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by

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# Employment and Hours over the Business Cycle in a Model with Search Frictions[1]

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**Abstract**

This paper presents a labor market search-matching model with multi-worker firms to investigate how firms utilize employment and hours of work over the business cycle. Each firm and its employees bargain over an *earnings schedule* that maps hours into earnings, and this schedule determines the costs of utilizing the two margins of labor adjustment. We calibrate the model for the Japanese labor market, in which fluctuations in hours of work account for 79 percent of the variations in the aggregate labor input. We find that the aggregate labor input in the model is as volatile as that in the data, and is *25 times* as volatile as that in the model without hours of work. Interestingly, with bargained hours of work, the model generates significantly small fluctuations in employment, unemployment, and vacancies, and is outperformed by the model with constant hours of work.

JEL classification: E32, J20, J64.

Keywords: search, hours of work, employment, business cycles, multi-worker firms.

# 1 Introduction

Firms adjust their labor inputs over the business cycle through the intensive margin (hours of work per employee) and the extensive margin (the number of employees). In their survey, Hall *et al.* (2000) found that 62 percent of firms consider overtime as the primary reaction to a demand boom. In a perfectly competitive labor market, firms do not need to utilize overtime because they can employ extra workers instantly at the going wage rate. This suggests the importance of *frictions* in understanding how firms utilize the intensive and extensive margins over the business cycle.

In this paper, we study the composition of labor demand over the business cycle by focusing on the fact that firms need to engage in time-consuming search-matching process when hiring new employees while changes in hours of work per employee are instantaneous. Our basic model builds on a recent development in labor market search models with *multi-worker firms*.[1] A novel property of our model is that firms facing productivity shocks choose both vacancies and hours of work per employees.

Introduction of endogenous working hours per employee changes the search-matching model in a nontrivial manner because total compensation depends on hours of work as well as the hourly wage rate. Further, the hourly wage rate itself may depend on hours of work. To capture the possibility of nonlinear compensation scheme, we assume that a firm and its employees bargain over a state-contingent contract in the form of an *earnings schedule* which maps hours of work per employee into earnings (Cooper *et al.*, 2007; Kudoh and Sasaki, 2011). The earnings schedule turns out to be a convex function of hours of work, and this specifies the *marginal hourly wage rate* for the firm of choosing hours of work.

In our framework, there is a clear distinction between earnings and the wage rate. The earnings equation is similar to the wage equation in the standard search-matching model, and thus

[1] A partial list of related contributions is Bertola and Caballero (1994), Smith (1999), Cahuc and Wasmer (2001), Koeniger and Prat (2007), Cahuc *et al.* (2008), Felbermayr and Prat (2011), Kudoh and Sasaki (2011), and Acemoglu and Hawkins (2014). Leading examples of business cycle studies using the multi-worker paradigm are Cooper *et al.* (2007), Elsby and Michaels (2013), Krause and Lubik (2013), and Fujita and Nakajima (2016).

depends on the current labor market tightness as well as productivity. However, the marginal hourly wage rate is independent of labor market tightness. As a result, the steady-state hours of work per employee are determined primarily by the level of TFP without any reference to the parameters related to search frictions. This suggests that search frictions are irrelevant for understanding the long-run trend of hours per worker. In contrast, the steady-state level of employment is influenced by both TFP and search frictions because the marginal gain from creating an additional vacancy depends directly on the vacancy-filling probability, which is determined by the degree of search frictions.

We calibrate our model to match the Japanese labor market facts. An important characteristic of labor market fluctuations in Japan is that the intensive margin accounts for a particularly large proportion of cyclical fluctuations in the aggregate labor input. In our empirical analysis, we find that variations in hours of work per employee account for *79 percent* of the variations in the aggregate labor input, while variations in the number of employees account for 21 percent of the variations.[2] Clearly, in understanding labor market fluctuations in Japan, it is a serious omission if one uses a model without variable hours of work. In our numerical analysis, we show that our model replicates much of the observed magnitudes of fluctuations in total labor input, hours of work per employee, employment, unemployment, and vacancies. In particular, variations in hours of work per employee account for 84 percent of the variations in the aggregate labor input, while variations in the number of employees account for 19 percent of the variations.[3]

To clarify the importance of hours of work, we study our model without hours of work, and find that the aggregate labor input in the model with variable hours of work fluctuates *25 times* as much as that in the model without hours of work. Introduction of hours of work also magnifies labor adjustment along the *extensive margin*: unemployment and vacancies in the model with variable hours of work fluctuate 4 times as much as those in the model without hours of work. These results are obtained without wage rigidity or additional shocks. This is in sharp contrast to the

[2]This is in sharp contrast with the labor market fluctuations in the U.S., in which 79 percent of the variations are accounted for by the *extensive* margin.

[3]Our decomposition of variations admits some discrepancy so that the sum of variations may deviate from unity.

existing literature, which suggests inability of the textbook search-matching model in generating the observed magnitudes of fluctuations in unemployment and vacancies in the US, known as the unemployment volatility puzzle (Shimer, 2005).

Using a model nearly identical to ours, but without hours of work, Krause and Lubik (2013) investigate whether multi-worker paradigm with Stole and Zwiebel's (1996) intra-firm bargaining helps resolve the unemployment volatility puzzle, and conclude that while there is an improvement, the effect is negligible. Why do the two similar models generate distinct results?

Our model's ability to generate high volatilities in unemployment and vacancies comes partly from the presence of disutility from hours of work. We find that disutility from hours of work plays a role similar to having a high target value of unemployment benefit in calibration because it raises the level of compensation and reduces the firm's profit, making the job-creation incentive more exposed to external shocks. In addition, introduction of hours of work introduces the *Frisch elasticity parameter* into the analysis, and this magnifies the link between hours of work and disutility. Indeed, we verify that the magnitudes of fluctuations in unemployment and vacancies depend significantly on the Frisch elasticity.

Interestingly, to generate fluctuations along the *extensive margin*, it matters who chooses hours of work. We show that if hours of work are determined in bargaining, then the volatilities in employment, unemployment, and vacancies become about 1/3 of those from the basic model. The model in which hours of work are chosen by workers shares the same property. Further, these models are outperformed by the model in which hours of work are held constant. We therefore conclude that our maintained assumption that firms choose hours of work from the bargained contract serves as a useful benchmark for understanding hours of work over the business cycle.

Since this paper deals with general issues, there is a long list of related studies. Therefore, the description of the literature below must necessarily be partial.

We intend our empirical analysis to be a contribution to the growing literature on re-examination of hours of work using new models and new datasets. This includes Rogerson (2006) and Ohanian and Raffo (2012), to name a few. Particularly relevant is Ohanian and Raffo (2012), who find that the intensive margin is increasingly more important for labor adjustment in 14 OECD countries they studied. Using the dataset we built for the Japanese labor market, our empirical analysis confirms the importance of the intensive margin.

In the labor hoarding literature and more generally the factor utilization literature, it is often argued that real business cycle models with costly labor adjustment display realistic cyclical fluctuations. Burnside *et al.* (1993), for instance, assume that it takes one period to adjust labor input. Our model naturally possesses the same property because the level of employment increases only after firms post vacancies and the matching process takes one period. Contrary to the factor utilization literature, in which the cost function for factor utilization is exogenously given, in our model, the cost that a firm faces when choosing longer hours of work is endogenously determined by intra-firm bargaining.

In terms of the structure of the model, particularly related to our study are Cooper *et al.* (2007) and Kudoh and Sasaki (2011), in which determination of hours of work is considered in the context of a search-matching model with multi-worker firms.[4] Cooper *et al.* (2007) study both employment and hours of work over the business cycle using a model similar to ours. They emphasize the importance of nonlinear cost of posting vacancies, and as a result, wage determination is simplified by assuming a take-it-or-leave-it offer protocol. While we restrict our analysis to a linear vacancy cost, our model is more general in that we adopt the Stole-Zwiebel bargaining, as in a steady-state economy developed by Kudoh and Sasaki (2011). The resulting earnings schedule reflects changes in the current state variables. As a result, our model closely replicates the magnitudes of fluctuations in both earnings and the hourly wage rate in the data, both of which fluctuate as much as TFP.

Our model is also closely related to the model developed by Fang and Rogerson (2009), who study the intensive and extensive margins using the framework of Merz (1995) and Andolfatto

[4]In a recent, independent contribution, Trapeznikova (2015) takes up the issue similar to ours. The model developed in Trapeznikova (2015) introduces on-the-job search with an intra-firm bargaining that is different from the Stole-Zwiebel to ensure efficiency of the bargaining outcome, and the model is calibrated to match the Danish labor market to replicate the firm-level facts.

(1996). In this model, the production unit is a matched worker-job pair, and this rules out the issue of intra-firm bargaining. In addition, the Merz-Andolfatto paradigm has a utility-maximizing household, who optimally chooses the level of consumption. Fang and Rogerson (2009) show that, with a concave utility function, there is a rich interaction between employment and hours through consumption, and as a result, an increase in the vacancy cost decreases employment and increases hours of work. This rich interaction comes from the labor-supply side. In contrast, we focus on the labor-demand side, and show that an increase in the vacancy cost decreases the steady-state level of employment, but it has no effect on hours of work in the steady state.

Using frictionless, competitive business cycle models, Kydland and Prescott (1991) and Cho and Cooley (1994) study hours and employment over the business cycle, and calibrate their models to match the US labor market facts. Braun *et al.* (2006) adopt the Cho-Cooley framework to study the labor market in Japan. In the Appendix, we compare a version of the Cho-Cooley model with our model, and show that, in the steady state, hours of work per employee are longer in our frictional economy than in the frictionless economy. Thus, while determination of the steady-state hours of work is independent of the parameters capturing the degree of search frictions, the very presence of frictions affects hours of work in the long-run.

The remainder of the paper is organized as follows. In Section 2, we empirically examine how labor inputs are adjusted over the business cycle in Japan, and present some other cyclical characteristics of the labor market in Japan. Section 3 describes our basic model, followed by characterization of equilibrium of the model in Section 4. In Section 5, we calibrate the model parameters and present the business cycle properties of our model. Section 6 presents the results under alternative parameter values and assumptions, and Section 7 concludes. Proofs and some additional results are found in the Appendix.

## 2 Labor Market Facts

### 2.1 Data

This section presents some empirical results highlighting the cyclical properties of the Japanese labor market. We obtain the series of the number of employed workers and the number of labor force from the Labour Force Survey (LFS), conducted by the Statistics Bureau and the Director-General for Policy Planning. The series of the average monthly hours worked per worker are obtained from the Monthly Labour Survey (MLS) conducted by the Ministry of Health, Labour and Welfare (MHLW). We construct our measure of the aggregate labor input as the product of the average monthly hours worked per worker and the number of employed workers, normalized by the labor force. These measures are consistent with those used in Ohanian and Raffo (2012).

The TFP series are from Braun *et al.* (2006).[5] We obtain the unemployment rate series from the LFS. The vacancy rate series are obtained from the Monthly Report on Employment Service (Shokugyo Antei Gyomu Tokei) conducted by the MHLW. Following Miyamoto (2011) and Lin and Miyamoto (2012), we construct the job finding and separation rates from the LFS. The series of real earnings and real wages are constructed from the nominal earnings series from the Monthly Labour Survey (MLS) conducted by the MHLW. Following the convention, we use the consumer price index (CPI) to obtain the real series.

Our data are quarterly, which, when necessary, are obtained by averaging or aggregating the corresponding monthly series. The sample covers 1980Q2–2010Q4. All series are seasonally adjusted using the Census Bureau's X12 ARIMA procedure and transformed by taking natural logarithms. Since our focus is on cyclical fluctuations in the series, the low-frequency movements in the data are filtered out by using the Hodrick-Prescott (HP) filter with smoothing parameter 1600.

### 2.2 Labor Market Fluctuations in Japan

Figure 1 plots the cyclical components of the aggregate labor input, hours of work per worker, and the number of employed workers. The figure shows that the aggregate labor input and its

[5]The TFP series are extended to 2010 by Nao Sudo, and we use the extended series.

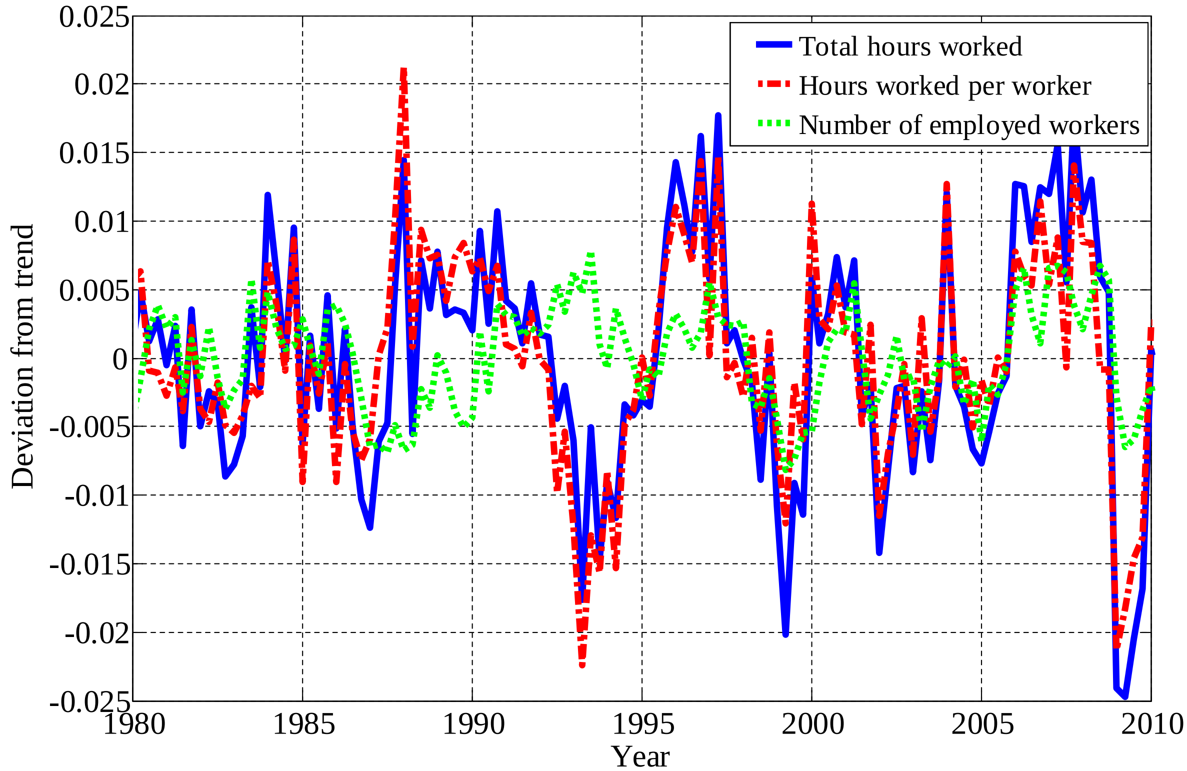


Figure 1: Total hours worked and its components over business cycles in Japan.

Note: The solid line indicates the cyclical component of total hours of worked. The dash-dotted line indicates the cyclical component of hours of worked per worker. The dashed line indicates the cyclical components of employed workers. The cyclical components are obtained by using the HP filter with smoothing parameter 1600. Sample covers 1980Q2-2010Q4.

components fluctuate significantly over the business cycle, and both hours and employment co-move with the aggregate labor input. It also indicates that the aggregate labor input comoves more closely with hours of work per worker than with employment, and that employment is less volatile than hours per worker.

Table 1: Summary statistics, quarterly Japanese data, 1980-2010

| Variables | $\hat{u}$ | $\hat{v}$ | $\hat{f}$ | $\hat{s}$ | $\hat{W}$ | $\hat{w}$ | $\hat{t}$ | $\hat{h}$ | $\hat{l}$ | $\hat{A}$ | $\hat{y}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Standard deviation | 0.059 | 0.096 | 0.091 | 0.092 | 0.012 | 0.010 | 0.009 | 0.008 | 0.004 | 0.011 | 0.016 |
| Correlation matrix | | | | | | | | | | | |
| $\hat{u}$ | 1 | -0.777 | -0.420 | 0.449 | -0.317 | -0.084 | -0.526 | -0.325 | -0.558 | -0.281 | -0.734 |
| $\hat{v}$ | - | 1 | 0.320 | -0.517 | 0.656 | 0.113 | 0.728 | 0.644 | 0.378 | 0.470 | 0.781 |
| $\hat{f}$ | - | - | 1 | -0.344 | 0.192 | 0.123 | 0.201 | 0.139 | 0.185 | 0.005 | 0.215 |
| $\hat{s}$ | - | - | - | 1 | -0.365 | 0.035 | -0.416 | -0.422 | -0.108 | -0.371 | -0.501 |
| $\hat{W}$ | - | - | - | - | 1 | 0.751 | 0.554 | 0.507 | 0.253 | 0.287 | 0.436 |
| $\hat{w}$ | - | - | - | - | - | 1 | -0.023 | -0.121 | 0.193 | 0.002 | 0.003 |
| $\hat{t}$ | - | - | - | - | - | - | 1 | 0.902 | 0.485 | 0.359 | 0.741 |
| $\hat{h}$ | - | - | - | - | - | - | - | 1 | 0.060 | 0.392 | 0.602 |
| $\hat{l}$ | - | - | - | - | - | - | - | - | 1 | 0.038 | 0.493 |
| $\hat{A}$ | - | - | - | - | - | - | - | - | - | 1 | 0.680 |
| $\hat{y}$ | - | - | - | - | - | - | - | - | - | - | 1 |

Table 1 quantifies what we see in Figure 1 and provides more information by summarizing cyclical characteristics of the key labor market variables such as the unemployment rate ($u$), the vacancy rate ($v$), the job-finding rate ($f$), the separation rate ($s$), total earnings ($W$), the real wage rate ($w$), total hours of work or the aggregate labor input ($t$), hours of work per employee ($h$), the number of employed workers ($l$), total factor productivity ($A$), and output ($y$). All variables are with hat, meaning that they are in percentage deviations from their trend levels. Since the series are in natural logarithms, the standard deviations can be interpreted as mean percentage deviations from their trend levels.

**Hours and Employment** Consistent with Figure 1, Table 1 shows a strong positive relationship between total hours worked and hours worked per worker. The correlation between them is 0.90. We also find a positive relationship between total hours and employment, with a correlation of 0.49. This is significantly smaller than the correlation between total hours and hours per worker. As Braun *et al.* (2006) pointed out, there is no strong correlation between hours worked per worker and employment. The correlation between them is 0.06. Table 1 also shows that the aggregate labor input and its components comove positively with TFP and output, and comove negatively with the unemployment rate, indicating that they are all pro-cyclical. Interestingly, the correlation between employment and TFP is 0.04 while the correlation between hours per worker and TFP is 0.39. The standard deviation of total hours worked is 0.9 percent, that of hours worked per worker is 0.8 percent, and that of employment is 0.4 percent. Thus, hours of work per worker are twice as volatile as employment. Since the standard deviations of TFP and output are 1.1 percent and 1.6 percent, respectively, the aggregate labor input and its components are less volatile than TFP or output.

**Unemployment and Vacancy** Table 1 shows that the unemployment rate is counter-cyclical and the vacancy rate is pro-cyclical. The correlation between the unemployment rate and TFP is -0.28 and the correlation between the unemployment rate and output is -0.73. The correlation between the vacancy rate and TFP is 0.47 and the correlation between the vacancy rate and output is 0.78. Since the unemployment rate is counter-cyclical and the vacancy rate is procyclical, these two series comove negatively. The correlation between them is -0.78, which implies the Beveridge curve. Both the unemployment rate and the vacancy rate are significantly more volatile than TFP or output. While the standard deviations of TFP and output is 1.1 percent and 1.6 percent, respectively, the standard deviations of the unemployment and vacancy rates are 5.9 percent and 9.6 percent, respectively.

**Job Finding Rate and Separation Rate** The job finding rate is acyclical or procyclical and the separation rate is counter-cyclical. The correlation between the job finding rate and TFP is 0.005, which is very weak. However, the correlation between the job finding rate and output is 0.22. The correlation between the separation rate and TFP is -0.37 and the correlation between the separation rate and output is -0.50. The standard deviations of the job finding rate and separation rate are 9.1 percent and 9.2 percent, respectively. Both the job finding rate and separation rate are more volatile than TFP or output, but the magnitudes are similar.

**Real Earnings and Real Wage Rate** In this paper, we clearly distinguish between earnings and the hourly wage rate. While real earnings are procyclical, the real hourly wage rate is acyclical. The correlation between real earnings and TFP is 0.29 and the correlation between real earnings and output is 0.44, indicating that real earnings are procyclical. However, the correlation between the real hourly wage rate and TFP is 0.002 and the correlation between the real hourly wage rate and output is 0.003. Real earnings and the real hourly wage rate are both positively correlated with employment. Interestingly, while real earnings are positively correlated with hours of work, the real hourly wage rate is *negatively* correlated with hours. This negative correlation between hours and hourly wage rate might look inconsistent with the presence of overtime payment. The key to understanding the result is that we can only compute the series of the *average* hourly wage rate, and the series of the *marginal* hourly wage rate are not available. The two measures do not coincide unless earnings per worker take the following form: $W = wh$.

## 2.3 Decomposition of Fluctuations

We now study the relative contributions of the intensive and extensive margins to fluctuations in the total hours worked. With $\hat{t} = \hat{h} + \hat{l}$, variance of total hours worked can be decomposed as

$$Var(\hat{t}) = Var(\hat{h}) + Var(\hat{l}) + 2Cov(\hat{h}, \hat{l}) = Cov(\hat{t}, \hat{h}) + Cov(\hat{t}, \hat{l}). \tag{1}$$

The term $Cov(\hat{t}, \hat{h})$ gives the amount of variations in $\hat{t}$ that derived from variations in $\hat{h}$ and through its comovement with $\hat{l}$. Similarly, the term $Cov(\hat{t}, \hat{l})$ is the amount of variations in $\hat{t}$ that derived from variations in $\hat{l}$ and through its comovement with $\hat{h}$. By dividing the both sides of (1) by $Var(\hat{t})$, we obtain

$$1 = \frac{Cov(\hat{t}, \hat{h})}{Var(\hat{t})} + \frac{Cov(\hat{t}, \hat{l})}{Var(\hat{t})} = \beta^h + \beta^l, \tag{2}$$

where $\beta^h$ and $\beta^l$ are the relative contributions of variations in $\hat{h}$ and $\hat{l}$ to variations in $\hat{t}$. These measures are an application of the "beta value" in finance.[6]

From the data, we find that $\beta^h = 0.79$ and $\beta^l = 0.21$. In other words, the intensive margin explains 79 percent of variations in total hours worked and the extensive margin accounts for 21 percent of the variations. This implies that over the business cycle, Japanese firms adjust labor inputs both by the intensive and extensive margins, but they use the intensive margin more heavily than the extensive margin.[7]

# 3 The Model

## 3.1 Environment

In this section, we present our basic model for explaining cyclical behaviors of employment and hours of work. The key feature of the model is that it captures the fact that while hours of work per worker can change within a period, firms need to open (costly) vacancies in a frictional labor market to hire new employees. To focus on the composition of the labor demand at each firm, we build a labor market search-matching model with multi-worker firms.

Consider an economy consisting of a large number of workers and firms. The measure of workers is normalized to unity. Both workers and firms are homogeneous. Time is discrete and all agents discount the future at the common discount rate $r$.

The production technology for each firm is given by $A_t L_t^{\alpha} k_t^{1-\alpha}$, where $0 < \alpha < 1$, $A_t$ denotes

[6] Petrongolo and Pissarides (2008) and Fujita and Ramey (2009) apply this measure to decompose unemployment fluctuations into inflow fluctuations and outflow fluctuations.

[7] This result is in sharp contrast with what we find from the U.S. data. We utilize the dataset constructed by Ohanian and Raffo (2012) and find that $\beta^h = 0.21$ and $\beta^l = 0.79$ for the U.S. labor market. Thus, U.S. firms adjust labor inputs mainly through the extensive margin.

the level of total factor productivity (TFP), $k_t$ denotes the stock of capital at each firm, and $L_t$ denotes the labor input (i.e., total hours worked). The level of TFP is stochastic. Assuming homogeneous labor, we postulate that $L_t \equiv h_t l_t$, where $h_t$ is hours of work per employee and $l_t$ is the number of employees at each firm. Thus, output $y_t$ is given by $y_t = A_t h_t^{\alpha} l_t^{\alpha} k_t^{1-\alpha}$.

It will be shown that, with this constant-returns-to-scale technology, the (steady-state) value of operating a firm exactly cancels out the cost of firm entry, and as a result, firms are indifferent between entry and exit, as in the zero-profit result in the standard neoclassical model (Proposition 2). Thus, we adopt the conventional remedy for indeterminacy of the number of firms by normalizing the measure of firms to be unity. In effect, we can ignore entry and exit of firms over the business cycle and focus on our main theme, employment versus hours over the business cycle. Note, however, that our aggregate data contains variations in employment due to changes in the number of firms.

Each firm possesses a technology that converts one unit of the final consumption good into a unit of investment good. Let $x_t$ be the level of investment made in period $t$. Then the stock of capital evolves according to $k_{t+1} = (1-\delta_k)k_t + x_t$, where $\delta_k$ is the rate of capital depreciation.

All workers are risk neutral, and maximize the expected lifetime utility, given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \left[ I_t - e\left(h_t\right) \right],$$

where $\delta \equiv 1/(1+r)$ is the discount factor, $I_t$ denotes income, and $e(h_t)$ represents disutility of work. We assume that $e'(\cdot) > 0$, $e''(\cdot) > 0$ and $\lim_{h\to\infty} e(h) = \infty$. Our specification of disutility function is

$$e\left(h\right) = e_0 \frac{h^{1+\mu}}{1+\mu}, \tag{3}$$

where $e_0 > 0$ and $1/\mu$ is the Frisch elasticity.

The labor market is frictional. The number of matches in period $t$ is determined by the matching technology $m_0 U^{\xi} V^{1-\xi}$, where $m_0 > 0$ and $0 < \xi < 1$ are parameters, $U_t$ is the total number of job seekers, and $V_t$ is the number of aggregate job vacancies. Let $V_t/U_t \equiv \theta_t$ denote the labor market tightness. A vacancy is matched to a worker during a period with probability $q_t$, where

$$q_t = m_0 U_t^{\xi} V_t^{1-\xi} / V_t = m_0 \theta_t^{-\xi} \equiv q\left(\theta_t\right). \tag{4}$$

This is referred to as the vacancy filling rate. It is easy to verify that an increase in labor market tightness $\theta_t$ decreases this probability. Similarly, the probability that a worker is matched with a vacancy, or the job finding rate, is given by $m_0 U_t^{\xi} V_t^{1-\xi} / U_t = m_0 \theta_t^{1-\xi} = \theta_t q(\theta_t)$. This probability is increasing in $\theta_t$.

We assume exogenous separations. At the end of each period, a fraction $\lambda$ of the current employees are assumed to leave the firm. Since the firm creates $v_t$ units of vacancies, the number of new employees for the next period is $q(\theta_t)v_t$. These new employees are not hit by the separation shock. Thus, the number of employees at each firm evolves according to $l_{t+1} = (1-\lambda)l_t + q(\theta_t)v_t$.

### 3.2 Timing

Let $S_t = (A_t, l_t, k_t, U_t)$ be the set of state variables. Among the state variables, the level of $A_t$ is revealed at the beginning of each period. Given the state variables, each firm and its employees bargain over a state-contingent *contract* that specifies an *earnings schedule*, which maps $h$ into an amount of compensation $W$ (Kudoh and Sasaski, 2011). At this point, the level of earnings $W$ is not realized because hours of work are determined only after the earnings schedule is agreed upon. The bargaining outcome is summarized by $W(h_t; S_t)$. With this schedule, the firm chooses hours of work per employee ($h_t$), vacancies to create ($v_t$), and the level of capital investment ($x_t$). Then, production takes place and output $y_t$ is realized. Finally, $\lambda l_t$ of the current employees leave the firm, and $q(\theta_t)v_t$ workers are newly employed. It is important to note that a firm cannot choose hours of work before the bargaining stage because the cost structure is realized only after the earnings schedule is agreed upon.[8]

### 3.3 Firms

We solve the firm's optimization problem by stationary dynamic programming, taking this period's bargaining outcome $W(h; S)$ as given, which reflects the fact that the set of state variables

[8] In addition, a firm has *no incentive* to choose $h$ in advance because doing so cannot manipulate the bargaining outcome. In contrast, a firm does have an incentive to manipulate the bargaining outcome by choosing a large $l$, generating the well-known overhiring result (Smith, 1999).

$S$ and the state-contingent contract $W(h;S)$ are known at the time of the firm's optimization. The instantaneous payoff to a firm is given by $y - W(h;S)l - \gamma v - x$, where $\gamma > 0$ is the (constant) cost of posting a vacancy. Let $l_{+1}$ and $k_{+1}$ denote the levels of employment and capital in the next period, respectively. The value of a firm $J(S)$ satisfies the following Bellman equation:

$$J(S) = \max_{h,v,x} \left\{ Ah^{\alpha} l^{\alpha} k^{1-\alpha} - W(h;S)l - \gamma v - x + \delta \mathbb{E} J(S_{+1}) \right\}, \tag{5}$$

where

$$l_{+1} = (1-\lambda)l + q(\theta)v, \tag{6}$$

$$k_{+1} = (1-\delta_k)k + x. \tag{7}$$

We assume that, as in Cooper *et al.* (2007), the firm chooses hours of work per employee.[9] Models with alternative assumptions regarding how hours of work are determined are presented in Section 5.3.

The first-order conditions with respect to $h$, $v$, and $x$ imply

$$\alpha A h^{\alpha-1} l^{\alpha-1} k^{1-\alpha} = W_h(h;S), \tag{8}$$

$$\delta \mathbb{E} J_l(S_{+1}) = \frac{\gamma}{q(\theta)}, \tag{9}$$

$$\delta \mathbb{E} J_k(S_{+1}) = 1. \tag{10}$$

The envelope conditions yield

$$J_l(S) = \alpha A h^{\alpha} l^{\alpha-1} k^{1-\alpha} - W(h;S) - W_l(h;S)l + (1-\lambda)\delta \mathbb{E} J_l(S_{+1}), \tag{11}$$

$$J_k(S) = (1-\alpha) A h^{\alpha} l^{\alpha} k^{-\alpha} - W_k(h;S)l + (1-\delta_k)\delta \mathbb{E} J_k(S_{+1}). \tag{12}$$

[9] Although direct evidence is hard to come by, we note here that there is a large discrepancy between the estimates of the labor supply elasticity. Namely, the micro elasticity, which primariry reflects the intensive margin of labor supply, is much smaller than the macro elasticity, which reflacts both the intensive and extensive margins. We intrepret this fact as indirect evidence that while labor market paricipation (extensive margin of labor supply) is chosen by each individual, labor supply conditional on employment (i.e., hours of work per employee) is driven by the demand side.

Substitute (9) and (10) into (11) and (12) to obtain

$$J_l(S) = \alpha A h^{\alpha} l^{\alpha-1} k^{1-\alpha} - W(h;S) - W_l(h;S) l + \frac{(1-\lambda)\gamma}{q(\theta)}, \tag{13}$$

$$J_k(S) = (1-\alpha) A h^{\alpha} l^{\alpha} k^{-\alpha} - W_k(h;S) l + 1 - \delta_k, \tag{14}$$

respectively.

### 3.4 Workers

The value of being employed, $J^E(S)$, satisfies

$$J^E(S) = W(h;S) - e(h) + \lambda\delta\mathbb{E}J^U(S_{+1}) + (1-\lambda)\delta\mathbb{E}J^E(S_{+1}), \tag{15}$$

where $J^U(S)$ is the value of being unemployed. Note that hours of work are determined by the firm, and therefore the worker takes $h$ and the level of disutility $e(h)$ as given. The value of being unemployed can be written as

$$J^U(S) = b + \theta q(\theta)\delta\mathbb{E}J^E(S_{+1}) + (1-\theta q(\theta))\delta\mathbb{E}J^U(S_{+1}), \tag{16}$$

where $b$ is the unemployment benefit. Since disutility from long working hours is captured by the disutility function, our $b$ does not include the value of leisure. Thus, it primarily reflects the unemployment insurance provided by the government.

### 3.5 Earnings Schedule

We assume that at the beginning of each period, workers and a firm bargain over a state-contingent *contract*, which takes in the form of an *earnings schedule* $W(h;S)$ that maps hours of work into earnings. It is important to emphasize that the exact amount of earnings $W$ and hours of work $h$ are not determined in the bargaining stage. We assume that workers are not unionized, and each worker is treated as a *marginal worker* (Stole and Zwiebel, 1996).

Consider a bargaining between a firm and a group of workers of measure $\Delta$. The threat point for the firm is $J(A, l-\Delta, k, U)$ because failing to agree on a contract implies losing the workers. The total match surplus is therefore $J(A,l,k,U) - J(A,l-\Delta,k,U) + \Delta(J^E(S) - J^U(S))$. If the firm's bargaining power is given by $1-\beta \in [0,1]$, then the bargaining outcome must satisfy $\beta[J(A,l,k,U) - J(A,l-\Delta,k,U)] = (1-\beta)\Delta[J^E(S) - J^U(S)]$. In the limit as $\Delta \to 0$,

$$\beta J_l(S) = (1-\beta)\left[J^E(S) - J^U(S)\right]. \tag{17}$$

This is the key equation for rent sharing. Note that this amounts to maximizing the asymmetric Nash product $[J_l(S)]^{1-\beta}[J^E(S) - J^U(S)]^\beta$ with respect to $W(h;S)$.

**Proposition 1** *The earnings schedule is given by*

$$W(h;S) = \frac{\alpha\beta A h^\alpha l^{\alpha-1} k^{1-\alpha}}{\alpha\beta + 1 - \beta} + (1-\beta)\left[e(h) + b\right] + \beta\gamma\theta. \tag{18}$$

**Proof.** See Appendix A. ■

The earnings schedule (18) is a natural extension of the one derived in Kudoh and Sasaki (2011). If the worker's bargaining power $\beta$ is zero, then (18) takes a very simple form: $W(h;S) = e(h) + b$. This makes the worker indifferent between the states of employment and unemployment. Similarly, if the firm's bargaining power is zero ($\beta = 1$), then we obtain $W(h;S)l = y + \gamma\theta l$, forcing the firm to pay more than what it produces. This suggests an upper bound on $\beta$ for existence of equilibrium.

From (18), we obtain

$$W_h(h;S) = \frac{\alpha^2\beta A h^{\alpha-1} l^{\alpha-1} k^{1-\alpha}}{\alpha\beta + 1 - \beta} + (1-\beta)e'(h) > 0, \tag{19}$$

$$W_{hh}(h;S) = -(1-\alpha)\frac{\alpha^2\beta A h^{\alpha-2} l^{\alpha-1} k^{1-\alpha}}{\alpha\beta + 1 - \beta} + (1-\beta)e''(h), \tag{20}$$

$$W_l(h;S) = -(1-\alpha)\frac{\alpha\beta A h^{\alpha} l^{\alpha-2} k^{1-\alpha}}{\alpha\beta + 1 - \beta} < 0, \tag{21}$$

$$W_k(h;S) = (1-\alpha)\frac{\alpha\beta A h^{\alpha} l^{\alpha-1} k^{-\alpha}}{\alpha\beta + 1 - \beta} > 0. \tag{22}$$

The key result here is that the marginal hourly wage rate given by (19) is nonlinear in hours of work, and is influenced by the marginal product of hours per worker ($\alpha A h^{\alpha-1} l^\alpha k^{1-\alpha}/l$) and the marginal disutility from longer hours of work. The influence of the former (latter) increases (decreases) with $\beta$. Expression (20) suggests that, when the disutility function is sufficiently convex and the worker's bargaining power $\beta$ is sufficiently small, the earnings schedule itself becomes convex in hours of work ($W_{hh} > 0$).

Another key result is that the level of earnings is decreasing in the number of employees ($W_l < 0$). This property induces the firm to employ too many workers in order to cut the wage rate, known as the overhiring effect (Smith, 1999). It is interesting to observe from (18) and (19) that labor market tightness $\theta$ has no effect on the marginal hourly wage rate, while it influences the level of earnings. This will imply that the labor market conditions summarized by $\theta$ has no direct impact on the choice of hours of work.

Using (19)–(22), we rewrite (13) and (14) as

$$J_l(S) = \frac{(1-\beta)\,\alpha}{\alpha\beta+1-\beta} A h^{\alpha} l^{\alpha-1} k^{1-\alpha} - (1-\beta)\left[e(h)+b\right] - \beta\gamma\theta + \frac{(1-\lambda)\,\gamma}{q(\theta)}, \tag{23}$$

$$J_k(S) = \frac{(1-\beta)\,(1-\alpha)}{\alpha\beta+1-\beta} A h^{\alpha} l^{\alpha} k^{-\alpha} + 1 - \delta_k. \tag{24}$$

Expression (23) determines the marginal value of the firm.

# 4 Equilibrium

## 4.1 Definition

We look for a rational expectations equilibrium in which TFP follows an exogenous stochastic process. Below, we define equilibrium of the model as a system of stochastic difference equations. From (8) and (19), we obtain

$$\frac{\alpha}{\alpha\beta+1-\beta} A_t h_t^{\alpha-1} l_t^{\alpha-1} k_t^{1-\alpha} = e'(h_t), \tag{25}$$

which governs $h_t$. Substitute (23) into (9) to obtain the job creation equation:

$$\mathbb{E}_t\left\{\frac{(1-\beta)\,\alpha}{\alpha\beta+1-\beta} A_{t+1} h_{t+1}^{\alpha} l_{t+1}^{\alpha-1} k_{t+1}^{1-\alpha} - (1-\beta)\left[e(h_{t+1})+b\right] - \beta\gamma\theta_{t+1} + \frac{(1-\lambda)\,\gamma}{q(\theta_{t+1})}\right\} = \frac{(1+r)\,\gamma}{q(\theta_t)}, \tag{26}$$

which determines the demand for $l_t$. Equations (25) and (26) summarize the firm's optimal choice regarding hours of work and employment. The evolution of employment follows $l_{t+1} = (1-\lambda)l_t + q(\theta_t)v_t$, which determines the the firm's vacancy $v_t$.

Similarly, substitute (24) into (10) to obtain

$$\mathbb{E}_t \left\{ \frac{(1-\beta)(1-\alpha)}{\alpha\beta + 1 - \beta} A_{t+1} h_{t+1}^{\alpha} l_{t+1}^{\alpha} k_{t+1}^{-\alpha} + 1 - \delta_k \right\} = 1 + r. \tag{27}$$

This determines the demand for capital. The evolution of capital stock is given by $k_{t+1} = (1-\delta_k) k_t + x_t$, which determines investment $x_t$.

The aggregate variables are determined as follows. In this economy, the number of the unemployed $U_t$ equals the the rate of unemployment because the labor force is normalized to unity. In each period, $\theta_t q(\theta_t) U_t$ job seekers find jobs. Similarly, the aggregate number of employees is $1 - U_t$, from which the aggregate number of separations is $\lambda(1-U_t)$. Thus, the number of the unemployed evolves according to

$$U_{t+1} - U_t = \lambda (1 - U_t) - \theta_t q(\theta_t) U_t. \tag{28}$$

In any steady state, the flow into employment $\theta q(\theta) U$ must equal the flow into unemployment $\lambda(1-U)$, or $m(U,V) = \lambda(1-U)$, which defines the Beveridge curve. Labor market tightness is given by $\theta_t = V_t / U_t$.

Since the number of firms is normalized to unity, we obtain

$$\frac{1 - U_t}{l_t} = 1, \tag{29}$$

where the numerator is the aggregate number of employees and the denominator is the number of employees at each firm, so the ratio defines the number of firms in the economy. For the same reason, the aggregate number of vacancies equals the number of vacancies created by each firm, or $V_t = v_t$. Similarly, the aggregate output, or GDP of the economy $Y_t$, is given by $Y_t = y_t$.

### 4.2 Steady State Equilibrium

From (25)-(29), we obtain the equations that determine a non-stochastic steady state:

$$\frac{(1-\beta)(1-\alpha)}{\alpha\beta+1-\beta}AK^{-\alpha} = r+\delta_k, \tag{30}$$

$$\frac{\alpha}{\alpha\beta+1-\beta}AK^{1-\alpha} = e'(h), \tag{31}$$

$$\frac{(r+\lambda)\gamma}{q(\theta)}+\beta\gamma\theta = (1-\beta)\left[e'(h)h-e(h)-b\right], \tag{32}$$

$$l = 1-\frac{\lambda}{\lambda+\theta q(\theta)}, \tag{33}$$

where $K = k/hl$ is the capital-labor ratio.[10] For existence of a steady-state equilibrium, $\beta$ must be strictly less than one, otherwise total compensation exceeds output. Since the right-hand side of (32) must be positive, parameters must be chosen to satisfy $e'(h)h - e(h) - b > 0$ (or $\mu(1+\mu)^{-1}e_0h^{1+\mu} > b$) and $\beta < 1$.

Uniqueness of the steady state is verified as follows. First, the steady-state capital-labor ratio $K$ is determined by (30), which comes from (27). Given $K$, the steady-state hours of work $h$ is determined by (31), which is from (25). Given $h$, (32), which is from (26), determines $\theta$. Given $\theta$, the steady state level of $l$ is determined by (33), which comes from (28) and (29). Finally, given the values of $K$, $h$, and $l$, we can derive the value of $k$ by $k = Khl$.

**Proposition 2** *For an appropriately chosen cost of entry, in any steady-state equilibrium, firms are indifferent between entry and exit.*

This result verifies that our assumption of a unit measure of firms causes no loss of generality. The cost of entry is chosen to make the size distribution of firms degenerate, as in Smith (1999) and Kudoh and Sasaki (2011). The detail is found in Appendix B.

**Proposition 3** *(a) An increase in $A$ increases $K$, $h$, $\theta$, $l$, and $k$. (b) An increase in $\beta$ decreases $K$, $h$, $\theta$, $l$, and $k$. (c) An increase in $\gamma$ has no effect on $K$ and $h$, and decreases $\theta$, $l$, and $k$. (d) An increase in $\lambda$ has no*

[10] Appendix C discusses the relashionship between our model and Cho and Cooley (1994), which we think serves as the frictionless counterpart of our model.

*effect on K and h, and decreases $\theta$, $l$, and $k$. (e) An increase in $m_0$ has no effect on K and h, and increases $\theta$, $l$, and $k$.*

Proof of Proposition 3 is straightforward, and is omitted. Among the results, the most notable is that changes in $\gamma$, $\lambda$, and $m_0$, which are the key parameters determining the labor market frictions, have no effect on the steady state level of hours of work, while these parameters influence the steady-state level of employment and labor market tightness. An important implication of the result is that search frictions are *irrelevant* for understanding the long-run trend of hours of work. To make this point clearer, eliminate $h$ from (30) and (31) to obtain

$$e'(h) = \alpha\left(a\beta + 1 - \beta\right)^{\frac{1}{\alpha}}\left[\frac{r + \delta_k}{(1-\beta)(1-\alpha)}\right]^{1-\frac{1}{\alpha}} A^{\frac{1}{\alpha}},$$

from which it is clear that the steady-state hours of work are determined by TFP, since all other variables are structural parameters.

# 5 Quantitative Analysis

## 5.1 Calibration

In this section, we study a quantitative version of the basic model. Specifically, we calibrate the model to match the selected long-run Japanese labor market facts summarized in Section 2. We then solve the quantitative model by approximating the equilibrium conditions around the non-stochastic steady state, and simulate it to obtain the model's cyclical properties.

We choose the model period to be a quarter and set the discount rate to be $r = 0.01$, which implies the discount factor to be $\delta = 1/(1+r) = 0.99$. This choice of the parameter is somewhat a priori, but is consistent with other studies such as Braun *et al.* (2006). In the production function, we set $\alpha = 2/3$ to target the labor share.[11] Following Esteban-Pretel *et al.* (2010), we set the deprecation rate to be $\delta_k = 0.028$.

[11] We aware that in our frictional economy, $\alpha$ is not necessarily the labor's share of national income. Nonetheless, we assume $\alpha$ to take the same value as in the perfectly competitive economy.

The matching function is Cobb-Douglas, given by $m(U, V) = m_0 U^{\xi} V^{1-\xi}$, where $m_0$ is the matching constant and $\xi$ is the matching elasticity with respect to the number of job-seekers. Lin and Miyamoto's (2014) estimate of the elasticity $\xi$ for the Japanese labor market is 0.6. We adopt the Lin-Miyamoto estimate to set $\xi = 0.6$. This value lies in the plausible range of 0.5–0.7, which is reported by Petrongolo and Pissarides (2001). Following the convention, we use the Hosios (1990) condition to pin down the worker's bargaining power, so $\beta = \xi = 0.6$.

Using the panel property of the monthly LFS, Miyamoto (2011) and Lin and Miyamoto (2012) construct the job-finding rate and the separation rate in Japan. Miyamoto (2011) also reports the mean value of the vacancy-unemployment ratio to be 0.78. Given this, we target the vacancy-unemployment ratio to be $\theta = 0.78$. We use the monthly job-finding rate 0.142 and the vacancy-unemployment ratio to pin down the scale parameter $m_0$. In particular, $m_0$ is the solution to $m_0(0.78)^{1-0.6} = 3 \times 0.142$. We also set the exogenous separation rate $\lambda = 0.014 = 3 \times 0.0048$ from Miyamoto (2011) and Lin and Miyamoto (2012).

We choose $\mu = 1.8$ or the Frisch elasticity is $1/\mu = 0.56$, which is consistent with the micro evidence that the Frisch elasticity is less than one.[12] Our parameter value is also consistent with the evidence that the Frisch elasticity for males in Japan is in the range of 0.2–0.7 (Kuroda and Yamamoto, 2008). To generate a realistic magnitude of fluctuations in hours of work per employee, we need a small $\mu$ (or, a large Frisch elasticity). At the same time, we need to constrain our choice of $\mu$ to be within the range of the set of parameters that supports a steady state equilibrium to exists (namely, $\mu(1+\mu)^{-1} e_0 h^{1+\mu} > b$), which requires a large $\mu$. We will discuss the sensitivity of the model to the choice of $\mu$.

We target the steady-state value of hours worked to be $1/3$.[13] With $h = 1/3$, (30) and (31) jointly determine the implied value of $e_0$, which is 12.576.

In our model, the value of $b$ reflects mostly the unemployment benefit provided by the government because the value of leisure is captured by the disutility from work $e(h)$. According to

[12] In a similar environment to ours, Cooper *et al.* (2007) calibrated the value of $\mu$ to be 1.9 for the US economy.

[13] In other words, we target $h$ to be 8 hours per day or 40 hours per week (or 5 business days). However, our quantitative results are independent of the choice of the target level for $h$.

Nickell, (1997), the benefit replacement rate in Japan is about 60 percent.[14] We thus adopt this estimate to target the unemployment benefit $b$ to satisfy $b = 0.6W$. Given this, we determine $b$ and $\gamma$ by solving (18) and (32) with targets $\theta = 0.78$ and $h = 1/3$. The implied values are $b = 0.348$ and $\gamma = 0.020$.

Finally, we assume that TFP follows a first order autoregressive process. Specifically, $\log A_t$ satisfies $\log A_t - \log A = \rho(\log A_{t-1} - \log A) + \varepsilon_t$,where $0 < \rho < 0$ and $\varepsilon_t \sim N(0, \sigma^2)$. We set $\rho = 0.612$ and $\sigma = 0.0085$ to match the first-order autocorrelation and standard deviation of (the business cycle component of) total factor productivity (TFP) in the data. Thus, for all simulations, the standard deviation of the percentage deviation of TFP from its steady-state level is 0.011, as in the data.

The parameter values for the benchmark analysis are summarized in Table 2. Note that the values of parameters $m_0$, $e_0$, $b$, and $\gamma$ are endogenous in the sense that the values of these parameters are re-calibrated to match the target moments for each set of purely exogenous parameters.

### 5.2 Main Results

We now compare the selected business cycle statistics from the simulated series with those from the corresponding time series data. To this end, we shall primarily focus on the magnitude of fluctuations in each variable measured by the standard deviation. Table 3 reports the standard deviations of the unemployment rate, the vacancy rate, earnings, the (average) hourly wage rate, the aggregate labor input, hours of work per employee, and employment, scaled by the standard deviation of TFP. We also report the relative contributions of the intensive and extensive margins calculated by formula (2).

For all variables listed in Table 3, the standard deviations obtained from our basic model are close to those obtained from the data. The relative standard deviations generated by the model of the aggregate labor input, hours per employee, and employment are 0.75, 0.63, and 0.15, respectively. The corresponding relative standard deviations from the data are 0.80, 0.70, and 0.35.

[14]See also Martin (2000).

Table 2: Parameter values

| Parameter | Description | Value | Source/Target |
|---|---|---|---|
| $r$ | Interest rate | 0.01 | Esteban-Pretel *et al.* (2010) |
| $\delta$ | Discount rate $1/(1+r)$ | 0.99 | - |
| $\alpha$ | Parameter in production function | 2/3 | Labor share |
| $m_0$ | Matching efficiency | 0.471 | Monthly job-finding rate |
| $\xi$ | Matching elasticity | 0.6 | Lin and Miyamoto (2014) |
| $\lambda$ | Exogenous separation rate | 0.014 | Monthly separation rate |
| $e_0$ | Parameter in disutility function | 12.576 | $h = 1/3$ |
| $\delta_k$ | Depreciation rate | 0.028 | Esteban-Pretel *et al.* (2010) |
| $\mu$ | Inverse of Frisch elasticity | 1.8 | Kuroda and Yamamoto (2008) |
| $b$ | Unemployment benefits | 0.348 | Replacement rate = 60 percent |
| $\beta$ | Worker's bargaining power | 0.6 | $\beta = \xi$ (Hosios condition) |
| $\gamma$ | Vacancy cost | 0.020 | $v - u$ ratio = 0.78 |
| $A$ | Productivity | 1.0 | Normalization |
| $\rho$ | AR-coefficient of shock | 0.612 | Data |
| $\sigma$ | Standard deviation of the shock | 0.0085 | Data |

While the magnitude of fluctuations in employment for the model is less than a half of that for the data, the model replicates much of fluctuations in the labor demand and its compositions. In particular, the model captures the observation that much of fluctuations in the aggregate labor input is accounted for by fluctuations in hours of work per employee. Indeed, by applying (2) to decompose the variations in the aggregate labor input, we find that $\beta^h = 0.84$ and $\beta^l = 0.19$.

The inability of our model in generating enough labor market fluctuations along the extensive margin can be partly explained by the absence of *the other extensive margin*, namely, changes in employment that are driven by entry and exit of firms over the business cycle. While this margin is beyond the scope of this paper, we speculate that a model with this third margin can generate a higher volatility in employment. With this limitation in mind, we conclude that our model

Table 3: Hours of Work and Labor Market Fluctuations

| | Relative standard deviations | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\hat{U}$ | $\hat{V}$ | $\hat{W}$ | $\hat{w}$ | $\hat{t}$ | $\hat{h}$ | $\hat{l}$ | $\beta^h$ | $\beta^l$ |
| **Data** | **5.45** | **8.92** | **1.16** | **0.91** | **0.80** | **0.70** | **0.35** | **0.79** | **0.21** |
| Basic model | 4.33 | 12.81 | 1.55 | 0.91 | 0.75 | 0.63 | 0.15 | 0.84 | 0.19 |
| Model w/o hours | 0.96 | 2.85 | 1.22 | 1.22 | 0.03 | - | 0.03 | - | - |
| Model with $h = 1/3$ | 4.33 | 12.81 | 0.96 | 0.96 | 0.15 | - | 0.15 | - | - |
| Bargained hours | 1.36 | 4.02 | 1.51 | 0.88 | 0.67 | 0.63 | 0.05 | 0.95 | 0.07 |

captures the labor market dynamics in Japan fairly well.[15]

It is often reported that the textbook search-matching model of the labor market fails to account for the observed high volatility of the unemployment and vacancy rates, often referred to as the unemployment volatility puzzle (Shimer, 2005).[16] Interestingly, Table 3 shows that our model does replicate fluctuations in the unemployment rate and the vacancy rate with realistic or even greater magnitudes. The relative standard deviation of the unemployment rate from the model is 4.33 while that from the data is 5.45, and the relative standard deviation of the vacancy rate from the model is 12.81 while that from the data is 8.92. The relative standard deviation of the labor market tightness, the ratio of the vacancy rate and the unemployment rate, for the model is 14.66 while that for the data is 12.92. We emphasize that these results are obtained without wage rigidity or additional shocks.

To clarify the importance of hours of work, we consider the polar case in which the intensive margin is shut down. To be more concrete, we study a model in which $h$ is removed. Thus, the level of labor input is now $L_t = l_t$ so that the production function is replaced with $A_t l_t^{\alpha} k_t^{1-\alpha}$. In

[15] We also note that there is a sizable proportion of employees under short-term contracts in Japan, referred to as non-regular employment (Miyamoto, 2016). Labor adjustment along the extensive margin for non-regular employment is considered to be less frictional. This partly explains why fluctuations along the extensive margin in the data are greater than those implied by the model.

[16] Esteban-Pretel *et al.* (2011) and Miyamoto (2011) show that the unemployment volatility puzzle holds for the Japanese economy.

addition, we remove disutility term $e(h_t)$ from the model. This corresponds to the model studied by Krause and Lubik (2013), in which the cyclical properties of a model with multi-worker firms are compared to those of a textbook search-matching model to clarify whether the multi-worker paradigm helps resolve the unemployment volatility puzzle and conclude that while the multi-worker paradigm helps increase the magnitudes of fluctuations in unemployment and vacancies, there is no sizable quantitative improvement.

The results are reported in Table 3. Without hours of work, the relative standard deviation of the aggregate labor input is 0.03. The corresponding value from the basic model is 0.75, which is about *25 times* as large as that from the model without hours of work. Similarly, in the model without hours of work, the relative standard deviations of unemployment and vacancies are 0.96 and 2.85, respectively. The corresponding values from the basic model are respectively 4.33 and 12.81, which are about 4 times as large as those from the model without hours of work. Thus, the extensive margin alone cannot account for fluctuations in the aggregate labor input. In addition, introduction of the intensive margin magnifies labor adjustment along the *extensive margin*.

To understand the impact of the intensive margin on the magnitude of fluctuations along the extensive margin, we report an additional result in Table 3. Instead of removing hours of work from the model, we study the model in which hours of work are fixed at our calibration target $h = 1/3$. Thus, in the steady state, this model is identical to the basic model, implying that workers incur disutility from long hours of work as in the basic model. However, the intensive margin is shut down in the sense that firms cannot change hours per employee in response to shocks. Interestingly, in this model, unemployment and vacancies fluctuate as much as those of the basic model.

From the comparison of the two models, we conclude that introduction of *disutility* from long hours of work per se helps generate high magnitudes of fluctuations in unemployment and vacancies, irrespective of whether the intensive margin is active or not. This is somewhat consistent with studies that emphasize the importance of including the value of leisure in calibrating the flow value of unemployment (Hagedorn and Manovskii, 2008; Hall and Milgrom, 2008). As is evident from (18), the level of disutility $e(h)$ works in the same direction as $b$ does: it increases the equilibrium level of total compensation, increasing the firm's exposure to external shocks.

### 5.3 Bargained Hours of Work

In the basic model, we have assumed that the firm and its employees bargain over an earnings schedule and that the firm chooses hours of work per employee from the state-contingent contract $W(h; S)$. A possible criticism on this setup is that, from a theoretical perspective, it is more natural to assume that both hours of work and earnings are determined in bargaining, as in Fang and Rogerson (2009). Although we have chosen our basic model to understand the labor market fluctuations through the lens of labor demand, it is nonetheless useful to present an alternative model in which hours of work are bargained.

We consider an alternative model in which hours of work per employee are determined in bargaining. Specifically, we assume that the *level* of compensation $W$ (not the earnings schedule) and hours per employee $h$ are determined so as to maximize the asymmetric Nash product: $[J_l(S)]^{1-\beta}[J^E((S) - J^U(S)]^{\beta}$. The first-order conditions with respect to $W$ and $h$ imply $\beta J_l(S) = (1-\beta)[J^E(S) - J^U(S)]$ and $\partial[J_l(S)]/\partial h + \partial[J^E(S)]/\partial h = 0$, where

$$\begin{aligned} \frac{\partial\left[J_l\left(S\right)\right]}{\partial h} &= \frac{\left(1-\beta\right)\alpha^2}{\alpha\beta+1-\beta} A h^{\alpha-1} l^{\alpha-1} k^{1-\alpha} - \left(1-\beta\right) e'\left(h\right), \\ \frac{\partial\left[J^E(S)\right]}{\partial h} &= \frac{\alpha^2 A h^{\alpha-1} l^{\alpha-1} k^{1-\alpha}}{\alpha + \frac{1-\beta}{\beta}} - \beta e'\left(h\right). \end{aligned}$$

Thus, (25) is replaced with

$$\frac{\alpha^2}{\alpha\beta+1-\beta} A_t h_t^{\alpha-1} l_t^{\alpha-1} k_t^{1-\alpha} = e'\left(h_t\right). \tag{34}$$

The difference comes from the evaluation of the marginal benefit of an additional hour of work: the marginal benefit for the worker is less than that for the firm by factor of $\alpha < 1$. Equation (34) is nearly identical to (25), making the qualitative properties of the model nearly identical to the basic model.

By comparing (34) with (25), one might argue that our basic model generates a small employment volatility because it overestimates the marginal benefit of an additional hour of work (because hours of work are chosen only to maximize the firm's value), making the extensive margin relatively less attractive.

Table 3 shows that the standard deviations of unemployment, vacancies, and employment obtained from the model with bargained hours of work are about $1/3$ of those from the basic model. The marginal product of an additional hour is reduced by factor of $\alpha$, which reduces the firm's incentive to utilize the intensive margin. However, for the model to match the target steady-state hours of work per employee, $h = 1/3$, the endogenous parameters need to be re-calibrated. The implied values are now $e_0 = 8.384$ and $\gamma = 0.075$. The smaller $e_0$ offsets the impact of the reduction in the marginal product, while the increase in $\gamma$ increases the marginal cost of posting a vacancy. As a result, the magnitudes of fluctuations along the extensive margin, namely, fluctuations in employment, vacancies, and unemployment, are all reduced, keeping the standard deviation of hours of work per employee virtually unchanged.

**Hours of Work Chosen by Workers** For completeness, we consider yet another scenario in which hours of work per employee are chosen by workers. As in the basic model, each firm and its employees bargain over an earnings schedule at the beginning of each period. Given the earnings schedule $W(h;S)$, each employee chooses hours of work in each period. For this model, we replace (15) with

$$J^E(S) = \max_h \left\{ W(h;S) - e(h) + \lambda\delta\mathbb{E}J^U(S_{+1}) + (1-\lambda)\delta\mathbb{E}J^E(S_{+1}) \right\}.$$

The first-order condition requires $W_h(h;S) = e'(h)$, from which we obtain the expression identical to (34). Thus, the model in which hours of work are chosen by workers and the model in which hours of work are bargained are observationally equivalent, as in a related steady-state economy studied by Kudoh and Sasaki (2011).

## 6 Robustness

### 6.1 Parameter Values

To clarify the extent to which the model's ability in generating the magnitude of fluctuations needed to match the data depends on the choice of parameters, we provide some sensitivity analyses in terms of the labor supply elasticity and the unemployment benefit. We take these two parameters for our sensitivity analyses because these parameters are known to have a range of estimates and calibrated values.

Table 4: Alternative Parameter Values

| | Relative standard deviations | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\hat{U}$ | $\hat{V}$ | $\hat{W}$ | $\hat{w}$ | $\hat{t}$ | $\hat{h}$ | $\hat{l}$ | $\beta^h$ | $\beta^l$ |
| **Data** | **5.45** | **8.92** | **1.16** | **0.91** | **0.80** | **0.70** | **0.35** | **0.79** | **0.21** |
| Basic model | 4.33 | 12.81 | 1.55 | 0.91 | 0.75 | 0.63 | 0.15 | 0.84 | 0.19 |
| $\mu = 1.9$ | 3.36 | 9.95 | 1.52 | 0.92 | 0.69 | 0.60 | 0.11 | 0.87 | 0.16 |
| $\mu = 1.7$ | 6.26 | 18.53 | 1.58 | 0.91 | 0.83 | 0.67 | 0.21 | 0.79 | 0.25 |
| $b/\bar{W} = 0.4$ | 0.77 | 2.28 | 1.56 | 0.92 | 0.65 | 0.63 | 0.03 | 0.97 | 0.04 |
| $b/\bar{W} = 0.8$, $\mu = 5.0$ | 5.53 | 16.35 | 1.18 | 0.94 | 0.39 | 0.24 | 0.19 | 0.57 | 0.48 |
| $\beta = 0.5$ | 3.89 | 11.50 | 1.46 | 0.83 | 0.73 | 0.63 | 0.13 | 0.86 | 0.18 |

For many countries, the Frisch labor supply elasticity, $1/\mu$, has a range of estimates. The empirical literature suggests that the elasticity from the micro data is much smaller than that from the macro data because the macro elasticity includes variations in labor market participation (or the extensive margin of labor supply). Further, the micro evidence suggests that the Frisch elasticity is less than one. Our benchmark model employs $\mu = 1.8$, which is close to the value used in Cooper *et al.* (2007), $\mu = 1.9$. Table 4 presents the results under two values of $\mu$, 1.9 and 1.7.

For $\mu = 1.9$ (or, $1/\mu = 0.53$) the values of $e_0$ is re-calibrated to match the target $h = 1/3$, to obtain $e_0 = 14.036$. Similarly, the values of $\gamma$ is re-calibrated to be 0.026. These values are greater than those under $\mu = 1.8$. While a higher $\mu$ implies a greater marginal hourly wage rate, a higher $\gamma$ implies a greater marginal cost of posting a vacancy. Thus, fluctuations in the aggregate labor input, hours per employee, and employment are all less than those from the basic model because the marginal costs for the both margins increased. Overall, the effect on the extensive margin dominates the other, and the relative importance of the intensive margin increases to 0.87. For $\mu = 1.7$ (or, $1/\mu = 0.59$), the mechanism is reversed, and the magnitudes of fluctuations in all measures are greater than those for the benchmark case.

In our benchmark calibration, we choose the value of $b$ so that $b = 0.6W$, where 60 percent is the actual replacement rate in Japan reported in Nickell (1997). The choice of the parameter value for $b$ has been the subject of discussion in the literature. For the US labor market, Shimer (2005) sets $b$ so that the replacement rate is 40 percent to target the actual replacement rate in the US, while Hagedorn and Manovskii (2008) argue that Shimer's $b$ is too low and that with a much higher $b$, search-matching models can replicate unemployment and vacancy fluctuations with realistic magnitudes. With this debate in mind, we consider alternative levels of $b$ that correspond to the replacement rates of 40 percent and 80 percent.

Under the replacement rate of 40 percent, the corresponding value of $b$ is $b = 0.230$, and the implied value for the vacancy cost is $\gamma = 0.112$. Since the vacancy cost is 5.6 times as large as that for the basic model, the incentive to utilize the extensive margin reduces significantly. As a result, the magnitude of fluctuations in employment decreases significantly, keeping the magnitude of fluctuations in hours per employee unchanged, as shown in Table 4. Consequently, the relative importance of the intensive margin increased from 0.84 to 0.97. Further, Table 4 suggests that a smaller $b$ cause unemployment and vacancies to be much less volatile. These results are in line with Hagedorn and Manovskii (2008).[17]

Hagedorn and Manovskii (2008) argued that, given that individuals typically have access to alternative sources of income such as home production, the flow value of unemployment should be close to the flow value of employment. With this in mind, we consider yet another set of pa-

[17] The relationship between the magnitude of fluctuations and the match surplus is clarified by Hornstein *et al.* (2005) and Elsby and Michaels (2013).

rameters which include a very high replacement rate within a set of parameters that support a steady state (i.e., $\mu(1+\mu)^{-1}e_0 h^{1+\mu} > b$, which implies that we need a higher $\mu$ to have a large replacement rate). Thus, we consider the replacement rate being 80 percent with $\mu = 5$ (or the Frisch elasticity to be 0.2). The implied parameter values are $e_0 = 422.978$ and $\gamma = 0.016$. Table 4 reports the results. With a smaller marginal cost of posting a vacancy, the magnitude of fluctuations in employment increases and gets closer to the data. However, the increased marginal hourly wage rate reduces the incentive to utilize the intensive margin significantly and reduced the magnitude of fluctuations in hours per employee. Thus, we conclude that while a model with a high replacement rate helps generate high volatilities in unemployment and vacancies, it cannot generate fluctuations in the aggregate labor input and hours per employee.

In the basic model, we follow the convention to choose $\beta = \xi$ to meet the Hosios condition. Another useful benchmark is to assume the symmetric Nash product, which implies $\beta = 0.5$ in our framework. With this value, the implied parameter values are $e_0 = 13.225$ and $\gamma = 0.031$. This case is somewhat similar to the case under a lower replacement rate. The implied values of $e_0$ and $\gamma$ are both greater than those under the basic model, making both the intensive and extensive margins more costly. However, the value of $e_0$ is only slightly greater than that for the basic model while the value of $\gamma$ is about 1.6 times as large as that for the basic model. This makes the extensive margin more costly for firms.

### 6.2 Extensions

In this section, we present the results from some extensions.

**Decreasing Efficiency** In the benchmark model, we have assumed that while longer hours of work cause greater disutility, there is no loss in productivity. Using the UK micro data, Pencavel (2014) found that the relationship between output and hours of work per worker is concave. This raises a question that our basic model might have overestimated the marginal benefit of an additional hour of work, making the extensive margin relatively less attractive. To capture the possibility that the productivity of hours per employee is concave, we modify our basic model

Table 5: Alternative Assumptions

| | Relative standard deviations | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\hat{U}$ | $\hat{V}$ | $\hat{W}$ | $\hat{w}$ | $\hat{t}$ | $\hat{h}$ | $\hat{l}$ | $\beta^h$ | $\beta^l$ |
| **Data** | **5.45** | **8.92** | **1.16** | **0.91** | **0.80** | **0.70** | **0.35** | **0.79** | **0.21** |
| Basic model | 4.33 | 12.81 | 1.55 | 0.91 | 0.75 | 0.63 | 0.15 | 0.84 | 0.19 |
| Decreasing efficiency | 1.62 | 4.80 | 1.39 | 0.81 | 0.62 | 0.58 | 0.05 | 0.93 | 0.09 |
| Capital adjustment | 3.01 | 8.22 | 1.18 | 0.69 | 0.56 | 0.49 | 0.10 | 0.87 | 0.17 |
| Stochastic separations | 7.39 | 13.04 | 1.55 | 0.91 | 0.81 | 0.63 | 0.25 | 0.76 | 0.26 |
| Contract rigidity | 5.87 | 17.37 | 1.48 | 0.56 | 1.07 | 0.92 | 0.20 | 0.86 | 0.18 |

such that the effective labor input satisfies $L = h^{\eta} l$, where $\eta \leq 1$ captures the efficiency of hours per employee. The model with $\eta = 1$ corresponds to the benchmark economy. The value of a firm is replaced with

$$J(S) = \max_{h,v,x} \left\{ A h^{\alpha\eta} l^{\alpha} k^{1-\alpha} - W(h;S) l - \gamma v - x + \delta \mathbb{E} J(S_{+1}) \right\},$$

and the optimization conditions need to be modified accordingly. The earnings function for this economy is

$$W(h;S) = \frac{\alpha\beta A h^{\alpha\eta} l^{\alpha-1} k^{1-\alpha}}{\alpha\beta + 1 - \beta} + (1-\beta)\left[e(h) + b\right] + \beta\gamma\theta.$$

For our numerical exercise, we adopt Pencavel's (2014) estimate, $\eta = 0.8$. As shown in Table 5, the cyclical properties of this model are similar to those of the model with bargained hours of work. The standard deviations of unemployment, vacancies, and employment obtained from this model are about 1/3 of those from the basic model. The marginal product of an additional hour is reduced by factor of $\eta$, which reduces the firm's incentive to utilize the intensive margin. This results in a smaller standard deviation of hours per employee (relative to the standard deviation of TFP): 0.58 from this model and 0.63 from the basic model. To replicate the same target moments, the endogenous parameters need to be re-calibrated, and the implied value for the vacancy cost is now $\gamma = 0.066$, which is three times as large as that for the basic model. The increased vacancy cost significantly reduces the volatility in employment, vacancies, and unemployment.

**Costly Capital Adjustment** Our benchmark model includes capital as a factor of production. With capital, production technology exhibits constant returns to scale, which safely rules out variations in vacancies along the margin of entry and exit. A possible side effect is that, since adjustment in capital is perfectly flexible, the very presence of a more flexible margin of adjustment might make the cost of labor adjustment relatively more costly, generating too little fluctuations in the aggregate labor input. It is therefore useful to investigate how introduction of costly capital adjustment influence the volatilities of labor input and its components.[18] In modeling the cost of capital adjustment, we follow the convention as much as possible. Namely, with costly capital adjustment, the value of the firm is given by

$$J(S) = \max_{h,v,x} \left\{ Ah^{\alpha} l^{\alpha} k^{1-\alpha} - W(h;S)l - \gamma v - x - C(k, k_{+1}) + \delta \mathbb{E} J(S_{+1}) \right\},$$

where the cost of capital adjustment takes the following standard form:

$$C(k, k_{+1}) = \frac{1}{2}\left(\frac{x}{k}\right)^2 k = \frac{1}{2}\left(\frac{k_{+1} - k}{k} + \delta_k\right)^2 k.$$

In this economy, adjustments of capital and employment are both costly. Table 5 shows that, with costly capital adjustment, the magnitudes of fluctuations in all variables are smaller than those from the basic model as well as those from the data. This result is due to the property that the cost of capital adjustment reduces the steady-state level of capital. With a smaller stock of capital, the marginal product of hours of work per employee is reduced, and this dominates the reduction in the marginal hourly wage rate. As a result, the firm's incentive to utilize the intensive margin declines. Similarly, a reduction in capital reduces the marginal product of employment, which reduces the firm's incentive to utilize the extensive margin. Overall, introduction of capital adjustment cost reduces the magnitudes of fluctuations in both the intensive and extensive margins. In other words, the presence of perfectly flexible capital does not make labor adjustment more costly.

[18]It is important to note that, in the basic model, there is little room for improving the magnitude of fluctuations in capital. In the data, the standard deviation of capital is 0.012 while that of the basic model is 0.017.

**Stochastic Separations** Recent empirical studies demonstrate that both unemployment inflow and outflow significantly contribute to the unemployment dynamics in Japan (Miyamoto, 2011; Lin and Miyamoto, 2012). In particular, in the data, TFP and the separation rate are negatively correlated. This tends to amplify fluctuations in the unemployment rate. Shimer (2005) finds that while introduction of separations shocks helps increase the magnitudes of fluctuations in unemployment and vacancies, it destroys the negative correlation between the unemployment rate and the vacancy rate, or the Beveridge curve. To assess the importance of the unemployment inflow channel in generating fluctuations in employment and unemployment within our framework and to see if the Beveridge curve is preserved, we study a model that incorporates the separation rate that follows a more realistic stochastic process. Specifically, we assume that the separation rate follows a first-order autoregressive process of $\log \lambda_t - \log \lambda = \rho_\lambda (\log \lambda_{t-1} - \log \lambda) + \varepsilon_{\lambda,t}$, where $0 < \rho_\lambda < 1$ and $\varepsilon_{\lambda,t} \sim N(0, \sigma_\lambda^2)$. From the data, we set $\rho_\lambda = 0.1575$ and $\sigma_\lambda = 0.0907$. Table 5 shows that introduction of stochastic separations significantly increases the standard deviations of unemployment and vacancies. However, the magnitudes of fluctuations are too large to match the observed standard deviations of unemployment and vacancies. Another finding is that the model with separation shocks accounts for much of the observed fluctuations in employment. The standard deviation of employment (relative to that of TFP) is 0.25 while that from the data is 0.35. However, as in the literature, stochastic separations weaken the negative correlation between unemployment and vacancies. An important implication of the results is that, while our model precludes the firing margin of labor adjustment, introduction of the firing margin per se may not improve the performance of the model.

**Contract Rigidity** Shimer (2005) suggests that introduction of some wage rigidity helps increase the magnitudes of fluctuations in unemployment and vacancies. Although our basic model perfectly replicates the observed volatility in hourly wage rate, earnings in the model fluctuate more than those in the data. Thus, there is a (small) room for improvement. A caveat here is that, with variable hours of work and a nonlinear earnings schedule, the concept of wage rigidity is not well-defined, as it assumes to have a model in which the labor contract takes in the form of hourly wage rate. Thus, we instead study the model with *contract rigidity*. Our modeling strategy is to modify the model as little as possible, rather than to write down a full-fledged micro-founded model of rigidity. To be more specific, we introduce an ad-hoc earnings schedule with rigidity that possesses the following two properties. One is that the rigid earnings schedule is identical to the benchmark earnings schedule (18) in the steady state. In other words, while ad-hoc, the rigid earnings schedule does not alter the steady state of the basic model. The other is that the current contract does not fully reflect the current TFP (Pissarides, 2009). Let $S_t^P = (A_t^P, l_t, k_t, U_t)$ be the *perceived* state of the economy in period $t$, where $A_t^P$ is the perceived level of TFP which is assumed to satisfy

$$A_t^P = \phi A_t + (1 - \phi) A. \tag{35}$$

Thus, the perceived TFP is given by the weighted average of the true current TFP and its steady-state level.[19] We assume that in the bargaining stage, the true state is only partially verifiable, and as a result, the contract is conditional only on the perceived state. The resulting earnings schedule in period $t$ is given by

$$W(h_t; S_t^P) = \frac{\alpha\beta A_t^P h_t^\alpha l_t^{\alpha-1} k_t^{1-\alpha}}{\alpha\beta + 1 - \beta} + (1 - \beta) [e(h_t) + b] + \beta\gamma\theta_t. \tag{36}$$

It is important to note that, while the contract is rigid, it does *not* guarantee that the wage rate is also rigid because changes in hours of work do change the level of earnings. For our numerical analysis, we choose $\phi$ to be 0.359 from a structural estimation by Lin and Miyamoto (2014). Table 5 shows that the model with contract rigidity increases the magnitudes of fluctuations in unemployment and vacancies. However, the magnitudes are too large. While the standard deviation of the unemployment rate is about the same as that of the data, the standard deviation of the vacancy rate is about twice as large as that of the data. Introduction of contract rigidity significantly decreases the standard deviation of the average hourly wage rate (relative to that of TFP) from 0.91 to 0.56. On the other hand, the standard deviation of earnings (relative to that of TFP) decreased

[19] Our formulation of wage rigidity is inspired by Hall (2005) and in particular Krause and Lubik (2007). These authors assume an ad-hoc wage equation in which the actual current wage rate is given by the weighted average of the Nash bargained wage rate and a reference wage rate, such as the past wage rate and the steady-state level.

only slightly, from 1.55 to 1.48. This is because the level of earnings fluctuates as hours of work per employee changes, and the magnitude of fluctuations in hours of work per employee is about 1.5 times as large as that from the basic model and about 1.3 times as large as that from the data. Since contract rigidity increases the magnitude of fluctuations in hours of work per employee too much, the relative importance of the extensive margin is reduced in the model with rigidity even if the standard deviation of employment is higher than that of the basic model.

# 7 Conclusion

In this paper, we showed empirically that while both employment and hours per employee are important for explaining cyclical fluctuations in the Japanese labor market, fluctuations in hours of work explain 79 percent of variations in the aggregate labor input. We then introduced variable hours of work into a search-matching model with multi-worker firms to investigate how firms utilize hours of work and vacancy creation over the business cycle, and calibrated the model's parameters to mimic the Japanese labor market facts. We showed that our model accounts for much of fluctuations in the aggregate labor input, hours of work, employment, unemployment, and vacancies, albeit it is less successful in generating the magnitude of fluctuations in employment. Overall, we conclude that introduction of hours of work is an important direction in the study of labor market dynamics over the business cycle.

The limitation of our model is that we did not consider *the other extensive margin*, namely, changes in employment that are driven by entry and exit of firms over the business cycle. For this reason, our framework is best used for economies in which fluctuations in employment associated with firm entry and exit play a relatively minor role. A possible future research is to introduce this margin into our framework to study how these three margins fluctuate over the business cycle.

An important line of future research is to identify the institutional characteristics across countries and provide a unified framework that captures the observed cross-country differences in the composition of labor demand over the business cycle. For this investigation, one needs a model with endogenous firing. With a micro-founded model of firing, one can study the impact of employment protection such as firing restriction on the importance of the intensive and extensive margins over the business cycle.[20] An important recent contribution along this line is Llosa *et al.* (2014), in which a frictionless model with firing costs is developed. Our framework, when modified accordingly, will provide a frictional counterpart of their model. A caveat is that, as we demonstrated, the model with separation fluctuations does not perform well; in particular, it does not produce a strong negative correlation between unemployment and vacancies.

Another important line of future research is to consider differences in types of employment. While we assumed a single employment contract for all workers, the labor market in Japan is best understood as being polarized into two groups of workers, those with well-protected long-term contracts and those with less-paid, less-protected "non-regular" employment contracts, under which firms may terminate contracts at will and hours of work are more flexible. Workers under such non-regular employment contracts have been increasing, and they amount to 40 percent of total employees in Japan (Miyamoto, 2016). Our framework has an advantage in investigating this important issue because it is designed to study the composition of labor demand.

[20] It is certainly easy to introduce a separation cost into our model. However, we believe that such a model cannot approximate the reality of an economy with firing costs. A useful model of firing costs must possess both involuntary separations (i.e., firing) and involuntary labor hoarding (i.e. restricted firing).

# Appendix

## A Proof of Proposition 1

Subtract (16) from (15) to obtain

$$J^E(S) - J^U(S) = W(S,h) - e(h) - b + [1-\lambda-\theta q(\theta)]\left[\delta \mathbb{E} J^E(S_{+1}) - \delta \mathbb{E} J^U(S_{+1})\right]. \tag{37}$$

Observe that, (9) and (17) imply

$$\frac{1-\beta}{\beta}\left[\delta \mathbb{E} J^E(S_{+1}) - \delta \mathbb{E} J^U(S_{+1})\right] = \frac{\gamma}{q(\theta)}.$$

Use this to rewrite (37) as follows:

$$J^E(S) - J^U(S) = W(S,h) - e(h) - b + \frac{1-\lambda-\theta q(\theta)}{1-\beta}\frac{\beta\gamma}{q(\theta)}. \tag{38}$$

Substitute (13) and (38) into (17) to obtain

$$\begin{aligned} &\beta\left[\alpha A h^{\alpha} l^{\alpha-1} k^{1-\alpha} - W(S,h) - W_l(S,h) l + \frac{(1-\lambda)\gamma}{q(\theta)}\right] \\ &= (1-\beta)\left[W(S,h) - e(h) - b\right] + [1-\lambda-\theta q(\theta)]\frac{\beta\gamma}{q(\theta)}, \end{aligned}$$

which reduces to $W(S,h) = \beta[\alpha A h^{\alpha} l^{\alpha-1} k^{1-\alpha} - W_l(S,h) l] + (1-\beta)[e(h)+b] + \beta\gamma\theta$, or

$$W_l(S,h) l + \frac{1}{\beta} W(S,h) = \alpha A h^{\alpha} l^{\alpha-1} k^{1-\alpha} + \frac{1-\beta}{\beta}[e(h)+b] + \gamma\theta. \tag{39}$$

This is a differential equation about the unknown earnings function. This equation satisfies for all $l \geq 0$, along with the condition that

$$W(S,h) l \leq A h^{\alpha} l^{\alpha} k^{1-\alpha}, \tag{40}$$

which requires that the total wage payment does not exceed the firm's revenue. It is useful to observe that

$$\begin{aligned} \frac{\partial}{\partial l}\left[W(S,h) l^{\frac{1}{\beta}}\right] &= \left[W_l(S,h) l + \frac{1}{\beta} W(S,h)\right] l^{\frac{1}{\beta}-1} \\ &= \left[\alpha A h^{\alpha} l^{\alpha-1} k^{1-\alpha} + \frac{1-\beta}{\beta}[e(h)+b] + \gamma\theta\right] l^{\frac{1}{\beta}-1} \\ &= \alpha A h^{\alpha} l^{\alpha+\frac{1}{\beta}-2} k^{1-\alpha} + \left[\frac{1-\beta}{\beta}[e(h)+b] + \gamma\theta\right] l^{\frac{1}{\beta}-1}. \end{aligned}$$

Since (40) implies $W(S,h)l^{\frac{1}{\beta}} \leq Ah^{\alpha}l^{\alpha+\frac{1}{\beta}-1}k^{1-\alpha}$, we have $\lim_{l\to 0} W(S,h)l^{\frac{1}{\beta}} = 0$. Thus, it follows that

$$\begin{aligned}
W(S,h)l^{\frac{1}{\beta}} &= \int_0^l \left\{ \alpha Ah^{\alpha} i^{\alpha+\frac{1}{\beta}-2}k^{1-\alpha} + \left[\frac{1-\beta}{\beta}\left[e\left(h\right)+b\right]+\gamma\theta\right] i^{\frac{1}{\beta}-1}\right\} di \\
&= \frac{\alpha Ah^{\alpha}k^{1-\alpha}}{\alpha+\frac{1}{\beta}-1} l^{\alpha+\frac{1}{\beta}-1} + \left[\left(1-\beta\right)\left[e\left(h\right)+b\right]+\beta\gamma\theta\right] l^{\frac{1}{\beta}}.
\end{aligned}$$

Thus, we finally obtain

$$W(S,h) = \frac{\alpha Ah^{\alpha}k^{1-\alpha}}{\alpha+\frac{1}{\beta}-1} l^{\alpha-1} + \left(1-\beta\right)\left[e\left(h\right)+b\right]+\beta\gamma\theta$$

as shown in the proposition.

# B Entry and Exit

We follow Smith (1999) and Kudoh and Sasaki (2011) to assume that each entrant must create vacancies so that it operates with the steady-state level of employment $l$ in the next period. This assumption rules out the size distribution of firms. Because the rate of filling a vacancy is $q(\theta)$, in order to achieve $l_{+1}$ in the next period, the firm must create exactly $l_{+1}/q(\theta)$ vacancies in the current period. Thus, the value of entry is given by

$$J\left(0\right) = -\frac{\gamma l_{+1}}{q(\theta)} - k_{+1} + \delta \mathbb{E} J\left(S_{+1}\right), \tag{41}$$

Therefore, the number of firms, $N_t$, is determined by $J(0) = 0$, or

$$\frac{\gamma l_{+1}}{q(\theta)} + k_{+1} = \delta \mathbb{E} J\left(S_{+1}\right). \tag{42}$$

In any steady state, the firm's value of operation (without imposing (42)) is

$$\begin{aligned}
\left(1-\delta\right) J\left(S\right) &= Ah^{\alpha}l^{\alpha}k^{1-\alpha} - W(S,h)l - \gamma v - x \\
&= \left[\frac{1-\beta}{\alpha\beta+1-\beta}\right] Ah^{\alpha}l^{\alpha}k^{1-\alpha} - \left(1-\beta\right)\left[e\left(h\right)+b\right] l - \beta\gamma\theta l - \gamma\frac{\lambda l}{q(\theta)} - \delta_k k \\
&= \frac{\left(1-\beta\right)\left(1-\alpha\right)}{\alpha\beta+1-\beta} Ah^{\alpha}l^{\alpha}k^{1-\alpha} + \frac{r\gamma}{q(\theta)} l - \delta_k k \\
&= r\frac{\gamma l}{q(\theta)} + rk,
\end{aligned}$$

from which we obtain

$$\delta J(S) = \frac{\gamma l}{q(\theta)} + k.$$

Thus, in any steady state, the value of entry is

$$J(0) = -\frac{\gamma l}{q(\theta)} - k + \delta J(S) = 0.$$

Thus, firms are indifferent between entry and exit. As a result, the number of firms will be *indeterminate* in this economy in the sense that a free entry condition cannot pin down the number of firms. The same result holds in the standard neoclassical economy in which firms profits are zero. The result did not arise in Smith (1999) or Kudoh and Sasaki (2011) because in these models, the production technology exhibits decreasing returns to scale.

# C Relation to the Frictionless Model

To understand the role of search frictions and wage bargaining, it is helpful to make a comparison between our model and a model with a perfectly competitive labor market. The benchmark for our comparison is a version of Cho and Cooley (1994), in which there is a representative household who maximizes

$$\sum_{t=0}^{\infty} \delta^t \left[u(c_t) - e(h_t) l_t + b(1 - l_t) + g(1 - l_t)\right]$$

subject to $k_{t+1} = Ah_t^\alpha l_t^\alpha k_t^{1-\alpha} + (1 - \delta_k) k_t - c_t$, where $g(.)$ is an increasing function of non-employment $1 - l_t$. In this model, $l_t$ measures the days of work or the number of family members who participate in the labor market. As $l_t$ increases, more workers incur the utility cost $e(h_t)$ and lose the opportunity to receive $b$. All other costs of labor market participation is summarized by $g(.)$. To make this frictionless economy comparable with our model economy, we assume that $g$ is log-linear: $g(x) = e_1(\ln x - x)$, where $e_1$ is a positive parameter.

**Proposition 4** *Suppose that utility is linear in consumption. (a) If $\beta > 0$, then the capital-labor ratio of the economy with search and wage bargaining is less than that for the frictionless economy, and hours of work per employee in the economy with search and wage bargaining is longer than that for the frictionless* *economy. (b) The steady-state levels of the capital-labor ratio, hours of work, and employment for the two economies coincide if $\beta = 0$, $\xi = 1/2$, and*

$$e_1 = \frac{r+\lambda}{m_0}\gamma\frac{\lambda}{m_0}. \tag{43}$$

**Proof.** From the first-order conditions, we obtain $u'(c_t) = \delta u'(c_{t+1})[(1-\alpha)A_{t+1}h_{t+1}^{\alpha}l_{t+1}^{\alpha}k_{t+1}^{-\alpha} + 1 - \delta_k]$, $e'(h_t)l_t = u'(c_t)\alpha A_t h_t^{\alpha-1}l_t^{\alpha}k_t^{1-\alpha}$, and $g'(1-l_t) = e'(h_t)h_t - e(h_t) - b$. Thus, a steady state is given by a set of $\{h, l, K\}$ that satisfy

$$(1-\alpha)\,AK^{-\alpha} = r + \delta_k, \tag{44}$$

$$\alpha AK^{1-\alpha} = e'(h), \tag{45}$$

$$g'\,(1-l) = e'\,(h)\,h - e\,(h) - b, \tag{46}$$

where we have imposed $u'\,(c) = 1$ to be consistent with our model, in which there is no consumption smoothing motive. It is interesting to observe that, (30) and (31) and coincide with (44) and (45) when $\beta = 0$. With $\beta > 0$, the capital-labor ratio ($K$) of the economy with search frictions and wage bargaining is less than the one without, and hours of work per employee ($h$) for the economy with search frictions and wage bargaining is longer than the one without. To facilitate comparison of the levels of employment for the two economies, we assume $g$ to be log-linear in non-employment: $g\,(x) = e_1(\ln x - x)$, where $e_1$ is a positive constant. Since $g'\,(x) = e_1(1-x)/x$, (46) reduces to

$$e_1\frac{l}{1-l} = e'\,(h)\,h - e\,(h) - b. \tag{47}$$

This determines the relationship between $h$ and $l$ for the frictionless economy. In what follows we let $\beta = 0$. Eliminate $\theta$ from (32) and (33) to obtain

$$\frac{r+\lambda}{m_0}\gamma\left(\frac{\lambda}{m_0}\frac{l}{1-l}\right)^{\frac{\xi}{1-\xi}} = e'\,(h)\,h - e\,(h) - b,$$

which reduces to

$$\frac{r+\lambda}{m_0}\gamma\frac{\lambda}{m_0}\frac{l}{1-l} = e'\,(h)\,h - e\,(h) - b \tag{48}$$

when $\xi = 1/2$. It is now clear that the steady-state levels of employment $l$ for the two economies coincide if

$$e_1 = \frac{r+\lambda}{m_0}\gamma\frac{\lambda}{m_0}.$$

(47) and (48) imply that if search frictions are sufficiently severe, then the level of employment in the economy with search frictions is below the one without frictions. ■

Condition (43) equates the marginal benefit of non-employment and an index of search costs. A higher discount rate, a higher separations rate, a higher vacancy cost, or a smaller matching coefficient, each of which implies greater search frictions, increases the term of the right-hand side of (43). If the right-hand side of (43) is greater than $e_1$, then the level of employment for the economy with search frictions is below the one for the frictionless economy even when $\beta = 0$ and $\xi = 1/2$. Proposition 4 indicates that our model contains a competitive, frictionless economy as a special case.

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