost, that is, c_L for a large firm and c_S for a small firm,*

(i) There exists a unique mixed market equilibrium if the following three conditions hold:

(i-1) $(\alpha - c_S - 2\sqrt{\beta F})\gamma_1 > (\alpha - c_L - 2\sqrt{\beta f})\gamma_3$*;*

(i-2) $(\alpha - c_L - 2\sqrt{\beta f})(M+1)\gamma_2 > (\alpha - c_S - 2\sqrt{\beta F})M\gamma_3$*;*

(i-3) $\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$*.*

(ii) The impact of a large firm's entry on firms' behavior is the same as in Propositions 2 and 4.

(iii) The entry of a large firm generates an ambiguous impact on welfare, depending on the same conditions as those in Proposition 3, except that E represents $(\alpha - c_S)\gamma_1 - (\alpha - c_L)\gamma_3 - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})$, which is positive according to condition (i-1).

In the second case, we extend the basic model by assuming that variable costs are quadratic in quantity. Specifically, the variable costs of the large and small firms can be represented by $t_L q_L^2$ and $t_S q_S^2$, respectively. Similar to the first case, our results are also robust under this modified setting.

In equilibrium, a small firm's quantity (q_S^*) and a large firm's per variety quantity (q_L^*) are respectively

$$q_S^* = \sqrt{f/(\beta + t_S)}, \text{ and } q_L^* = \sqrt{F/(\beta + t_L)}.$$

The competitive fringe and a large firm's product range are

$$N^* = \sqrt{\frac{\beta + t_S}{f}} \frac{\alpha[\gamma_2(M+1) - \gamma_3 M] - 2[\gamma_2(M+1)\sqrt{f(\beta + t_S)} - \gamma_3 M\sqrt{F(\beta + t_L)}]}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M},$$

$$\overline{\omega}^* = \sqrt{\frac{\beta + t_L}{F}} \frac{\alpha(\gamma_1 - \gamma_3) - 2(\gamma_1\sqrt{F(\beta + t_L)} - \gamma_3\sqrt{f(\beta + t_S)})}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}.$$

In this case, the conditions for a unique mixed market equilibrium and comparative analysis are also qualitatively the same as in the basic model. Specifically,

Proposition 8 *When large and small firms incur a variable cost that is quadratic in quantity, that is, $t_L q_L^2$ for a large firm and $t_S q_S^2$ for a small firm,*

(i) There exists a unique mixed market equilibrium if the following three conditions hold:

(i-1) $(\alpha - 2\sqrt{(\beta + t_L)F})\gamma_1 > (\alpha - 2\sqrt{(\beta + t_S)f})\gamma_3$;

(i-2) $(\alpha - 2\sqrt{(\beta + t_S)f})(M+1)\gamma_2 > (\alpha - 2\sqrt{(\beta + t_L)F})M\gamma_3$;

(i-3) $\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$.

(ii) The impact of a large firm's entry on firms' behavior is the same as in Propositions 2 and 4.

(iii) The entry of a large firm generates an ambiguous impact on welfare, depending on the same conditions as those in Proposition 3, except that E represents $\alpha(\gamma_1 - \gamma_3) - 2(\gamma_1\sqrt{(\beta + t_L)F} - \gamma_3\sqrt{(\beta + t_S)f})$, which is positive according to condition (i-1).

To sum up, the entry of a large firm will qualitatively exert the same impact achieved by Propositions 1, 2, 3, and 4 in all of the above four extensions.

5 Conclusion

In this paper, we considered a market with large and small firms that offer products that may have different levels of substitution. In this market structure, we investigated the impact of a large firm's entry on incumbent firms' behavior and welfare. As we noted at the beginning of the article, different industries featuring this mixed market structure exhibit distinct impacts on large and small firms. We focus on the relative strength of two opposing effects on the incumbent large firms. The first effect is the negative substitution effect, and the second effect is the positive squeezing-out (expanding) effect due to the shrinkage (expansion) of small firms when the products are substitutes (complements) across big and small firms. Which of these two effects dominates hinges on the different substitutabilities between large and small firms. The welfare effect is also ambiguous, depending on the different levels of substitution and technology.

Many countries enforce laws to restrict the entry of large firms' entry. However, our analysis indicates that the government should be cautious in adopting and enforcing such laws because such restrictions may not always be welfare-improving. If small firms are able to differentiate themselves from large firms or are essentially complementary to large firms, the entry of a large firm into the local market may improve consumer welfare and social welfare. Our finding suggests that policy makers should account for the specific demand characteristics in different industries or markets when implementing zoning laws.

Despite the prevalence of markets with large and small firms, only a few theoretical studies have been conducted on this topic thus far, such as Shimomura and Thisse (2012) and Parenti (2016). This paper attempts to explore this market structure from the perspective of demand substitutability. Future research is advisable in several directions. This paper assumes symmetric technology among large firms and among small firms, but the heterogeneity of technology may also be present within large firms or among small firms. In addition, this paper employs quasi-linear utility with quadratic subutility to express the representative consumer's preference. Considering a more general framework would be another interesting project. Finally, this paper investigates the change in social welfare caused by a large firm's entry, but a comprehensive welfare analysis of this mixed market structure – such as the optimal number of small firms given the number of large firms or the optimal number of large firms – could be an important project for future research.

Appendix

A Concavity of the Quadratic Subutility Function

Before we show the concavity of the quadratic subutility function, several notations must be introduced.[25]

Let $\mathbf{q}_S : [0, N] \to \mathbb{R}$ and $\mathbf{q}'_S : [0, N] \to \mathbb{R}$ be two square-integrable functions. Let

$$\langle \mathbf{q}_S, \mathbf{q}'_S \rangle_S \equiv \int_0^N q_S(i) q'_S(i) di.$$

Indeed, $\langle \cdot, \cdot \rangle_S$ is an inner product.

For each $m = 1, \cdots, M$, let $\mathbf{q}_L^m : [0, \bar{\omega}_m] \to \mathbb{R}$ and $\mathbf{q}_L'^m : [0, \bar{\omega}_{m'}] \to \mathbb{R}$ be square-integrable functions. Let $\mathbf{q}_L \equiv (\mathbf{q}_L^1, \cdots, \mathbf{q}_L^M)$ and $\mathbf{q}'_L \equiv (\mathbf{q}_L'^1, \cdots, \mathbf{q}_L'^M)$. Let

$$\langle \mathbf{q}_L, \mathbf{q}'_L \rangle_L \equiv \sum_{m=1}^{M} \int_0^{\bar{\omega}_m} q_L^m(\omega) q_L'^m(\omega) d\omega.$$

Also, $\langle \cdot, \cdot \rangle_L$ is an inner product.

Then, our utility function, that is, equation (1), can be written as

$$\begin{aligned} U &= u(\mathbf{q}_S, \mathbf{q}_L) + q_0 \\ &= \alpha(\langle \mathbf{q}_S, \mathbf{1} \rangle_S + \langle \mathbf{q}_L, \mathbf{1} \rangle_L) - \frac{\beta}{2}(\langle \mathbf{q}_S, \mathbf{q}_S \rangle_S + \langle \mathbf{q}_L, \mathbf{q}_L \rangle_L) \\ &- \frac{1}{2}(\gamma_1 \langle \mathbf{q}_S, \mathbf{1} \rangle_S{}^2 + \gamma_2 \langle \mathbf{q}_L, \mathbf{1} \rangle_L{}^2 + 2\gamma_3 \langle \mathbf{q}_S, \mathbf{1} \rangle_S \langle \mathbf{q}_L, \mathbf{1} \rangle_L) + q_0, \end{aligned}$$

where $\mathbf{1}$ denotes the constant function, which is equal to 1 for each $i \in [0, N]$, each $m = 1, \cdots, M$, and each $\omega \in [0, \bar{\omega}_m]$. Recall that q_0 is the consumption of the numeraire.

In the following, we show the concavity of the quadratic subutility function: if
(i) $\beta/N + \gamma_1 > 0$,
(ii) $\beta/(\sum_m \overline{\omega}_m) + \gamma_2 > 0$, and
(iii) $(\beta/N + \gamma_1)[\beta/(\sum_m \overline{\omega}_m) + \gamma_2] > \gamma_3^2$,

then, for each $\lambda \in (0, 1)$, each $\mathbf{q} \equiv (\mathbf{q}_S, \mathbf{q}_L)$, and each $\mathbf{q}' \equiv (\mathbf{q}'_S, \mathbf{q}'_L)$,

$$u(\lambda \mathbf{q} + (1 - \lambda)\mathbf{q}') - \lambda u(\mathbf{q}) - (1 - \lambda)u(\mathbf{q}') > 0.$$

By simple manipulation,

$$\begin{aligned} &u(\lambda \mathbf{q} + (1 - \lambda)\mathbf{q}') - \lambda u(\mathbf{q}) - (1 - \lambda)u(\mathbf{q}') \\ &= \frac{\lambda(1 - \lambda)}{2}\{\beta(\|\Delta \mathbf{q}_S\|_S^2 + \|\Delta \mathbf{q}_L\|_L^2) + \gamma_1 \langle \Delta \mathbf{q}_S, \mathbf{1} \rangle_S{}^2 \\ &+ \gamma_2 \langle \Delta \mathbf{q}_L, \mathbf{1} \rangle_L{}^2 + 2\gamma_3 \langle \Delta \mathbf{q}_S, \mathbf{1} \rangle_S \langle \Delta \mathbf{q}_L, \mathbf{1} \rangle_L\}, \end{aligned}$$

[25] We thank an anonymous referee for suggesting a rigorous approach to show the concavity of the utility function on which this proof is based.

where $\Delta\mathbf{q}_S \equiv \mathbf{q}_S - \mathbf{q}'_S$ and $\Delta\mathbf{q}_L \equiv \mathbf{q}_L - \mathbf{q}'_L$, and $\|\cdot\|_S$ and $\|\cdot\|_L$ are the norms associated with the inner products.

By the Cauchy-Schwarz inequality,

$$\|\Delta\mathbf{q}_S\|_S^2 \, \|\mathbf{1}\|_S^2 \geq \langle\Delta\mathbf{q}_S, \mathbf{1}\rangle_S{}^2,$$
$$\|\Delta\mathbf{q}_L\|_L^2 \, \|\mathbf{1}\|_L^2 \geq \langle\Delta\mathbf{q}_L, \mathbf{1}\rangle_L{}^2.$$

Since

$$\|\mathbf{1}\|_S^2 = \langle\mathbf{1}, \mathbf{1}\rangle_S = \int_0^N 1^2 di = N,$$
$$\|\mathbf{1}\|_L^2 = \langle\mathbf{1}, \mathbf{1}\rangle_L = \sum_{m=1}^M \int_0^{\bar{\omega}_m} 1^2 di = \sum_{m=1}^M \bar{\omega}_m,$$

then

$$\|\Delta\mathbf{q}_S\|_S^2 + \|\Delta\mathbf{q}_L\|_L^2 \geq \frac{\langle\Delta\mathbf{q}_S, \mathbf{1}\rangle_S{}^2}{N} + \frac{\langle\Delta\mathbf{q}_L, \mathbf{1}\rangle_L{}^2}{\sum_m \bar{\omega}_m}.$$

Therefore,

$$\begin{aligned}
&u(\lambda\mathbf{q} + (1-\lambda)\mathbf{q}') - \lambda u(\mathbf{q}) - (1-\lambda)u(\mathbf{q}') \\
&\geq \frac{\lambda(1-\lambda)}{2}\left[\left\{\frac{\beta}{N} + \gamma_1\right\}\langle\Delta\mathbf{q}_S, \mathbf{1}\rangle_S{}^2 + \right. \\
&\left.\left\{\frac{\beta}{\sum_m \bar{\omega}_m} + \gamma_2\right\}\langle\Delta\mathbf{q}_L, \mathbf{1}\rangle_L{}^2 + 2\gamma_3\langle\Delta\mathbf{q}_S, \mathbf{1}\rangle_S\langle\Delta\mathbf{q}_L, \mathbf{1}\rangle_L\right].
\end{aligned}$$

Since $\lambda \in (0, 1)$, then $\lambda(1-\lambda) > 0$. Note that conditions (i), (ii), and (iii) imply that for each $\langle\Delta\mathbf{q}_S, \mathbf{1}\rangle_S$ and each $\langle\Delta\mathbf{q}_L, \mathbf{1}\rangle_L$,

$$\left\{\frac{\beta}{N} + \gamma_1\right\}\langle\Delta\mathbf{q}_S, \mathbf{1}\rangle_S{}^2 + \left\{\frac{\beta}{\sum_m \bar{\omega}_m} + \gamma_2\right\}\langle\Delta\mathbf{q}_L, \mathbf{1}\rangle_L{}^2 + 2\gamma_3\langle\Delta\mathbf{q}_S, \mathbf{1}\rangle_S\langle\Delta\mathbf{q}_L, \mathbf{1}\rangle_L > 0.$$

Thus, the concavity of the quadratic subutility function is shown. **Q.E.D.**

B Proofs

B.1 Proof of Proposition 1

In the following, we conduct the dynamic adjustment process to identify the conditions for the (local) stability of the mixed market equilibrium.

Large firms choose their behavior non-cooperatively. Within the firm, we assume that each large firm coordinates among the per-variety quantity q_L and product range ω. Thus, the per-variety quantity in equilibrium is by equations (12) and (13). Denote

$Q_L^m \equiv \overline{\omega}_m q_L^*$ as the total quantity of large firm m ($m = 1, ..., M$). We conduct the dynamic adjustment of large firms with respect to the aggregate behavior of each large firm, that is, Q_L^m ($m = 1, ..., M$).

To simplify the analysis, we assume collective behavior among small firms, and the dynamic adjustment of small firms is in terms of the aggregate quantity, Q_S. A small firms's quantity in equilibrium is $q_S^* = \sqrt{f/\beta}$ by equations (8) and (10). The total output of small firms, Q_S, is adjusted by free entry and exit.

Gathering large and small firms, we derive the stability conditions through the dynamic adjustment process with respect to the total quantity of small firms, Q_S, and the total quantity of large firm m, Q_L^m ($m = 1, ..., M$).

To be precise, we have the following dynamic adjustment system in terms of Q_S, and $Q_L^m (= \overline{\omega}_m q_L^*)$ ($m = 1, ..., M$):

$$
\begin{aligned}
\dot{Q}_S(Q_S, Q_L^1, ..., Q_L^M) &= d_0[\alpha - 2\sqrt{\beta f} - \gamma_1 Q_S - \gamma_3 \sum_{m=1}^{M} Q_L^m], \\
\dot{Q}_L^1(Q_S, Q_L^1, ..., Q_L^M) &= d_1[\alpha - 2\sqrt{\beta F} - \gamma_3 Q_S - \gamma_2(2Q_L^1 + \sum_{m=2}^{M} Q_L^m)], \\
&\vdots \\
\dot{Q}_L^M(Q_S, Q_L^1, ..., Q_L^M) &= d_M[\alpha - 2\sqrt{\beta F} - \gamma_3 Q_S - \gamma_2(2Q_L^M + \sum_{m=1}^{M-1} Q_L^m)].
\end{aligned}
$$

where "$\cdot$" denotes the differentiation with respect to time, i.e., $\dot{Q}_S = dQ_S/dt$, and $\dot{Q}_L^m = dQ_L^m/dt$, ($m = 1, ..., M$); and d_i ($i = 0, 1, \cdots, M$) is the positive coefficient measuring the speed of dynamic adjustment. Without loss of generality, set $d_i = 1$, $i = 0, 1, \cdots, M$.

The Jacobian matrix of the above dynamic adjustment system is

$$
\mathbf{J}(Q_S, Q_L^1, ..., Q_L^M) = \begin{pmatrix} \partial\dot{Q}_S/\partial Q_S & \partial\dot{Q}_S/\partial Q_L^1 & \cdots & \partial\dot{Q}_S/\partial Q_L^M \\ \partial\dot{Q}_L^1/\partial Q_S & \partial\dot{Q}_L^1/\partial Q_L^1 & \cdots & \partial\dot{Q}_L^1/\partial Q_L^M \\ \vdots & \vdots & \ddots & \vdots \\ \partial\dot{Q}_L^M/\partial Q_S & \partial\dot{Q}_L^M/\partial Q_L^1 & \cdots & \partial\dot{Q}_L^M/\partial Q_L^M \end{pmatrix}
= \begin{pmatrix} -\gamma_1 & -\gamma_3 & \cdots & -\gamma_3 \\ -\gamma_3 & -2\gamma_2 & \cdots & -\gamma_2 \\ \vdots & \vdots & \ddots & \vdots \\ -\gamma_3 & -\gamma_2 & \cdots & -2\gamma_2 \end{pmatrix}.
$$

To guarantee the stability of the system, the Jacobian matrix should be negative definite. That is, for any $x \equiv (x_0, x_1, x_2, ..., x_M) \neq \underbrace{(0, 0, ..., 0)}_{M+1}$, $-x^T \mathbf{J} x > 0$.

Let $X \equiv \sum_{i=1}^{M} x_i$. Then,

$$\begin{aligned} -x^T \mathbf{J} x &= \gamma_1 x_0^2 + 2\gamma_2 \sum_{i=1}^{M} x_i^2 + 2\gamma_3 x_0 \sum_{i=1}^{M} x_i + 2\gamma_2 \sum_{i=1, i\neq j}^{M} x_i x_j \\ &= \gamma_1 x_0^2 + 2\gamma_2 \sum_{i=1}^{M} x_i^2 + 2\gamma_3 x_0 X + 2\gamma_2 \sum_{i=1, i\neq j}^{M} x_i x_j \end{aligned}$$

By Cauchy-Schwarz inequality, $\sum_{i=1}^{M} x_i^2 \geqq X^2/M$. Thus,

$$\begin{aligned} -x^T \mathbf{J} x &\geqq \gamma_1 x_0^2 + 2\gamma_3 x_0 X + \frac{M+1}{M}\gamma_2 X^2 \\ &= \begin{pmatrix} x_0 & X \end{pmatrix} \begin{pmatrix} \gamma_1 & \gamma_3 \\ \gamma_3 & \gamma_2(M+1)/M \end{pmatrix} \begin{pmatrix} x_0 \\ X \end{pmatrix} \\ &= \begin{pmatrix} x_0 & X \end{pmatrix} \mathbf{A} \begin{pmatrix} x_0 \\ X \end{pmatrix}, \end{aligned}$$

where $\mathbf{A} \equiv \begin{pmatrix} \gamma_1 & \gamma_3 \\ \gamma_3 & \gamma_2(M+1)/M \end{pmatrix}$.

Note that $\mathbf{J}$ is negative definite if and only if $\mathbf{A}$ is positive definite, which requires that $\gamma_1 > 0$, $\gamma_2 > 0$, and $\gamma_1\gamma_2(M+1)/M > \gamma_3^2$.

Therefore, the mixed market equilibrium is stable when $\gamma_1 > 0$, $\gamma_2 > 0$, and $\gamma_1\gamma_2(M+1) > \gamma_3^2 M$.

Applying the stability condition, the denominator of N^* and $\overline{\omega}^*$ is positive. Therefore, $N^* > 0$ if

$$(\alpha - 2\sqrt{\beta f})(M+1)\gamma_2 > (\alpha - 2\sqrt{\beta F})M\gamma_3,$$

and $\overline{\omega}^* > 0$ if

$$(\alpha - 2\sqrt{\beta F})\gamma_1 > (\alpha - 2\sqrt{\beta f})\gamma_3.$$

Therefore, to ensure the existence of a unique equilibrium that is stable and features a mixed market, the following three conditions should be satisfied[26]:

(i) $(\alpha - 2\sqrt{\beta F})\gamma_1 > (\alpha - 2\sqrt{\beta f})\gamma_3$,
(ii) $(\alpha - 2\sqrt{\beta f})(M+1)\gamma_2 > (\alpha - 2\sqrt{\beta F})M\gamma_3$, and
(iii) $\gamma_1\gamma_2(M+1) > \gamma_3^2 M$. **Q.E.D.**

B.2 Proof of Proposition 2.

Let $D(M) \equiv \gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M$, and $E \equiv \alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})$. By assuming that a unique mixed market equilibrium arises before and after a large firm's entry, $D(M) > 0$, $D(M+1) > 0$, and $E > 0$ (Proposition 1).

[26]These three conditions imply that $\gamma_1 > 0$ and $\gamma_2 > 0$.

Deriving from the equilibrium values, the changes of the variables when the number of large firms increase from M to $M+1$ are as follows:

$$\Delta q_S^* = 0, \Delta p_S^* = 0, \Delta q_L^* = 0,$$
$$\Delta N^* = \frac{-\gamma_2\gamma_3 E}{D(M)D(M+1)}\sqrt{\frac{\beta}{f}} < 0.$$

And

$$\Delta p_L^* = \frac{\gamma_2(\gamma_3^2 - \gamma_1\gamma_2)E}{D(M)D(M+1)},$$
$$\Delta \overline{\omega}^* = \frac{(\gamma_3^2 - \gamma_1\gamma_2)E}{D(M)D(M+1)}\sqrt{\frac{\beta}{F}},$$
$$\Delta \Pi_L^* = \frac{\gamma_2(\gamma_3^2 - \gamma_1\gamma_2)E^2}{D(M)D(M+1)}(\frac{1}{D(M)} + \frac{1}{D(M+1)}).$$

Thus, if $\gamma_1\gamma_2 < \gamma_3^2$, then $\Delta p_L^* > 0$, $\Delta\overline{\omega}^* > 0$, and $\Delta\Pi_L^* > 0$. If $\gamma_1\gamma_2 > \gamma_3^2$, then $\Delta p_L^* < 0$, $\Delta\overline{\omega}^* < 0$, and $\Delta\Pi_L^* < 0$. If $\gamma_1\gamma_2 = \gamma_3^2$, then $\Delta p_L^* = 0$, $\Delta\overline{\omega}^* = 0$, and $\Delta\Pi_L^* = 0$.

Recall that Q^* is the total quantity in the differentiated goods market. Thus,

$$\Delta Q^* = \frac{\gamma_2(\gamma_1 - \gamma_3)E}{D(M)D(M+1)}.$$

Thus, if $\gamma_1 > \gamma_3$, then $\Delta Q^* > 0$. If $\gamma_1 < \gamma_3$, then $\Delta Q^* < 0$. If $\gamma_1 = \gamma_3$, then $\Delta Q^* = 0$. **Q.E.D.**

B.3 Proof of Proposition 3.

Consumer welfare, producer surplus, and social welfare can be expressed as

$$\begin{aligned}
CW(M) &= \alpha Q^* - \frac{\beta}{2}(N^* q_S^{*2} + M\overline{\omega}^* q_L^{*2}) - \frac{\gamma_1}{2}Q_S^{*2} - \frac{\gamma_2}{2}Q_L^{*2} - \gamma_3 Q_S^* Q_L^* - p_S^* Q_S^* - p_L^* Q_L^* \\
&= \frac{\beta}{2}(N^* q_S^{*2} + M\overline{\omega}^* q_L^{*2}) + \frac{\gamma_1}{2}Q_S^{*2} + \frac{\gamma_2}{2}Q_L^{*2} + \gamma_3 Q_S^* Q_L^*, \\
PS(M) &= p_L^* Q_L^* - MF\overline{\omega}^* = \frac{\gamma_2 M E^2}{D^2(M)}, \\
SW(M) &= CW(M) + PS(M).
\end{aligned}$$

The impact of an increase from M to $M+1$ on consumer welfare is

$$\begin{aligned}
\Delta CW &= \frac{\beta q_S^{*2}}{2}(N^*(M+1) - N^*(M)) + \frac{\beta q_L^{*2}}{2}[(M+1)\overline{\omega}^*(M+1) - M\overline{\omega}^*(M)] \\
&\quad + \frac{\gamma_1}{2}(Q_S^{*2}(M+1) - Q_S^{*2}(M)) + \frac{\gamma_2}{2}(Q_L^{*2}(M+1) - Q_L^{*2}(M)) \\
&\quad + \gamma_3(Q_S^*(M+1)Q_L^*(M+1) - Q_S^*(M)Q_L^*(M)) \\
&= \frac{\beta}{2}(q_S^{*2} - q_L^{*2})\Delta V_S + \frac{\beta q_L^{*2}}{2}\Delta V + \frac{Q_S^*(M+1) + Q_S^*(M)}{2}(\gamma_1 \Delta Q_S + \gamma_3 \Delta Q_L) \\
&\quad + \frac{Q_L^*(M+1) + Q_L^*(M)}{2}(\gamma_2 \Delta Q_L + \gamma_3 \Delta Q_S) \\
&= \frac{\beta}{2}(q_S^{*2} - q_L^{*2})\Delta V_S + \frac{\beta q_L^{*2}}{2}\Delta V + \frac{Q_S^*(M+1) + Q_S^*(M)}{2}\Delta p_S \\
&\quad + \frac{Q_L^*(M+1) + Q_L^*(M)}{2}\Delta p_L \\
&= \frac{\beta}{2}(q_S^{*2} - q_L^{*2})\Delta V_S + \frac{\beta q_L^{*2}}{2}\Delta V - \frac{Q_L^*(M+1) + Q_L^*(M)}{2}\Delta p_L,
\end{aligned}$$

where $\Delta V_S = N^*(M+1) - N^*(M)$, and $\Delta V = (N^*(M+1) + (M+1)\overline{\omega}^*(M+1)) - (N^*(M) + M\overline{\omega}^*(M))$.

By substituting the equilibrium values of the endogenous variables into the above equation,

$$\Delta CW = \frac{\gamma_2 E}{2D(M)D(M+1)}\{\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f}) + E(\gamma_1\gamma_2 - \gamma_3^2)[\frac{M}{D(M)} + \frac{M+1}{D(M+1)}]\}.$$

If $\sqrt{\beta}(\gamma_1\sqrt{F}-\gamma_3\sqrt{f})+E(\gamma_1\gamma_2-\gamma_3^2)[M/D(M)+(M+1)/D(M+1)] > 0$, then $\Delta CW > 0$. Otherwise, if $\sqrt{\beta}(\gamma_1\sqrt{F}-\gamma_3\sqrt{f})+E(\gamma_1\gamma_2-\gamma_3^2)[M/D(M)+(M+1)/D(M+1)] < 0$, then $\Delta CW < 0$. If $\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f}) + E(\gamma_1\gamma_2 - \gamma_3^2)[M/D(M) + (M+1)/D(M+1)] = 0$, then $\Delta CW = 0$.

The impact of an increase from M to $M+1$ on producer surplus is

$$\Delta PS = \gamma_2 E^2[\frac{M+1}{D^2(M+1)} - \frac{M}{D^2(M)}].$$

Thus if $(M+1)/D^2(M+1) - M/D^2(M) > 0$, then $\Delta PS > 0$. Otherwise, if $(M+1)/D^2(M+1) - M/D^2(M) < 0$, then $\Delta PS < 0$. If $(M+1)/D^2(M+1) - M/D^2(M) = 0$, then $\Delta PS = 0$

The impact of an increase from M to $M+1$ on social welfare is

$$\Delta SW = \frac{\gamma_2 E}{2D(M)D(M+1)}[\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f}) + \gamma_1\gamma_2 E(\frac{1}{D(M)} + \frac{1}{D(M+1)})] .$$

Thus, if $\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f}) + \gamma_1\gamma_2 E[1/D(M) + 1/D(M+1)] > 0$, then $\Delta SW > 0$. Otherwise, if $\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f}) + \gamma_1\gamma_2 E[1/D(M) + 1/D(M+1)] < 0$, then $\Delta SW < 0$. If $\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f}) + \gamma_1\gamma_2 E[1/D(M) + 1/D(M+1)] = 0$, then $\Delta SW = 0$. **Q.E.D.**

B.4 Proof of Proposition 5.

(i) When the large firm supplies one variety, the computation shows that

$$
\begin{aligned}
q_S^* &= \sqrt{\frac{f}{\beta}},\\
q_L^* &= \frac{\alpha(\gamma_1-\gamma_3)+2\gamma_3\sqrt{\beta f}}{\gamma_1(2\beta+\gamma_2)+(\gamma_1\gamma_2-\gamma_3^2)M},\\
p_L^* &= (\beta+\gamma_2)\frac{\alpha(\gamma_1-\gamma_3)+2\gamma_3\sqrt{\beta f}}{\gamma_1(2\beta+\gamma_2)+(\gamma_1\gamma_2-\gamma_3^2)M},\\
N^* &= \sqrt{\frac{\beta}{f}}\frac{\alpha[2\beta+\gamma_2(M+1)-\gamma_3 M]-2\sqrt{\beta f}[2\beta+\gamma_2(M+1)]}{\gamma_1(2\beta+\gamma_2)+(\gamma_1\gamma_2-\gamma_3^2)M}.
\end{aligned}
$$

To ensure the existence of a unique mixed market equilibrium, we should have the stability of the equilibrium, $N^* > 0$ and $\Pi_L^* > 0$.

Following the same approach in Appendix (B.1), we have the following dynamic adjustment system in terms of Q_S, and q_L^m $(m = 1, ..., M)$:

$$
\begin{aligned}
\dot{Q}_S(Q_S, q_L^1, ..., q_L^M) &= d_2[\alpha - 2\sqrt{\beta f} - \gamma_1 Q_S - \gamma_3\sum_{m=1}^{M} q_L^m - \frac{f}{q_S}],\\
\dot{q}_L^1(Q_S, q_L^1, ..., q_L^M) &= d_3[\alpha - 2\beta q_L^1 - \gamma_3 Q_S - \gamma_2(2q_L^1 + \sum_{m=2}^{M} q_L^m)],\\
&\vdots
\end{aligned}
$$

$$
\dot{q}_L^M(Q_S, q_L^1, ..., q_L^M) = d_{M+2}[\alpha - 2\beta q_L^M - \gamma_3 Q_S - \gamma_2(2q_L^M + \sum_{m=1}^{M-1} q_L^m)].
$$

where "$\cdot$" denotes the differentiation with respect to time, i.e., $\dot{Q}_S = dQ_S/dt$, and $\dot{q}_L^m = dq_L^m/dt$, $(m = 1, ..., M)$; and d_i $(i = 0, 1, \cdots, M)$ is the positive coefficient measuring the speed of dynamic adjustment. Without loss of generality, set $d_i = 1$, $i = 0, 1, \cdots, M$.

The Jacobian matrix of the above dynamic adjustment system is

$$
\mathbf{J}(Q_S, q_L^1, ..., q_L^M) = \begin{pmatrix} \partial\dot{Q}_S/\partial Q_S & \partial\dot{Q}_S/\partial q_L^1 & \cdots & \partial\dot{Q}_S/\partial q_L^M \\ \partial\dot{q}_L^1/\partial Q_S & \partial\dot{q}_L^1/\partial q_L^1 & \cdots & \partial\dot{q}_L^1/\partial q_L^M \\ \vdots & \vdots & \ddots & \vdots \\ \partial\dot{q}_L^M/\partial Q_S & \partial\dot{q}_L^M/\partial q_L^1 & \cdots & \partial\dot{q}_L^M/\partial q_L^M \end{pmatrix}
$$

$$
= \begin{pmatrix} -\gamma_1 & -\gamma_3 & \cdots & -\gamma_3 \\ -\gamma_3 & -2(\beta+\gamma_2) & \cdots & -\gamma_2 \\ \vdots & \vdots & \ddots & \vdots \\ -\gamma_3 & -\gamma_2 & \cdots & -2(\beta+\gamma_2) \end{pmatrix}.
$$

To guarantee the stability of the system, the Jacobian matrix should be negative definite. That is, for any $x \equiv (x_0, x_1, x_2, ..., x_M) \neq \underbrace{(0, 0, ..., 0)}_{M+1}$, $-x^T\mathbf{J}x > 0$.

Let $X \equiv \sum_{i=1}^{M} x_i$. Then,

$$\begin{aligned} -x^T\mathbf{J}x &= \gamma_1 x_0^2 + 2(\beta+\gamma_2)\sum_{i=1}^{M} x_i^2 + 2\gamma_3 x_0 \sum_{i=1}^{M} x_i + 2\gamma_2 \sum_{i=1, i\neq j}^{M} x_i x_j \\ &= \gamma_1 x_0^2 + 2(\beta+\gamma_2)\sum_{i=1}^{M} x_i^2 + 2\gamma_3 x_0 X + 2\gamma_2 \sum_{i=1, i\neq j}^{M} x_i x_j \end{aligned}$$

By Cauchy-Schwarz inequality, $\sum_{i=1}^{M} x_i^2 \geqq X^2/M$. Thus,

$$\begin{aligned} -x^T\mathbf{J}x &\geqq \gamma_1 x_0^2 + 2\gamma_3 x_0 X + (2\beta + \frac{M+1}{M}\gamma_2)X^2 \\ &= \begin{pmatrix} x_0 & X \end{pmatrix} \begin{pmatrix} \gamma_1 & \gamma_3 \\ \gamma_3 & 2\beta + \gamma_2(M+1)/M \end{pmatrix} \begin{pmatrix} x_0 \\ X \end{pmatrix} \\ &= \begin{pmatrix} x_0 & X \end{pmatrix} \mathbf{B} \begin{pmatrix} x_0 \\ X \end{pmatrix}, \end{aligned}$$

where $\mathbf{B} \equiv \begin{pmatrix} \gamma_1 & \gamma_3 \\ \gamma_3 & 2\beta + \gamma_2(M+1)/M \end{pmatrix}$.

Note that $\mathbf{J}$ is negative definite if and only if $\mathbf{B}$ is positive definite, which requires that $\gamma_1 > 0$ and $\gamma_1(2\beta+\gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$.

Therefore, the mixed market equilibrium is stable when $\gamma_1 > 0$ and $\gamma_1(2\beta+\gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$.

In addition, we should have $N^* > 0$, and $\Pi_L^* > 0$, i.e., $F < p_L^* q_L^*$. Accordingly,

$$\alpha[2\beta + \gamma_2(M+1) - \gamma_3 M] > 2\sqrt{\beta f}[2\beta + \gamma_2(M+1)],$$

$$\alpha(\gamma_1 - \gamma_3) + 2\gamma_3\sqrt{\beta f} > [\gamma_1(2\beta+\gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M]\sqrt{\frac{F}{(\beta+\gamma_2)}}.$$

(ii) Let $J \equiv \alpha(\gamma_1 - \gamma_3) + 2\gamma_3\sqrt{\beta f}$, and $K(M') \equiv \gamma_1(2\beta+\gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M'$. By (4-i), $J > 0$, $K(M) > 0$ and $K(M+1) > 0$. From the equilibrium, the changes of the variables when the number of large firms increase from M to $M+1$ are derived as follows:

$$\Delta q_S^* = 0,$$
$$\Delta p_S^* = 0,$$
$$\Delta N^* = -\frac{(2\beta+\gamma_2)\gamma_3 J}{K(M)K(M+1)}\sqrt{\frac{\beta}{f}} < 0 \text{ if } \gamma_3 > 0, \text{ and } > 0 \text{ if } \gamma_3 < 0.$$

Moreover,

$$\Delta q_L^* = \frac{(\gamma_3^2 - \gamma_1\gamma_2)J}{K(M)K(M+1)},$$
$$\Delta p_L^* = \frac{(\beta+\gamma_2)(\gamma_3^2 - \gamma_1\gamma_2)J}{K(M)K(M+1)},$$
$$\Delta \Pi_L^* = \frac{(\beta+\gamma_2)(\gamma_3^2 - \gamma_1\gamma_2)J^2}{K(M)K(M+1)}[\frac{1}{K(M)} + \frac{1}{K(M+1)}].$$

which are positive if $\gamma_1\gamma_2 < \gamma_3^2$, negative if $\gamma_1\gamma_2 > \gamma_3^2$, and constant if $\gamma_1\gamma_2 = \gamma_3^2$. Also

$$\Delta Q^* = \frac{(2\beta+\gamma_2)(\gamma_1 - \gamma_3)J}{K(M)K(M+1)},$$

which is positive if $\gamma_1 > \gamma_3$, negative if $\gamma_1 < \gamma_3$, and constant if $\gamma_1 = \gamma_3$.

(iii) The associated consumer welfare, producer surplus and social welfare are

$$CW(M)^* = \alpha Q^* - \frac{\beta}{2}(N^* q_S^{*2} + M\,|\Omega^*|\, q_L^{*2}) - \frac{\gamma_1}{2}Q_S^{*2} - \frac{\gamma_2}{2}Q_L^{*2} - \gamma_3 Q_S^* Q_L^* - p_S^* Q_S^* - p_L^* Q_L^*,$$
$$PS(M)^* = \frac{\gamma_2 M J^2}{K^2(M)},$$
$$SW(M)^* = CW(M)^* + PS(M)^* - FM.$$

The impact of an increase from M to $M+1$ on consumer welfare is

$$\Delta CW^* = \frac{J}{2K(M)K(M+1)}[\Psi + (\beta+\gamma_2)(\gamma_1\gamma_2 - \gamma_3^2)J(\frac{M}{K(M)} + \frac{M+1}{K(M+1)})].$$

where $\Psi = \alpha\beta(\gamma_1 - \gamma_3) - \gamma_2\gamma_3\sqrt{\beta f}$. ΔCW^* is positive if $\Psi + (\beta+\gamma_2)(\gamma_1\gamma_2 - \gamma_3^2)J[M/K(M) + (M+1)/K(M+1)] > 0$, and is negative otherwise.

The impact of a marginal increase of M on producer surplus is

$$\Delta PS^* = \frac{(\beta+\gamma_2)J^2}{K(M)K(M+1)}[\frac{M+1}{K^2(M+1)} - \frac{M}{K^2(M)}].$$

which is positive if $(M+1)/K^2(M+1) - M/K^2(M) > 0 > 0$, and is negative otherwise.

The impact of a marginal increase of M on social welfare is

$$\Delta SW^* = \frac{J}{2K(M)K(M+1)}[\Psi + \gamma_1(\beta+\gamma_2)(2\beta+\gamma_2)J(\frac{1}{K(M)} + \frac{1}{K(M+1)})] - F.$$

which is positive if $\Psi + \gamma_1(\beta+\gamma_2)(2\beta+\gamma_2)J[1/K(M) + 1/K(M+1)] > 2FK(M)K(M+1)/J$, and is negative otherwise. **Q.E.D.**

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Figures

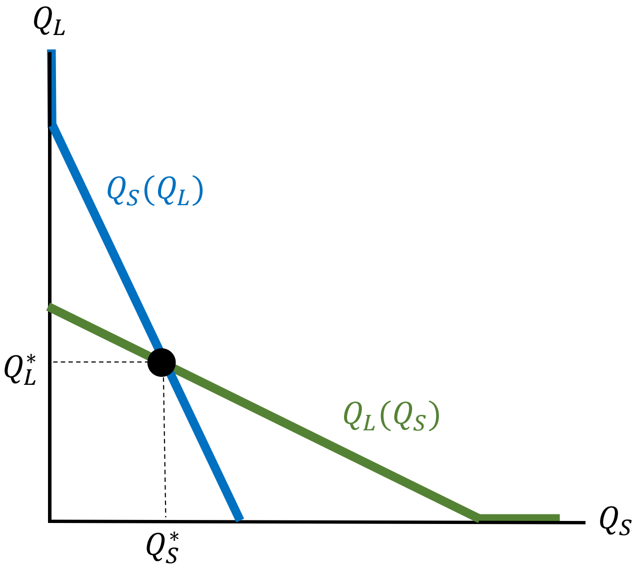


Figure 1a
A Stable Mixed Market Equilibrium
(Substitutes)

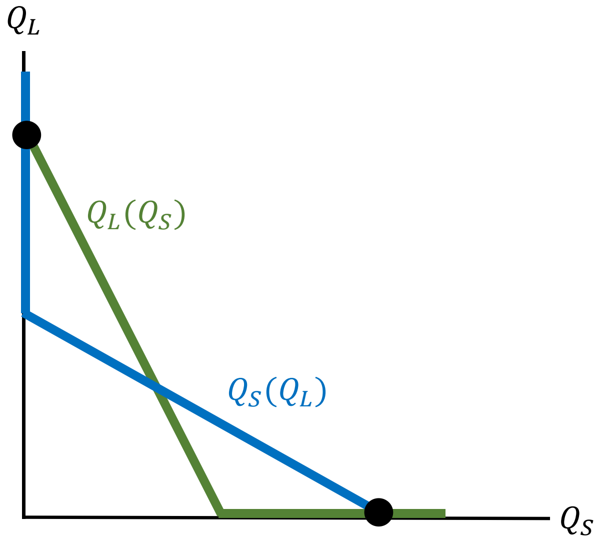


Figure 1b
An Unstable Mixed Market Equilibrium
(Substitutes)

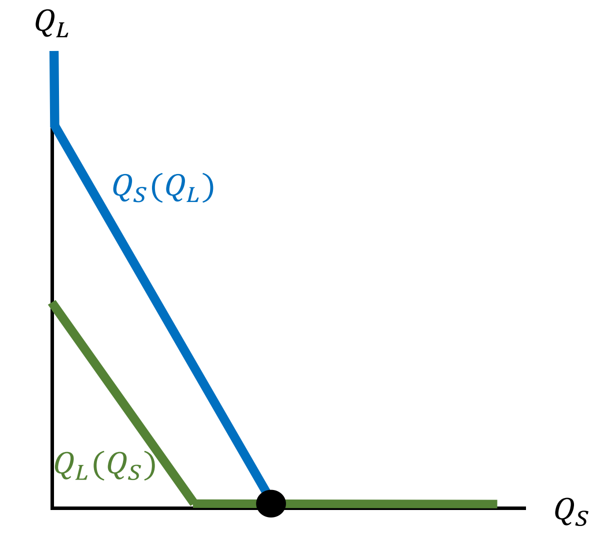


Figure 1c
No Mixed Market Equilibrium
(Substitutes)

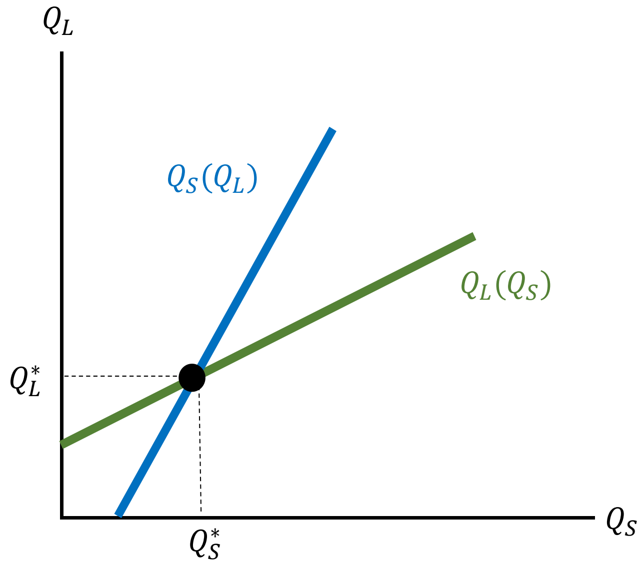


Figure 1d
A Stable Mixed Market Equilibrium
(Complements)

Figure 1 Existence and Stability of a Mixed Market Equilibrium

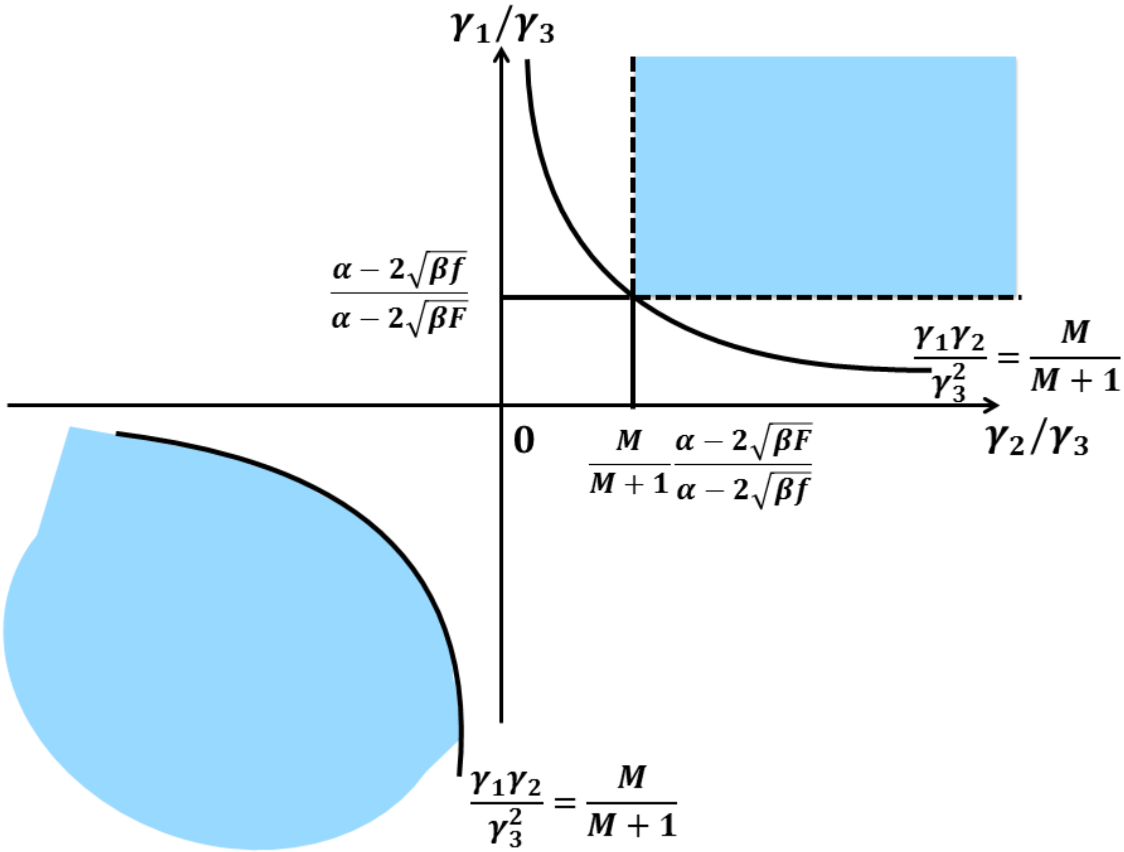
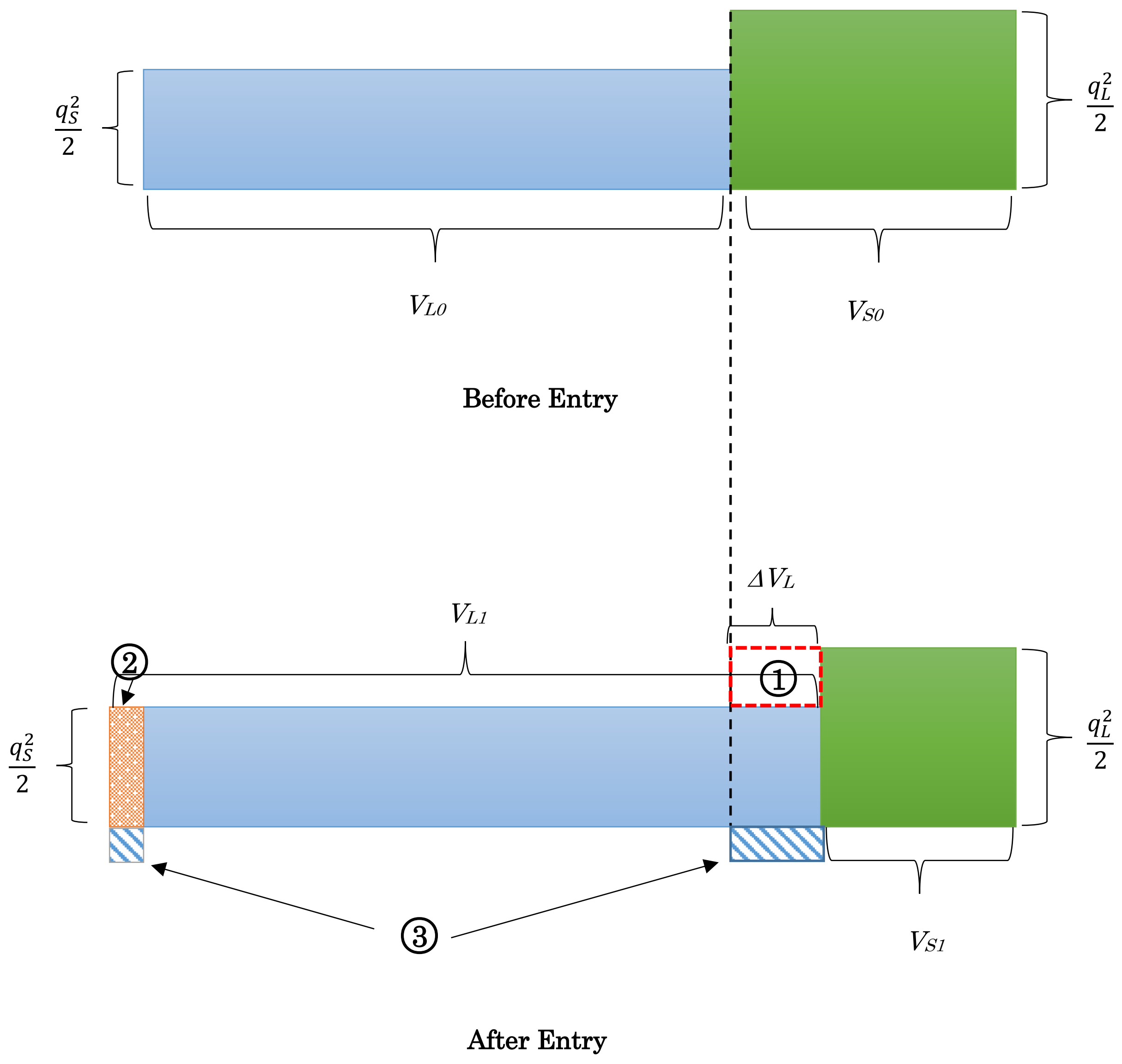


Figure 2. Conditions for a Unique Mixed Market Equilibrium



① Replacement Effect
② Variety Effect
③ Quantity Effect (on Producer Surplus)

Figure 3 Social Welfare Change (Assume f > F)