ECONOMIC RESEARCH CENTER
DISCUSSION PAPER

E-Series

No.E16-9

Long-Run Growth and Welfare in an Endogenous Growth Model with Productive Public Goods and Spillover Effects

by

Toshiki Tamai
Koji Kitaura

November 2016

ECONOMIC RESEARCH CENTER
GRADUATE SCHOOL OF ECONOMICS
NAGOYA UNIVERSITY

Long-Run Growth and Welfare in an Endogenous Growth Model with Productive Public Goods and Spillover Effects

Toshiki Tamai[a*] and Koji Kitaura[b]

[a] *Graduate School of Economics, Nagoya University*

Furo-cho, Chikusa-ku, Nagoya, Aichi 464-8601, Japan

[b] *Faculty of Social Sciences, Hosei University,*

4342 Aihara-cho, Machida-shi, Tokyo 194-0298, Japan

Abstract

This paper develops a two-region model of endogenous growth with productive public goods, perfectly mobile capital, and immobile labor. Productive public goods have cross-border spillover effects, which mean that the production of one region is affected by the productive public good of another region. We investigate the interaction between regional fiscal policy, policy tasks of regional governments, and spillover effects under conditions of capital flight. Our analysis shows that regional governments, which have the task of maximizing regional welfare, fail to achieve this because of spillover effects. Further, it is shown that fiscal policy, with a common central government tax rate, also fails to maximize social welfare. Therefore, the *Barro tax rule* does not hold.

Keywords: Productive public goods; Economic growth; Welfare

JEL Classifications: O41, H20, E62

[*] Corresponding author: tamai@soec.nagoya-u.ac.jp (T. Tamai).

1. Introduction

Empirical literature has investigated the relationship between the fiscal structures of state and local governments and their economic growth. Miller and Russek (1997) found that state and local taxes affect economic growth if revenue finances public services, such as transportation. Cohen and Paul (2004), among others, suggested a positive relationship between regional core infrastructure (e.g., roads, railways, airports, sewage and water facilities), and output in the US data. More recently, Bom and Ligthart (2014) found that core infrastructure owned by local governments may be more productive than public capital supplied by the federal government, using data from selected OECD countries. The effects of public investments at the regional level have traditionally been unable to replicate the effects of public investments at the national level (See, for example, Pereira and Andraz (2013) for an excellent survey). One possible explanation for this paradox is that spillover effects captured by national level studies are not captured at the regional level (Boarnet, 1998; Mikelbank and Jackson, 2000). These authors posit that the empirical relevance of spillover effects across regions is largely an unresolved issue in theoretical studies. In this paper, we investigate the interaction between regional fiscal policy, policy tasks of regional governments, and spillover effects, using a two-region model of endogenous growth with productive public goods.

Existing endogenous growth models with productive public goods has mostly focused on the growth and welfare effects of fiscal policies, using only one sector. As shown in Barro (1990), among others, this increases the marginal product of private capital and leads to economic growth, using a representative agent framework (e.g., Futagami et al., 1993; Turnovsky and Fisher, 1995). Little theoretical research has attempted to explain the relationship between regional productive public goods and economic growth. Figuières et al. (2013) is the only theoretical study where infrastructure effects produced externalities, using a two country model. However, the above authors did not consider regional growth effects. This study, therefore, analyzes the effect of regional productive public goods on economic growth and welfare, incorporating tax competition with perfectly mobile capital and immobile labor.

The analysis of tax competition were pioneered by Wilson (1986) and Zodrow and Miezkowski (1986) and has been developed by many other authors.[1] Recently, theoretical studies have considered the relationship between tax competition and economic growth (Lejour and Verbon, 1997; Becker and Rauscher, 2013; Hatfield, 2015), comparing fiscal centralization to decentralization with imperfect capital mobility. Using perfect mobility, Rauscher (2005) examined the effects of a more mobile tax base on economic growth. This issue still seems empirically controversial (e.g., Thornton, 2007).

Reflecting above arguments, our model has two main features. First, and most importantly, we

[1] Wildasin and Wilson (2004) and Zodrow (2010) survey recent studies of tax competition.

assume that the growth engine, productive public goods, has spillover effects. In this paper, following Bronzini and Piselli (2009) and Figuières et al. (2013), we assumed that productive public goods have spillovers. Second, we considered maximizing equilibrium growth rate and regional welfare by regional governments and compared these results with the benchmark case derived from social welfare maximization or the Barro tax rule. Recently, Chu and Yang (2012) compared the performance of fiscal decentralization with fiscal centralization in terms of both economic growth and social welfare, using the standard AK model.

The results of this study show that growth-maximizing and regional welfare-maximizing tax rates set by regional governments are not equal to welfare-maximizing tax rates set by central governments when there are asymmetric regions in the economy with mobile capital. Although the welfare-maximizing tax rate is equal to the output elasticity of public input and is the same as the growth-maximizing tax rate for symmetric regions with same initial inputs, the welfare-maximizing tax rate set by central governments differs from the output elasticity of public input for asymmetric regions. Initial capital and labor endowments of each region are different for asymmetric regions. Capital moves across regions, while labor stays in each region. The government of each region attracts mobile capital to maximize regional welfare, because capital inflow increases current income levels. Therefore, the regional government sets the tax rate lower than the efficient level.

The remainder of this paper is organized as follows. In Section 2, a model is presented, and decentralized equilibrium under fiscal policy by regional governments is characterized in Section 3. In Section 4, the relationship between decentralized equilibria with different policy tasks is examined. In particular, we focus on the relationship between regional and social welfare maximization. Section 5 offers some conclusions.

2. The Model

Consider an economy that consists of two regions denoted by subscript numbers $i = 1,2$. Time is continuous and indexed by *t*. The population of region *i* is constant over periods and denoted by L_i. Each region has a representative immobile resident who lives forever. The capital market is integrated into the economy, but the labor market is not, and its participants are composed of workers living in their own region. We assume that the population of the economy is normalized to unity, i.e., $L = L_1 + L_2 = 1$. Let $K(t)$ be aggregate private capital: $K(t) = K_1(t) + K_2(t)$.

The lifetime utility function of a representative resident is

$$\int_0^{\infty} \frac{\left(C_i(t)\right)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt,$$

where $C_i(t)$ is private consumption, θ is the inverse of the elasticity of intertemporal substitution, and ρ is the subjective discount rate. The budget constraint of the resident is

$$\dot{A}_i(t) = r(t)A_i(t) + w_i(t)L_i - C_i(t), \tag{1}$$

where $A_i(t)$ is the asset, $r_i(t)$ is the equilibrium interest rate, and $w_i(t)$ is the after-tax wage rate of region *i*. Solving the optimization problem of the resident, we obtain

$$\frac{\dot{C}_i(t)}{C_i(t)} = \frac{r(t) - \rho}{\theta}, \tag{2}$$

and the transversality condition.

We assume that private goods are produced in each region by perfectly competitive firms using capital and labor inputs. Regional output is affected by spillover effects of productive public goods; the production function takes the form of $Y_i = F_i\big(K_i, h_i(G_i, G_j)L_i\big)$, where $h_i(G_i, G_j)$ captures the productivity effect of public goods includes spillover effect.

Suppose that regions have same production technology. Specifically, the production function in region *i* is assumed to be

$$Y_i(t) = \xi[K_i(t)]^{1-\alpha}\,[G(t)L_i(t)]^{\alpha}, \tag{3}$$

where $\xi > 0$ is a positive constant, $G(t)$ represents productive and spillover effects of public goods, and $0 < \alpha < 1$.

The term of $G(t)$ is also specified as

$$G(t) = \frac{[(H_1(t))^{-\sigma} + (H_2(t))^{-\sigma}]^{-\frac{1}{\sigma}}}{2}, \tag{4}$$

where G_i is the productive public goods provided by region *i*'s government and $\sigma \geq -1$. Thus, we obtain the elasticity of substitution between local government expenditures as $\epsilon = 1/(1+\sigma)$. In this specification, the productive public goods of one region positively affects not only its own production but also that of the other region. When $\sigma = 0$, this formulation is consistent with findings by Bronzini and Piselli (2009), who estimated the long-run relationship between total factor productivity and public infrastructure between 1980 and 2001 across regions in Italy. They found that regional productivity was positively affected by the public infrastructure of neighboring regions. A similar specification was considered by Figuieres et al. (2013).[2]

The regional government taxes capital input and uses the tax revenue for productive public goods. The budget constraint of region *i* becomes

$$G_i(t) = \tau_i Y_i(t), \tag{5}$$

where τ_i denotes the capital tax rate in region *i*.

The profit maximization conditions for firms are

$$r_i(t) = (1-\tau_i)(1-\alpha)\xi[K_i(t)]^{-\alpha}\,[G(t)L_i(t)]^{\alpha}, \tag{6}$$

$$w_i(t) = (1-\tau_i)\alpha\xi[K_i(t)]^{1-\alpha}\,[G(t)L_i(t)]^{\alpha}/L_i. \tag{7}$$

[2] Hirshleifer (1983, 1985) assumed the weakest link technology. Although we exclude the case of this extreme case (Leontief case), similar situation are considerable.

The factor price equalization holds: $r_1 = r_2$ because the capital market is integrated. In other words, we have

$$(1-\tau_1)(1-\alpha)\xi[K_1(t)]^{-\alpha}\,[G(t)L_1(t)]^{\alpha} = (1-\tau_2)(1-\alpha)\xi[K_2(t)]^{-\alpha}\,[G(t)L_2(t)]^{\alpha}. \tag{8}$$

Let us denote the equilibrium interest rate by r. Hence, $r(t) \equiv r_i(t)$ holds for $i = 1,2$ in equilibrium.

Let $\kappa_i := K_i(t)/K(t)$ and $\lambda_i := L_i/L$. Note that $\kappa_1 + \kappa_2 = 1$ and $\lambda_1 + \lambda_2 = 1$ hold. Using equations (4) and (5), the ratio of public goods to private capital satisfies

$$g(t) = \frac{\left[\{\tau_1\xi\kappa_1^{1-\alpha}g(t)^{\alpha}\,\lambda_1^{\alpha}\}^{\frac{\epsilon-1}{\epsilon}} + \{\tau_2\xi\kappa_2^{1-\alpha}g(t)^{\alpha}\,\lambda_2^{\alpha}\}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{(\epsilon-1)}}}{2}, \tag{9}$$

where $g(t) \equiv G(t)/K(t)$. Furthermore, the condition for factor price equalization (7) is rewritten as

$$(1-\tau_1)\kappa_1^{-\alpha}\lambda_1^{\alpha} = (1-\tau_2)\kappa_2^{-\alpha}\lambda_2^{\alpha}. \tag{10}$$

Equation (10) leads to

$$\kappa_i = \frac{(1-\tau_i)^{\frac{1}{\alpha}}\lambda_i}{(1-\tau_i)^{\frac{1}{\alpha}}\lambda_i + \left(1-\tau_j\right)^{\frac{1}{\alpha}}\lambda_j} = \frac{\lambda_i}{\lambda_i + \left(\frac{1-\tau_j}{1-\tau_i}\right)^{\frac{1}{\alpha}}\lambda_j} = \kappa_i(\tau_1,\tau_2), \tag{11}$$

where

$$\frac{\tau_i}{\kappa_i}\frac{\partial\kappa_i}{\partial\tau_i} = -\frac{\tau_i}{(1-\tau_i)\alpha}\kappa_j,$$

$$\frac{\tau_j}{\kappa_i}\frac{\partial\kappa_i}{\partial\tau_j} = \frac{\tau_j}{(1-\tau_j)\alpha}\kappa_j \qquad (i,j = 1,2, i \neq j).$$

For a given τ_i, the ratio of public input to private capital g is uniquely determined by equations (9) and (11):

$$g = \left[\frac{\{\tau_1\xi\kappa_1^{1-\alpha}\,\lambda_1^{\alpha}\}^{\frac{\epsilon-1}{\epsilon}} + \{\tau_2\xi\kappa_2^{1-\alpha}\,\lambda_2^{\alpha}\}^{\frac{\epsilon-1}{\epsilon}}}{2}\right]^{\frac{\epsilon}{(\epsilon-1)(1-\alpha)}} = g(\tau_1,\tau_2), \tag{12}$$

$$\frac{\tau_i}{g}\frac{\partial g}{\partial\tau_i} = \frac{\phi_i}{\phi_i+\phi_j}\left[\frac{\alpha-\tau_i}{\alpha(1-\alpha)(1-\tau_i)}\right] + \frac{\tau_i\kappa_i}{(1-\tau_i)\alpha}.$$

where $\phi_1 \equiv \{\tau_1\xi\kappa_1^{1-\alpha}\,\lambda_1^{\alpha}\}^{\frac{\epsilon-1}{\epsilon}}/2$ and $\phi_2 \equiv \{\tau_2\xi\kappa_2^{1-\alpha}\,\lambda_2^{\alpha}\}^{\frac{\epsilon-1}{\epsilon}}/2$.

Equations (6), (8), (11), and (12) yield

$$r = (1-\tau_1)(1-\alpha)\xi\kappa_1^{-\alpha}\lambda_1^{\alpha}g^{\alpha} = (1-\tau_2)(1-\alpha)\xi\kappa_2^{-\alpha}\lambda_2^{\alpha}g^{\alpha}$$

$$= (1-\alpha)\left[\frac{\left\{\tau_1(1-\tau_1)^{\frac{1-\alpha}{\alpha}}\,\lambda_1\right\}^{\frac{\epsilon-1}{\epsilon}} + \left\{\tau_2(1-\tau_2)^{\frac{1-\alpha}{\alpha}}\,\lambda_2\right\}^{\frac{\epsilon-1}{\epsilon}}}{2}\right]^{\frac{\alpha\epsilon}{(1-\alpha)(\epsilon-1)}}\xi^{\frac{1}{1-\alpha}}. \tag{13}$$

Using equations (3) and (9), the production function in equilibrium is

$$Y_i(t) = \xi\kappa_i^{1-\alpha}\lambda_i^{\alpha}g^{\alpha}K(t). \tag{14}$$

The asset market clearing condition is

$$K(t) \coloneqq K_1(t) + K_2(t) = A_1(t) + A_2(t). \tag{15}$$

By equations (1), (6), (7), (14), and (15), the resource constraint becomes

$$\dot{K}(t) = rK(t) + \sum_{i=1}^{2} w_i(t)L_i - C(t) = \left\{r + \alpha\xi g^{\alpha} \sum_{i=1}^{2} (1-\tau_i)\kappa_i^{1-\alpha}\lambda_i^{\alpha}\right\} K(t) - C(t),$$

where

$$C(t) \coloneqq C_1(t) + C_2(t).$$

Using equations (2), (6), (13), and the definition of $C(t)$, we obtain the equilibrium growth rate of private consumption as

$$\frac{\dot{C}(t)}{C(t)} = \frac{\dot{C}_i(t)}{C_i(t)} = \frac{(1-\tau_i)(1-\alpha)\xi\kappa_i^{-\alpha}\lambda_i^{\alpha}g^{\alpha} - \rho}{\theta} \equiv \gamma. \tag{16}$$

Proposition 1. *There exists a unique, stationary equilibrium, and the economy is always in equilibrium.*

(Proof) See Appendix A.

In stationary equilibrium, all economic variables except for prices and ratios grow at same rate, giving a balanced growth equilibrium. By Proposition 1, equation (16) stands for the balanced growth rate. In the balanced growth equilibrium with equation (16), the indirect utility of region i is derived as the following (See Appendices A and B):

$$V_i(0) = \frac{1}{1-\theta}\left[\frac{\left(C_i(0)\right)^{1-\theta}}{\rho + (\theta-1)\gamma} - \frac{1}{\rho}\right], \tag{17}$$

where

$$C_i(0) = \left[1 + \left(\frac{1-\alpha}{\alpha}\right)\mu_i\right] rA_i(0) = \left[\eta_i + \left(\frac{1-\alpha}{\alpha}\right)\kappa_i\right] rK(0), \tag{18}$$

$$\eta_i \equiv \frac{A_i(0)}{K(0)}, \mu_i \equiv \frac{K_i(0)}{A_i(0)} = \frac{\kappa_i}{\eta_i}.$$

Hereafter, we omit the script *t*, except for calling readers' attention to it.

Differentiating equation (17) with respect to τ_i and τ_j, the effects of a change in τ_i and τ_j on the welfare of region i are

$$\frac{\partial V_i(0)}{\partial \tau_i} = \frac{1}{[\rho + (\theta-1)\gamma]^2}\left[\left(C_i(0)\right)^{-\theta}\frac{\partial C_i(0)}{\partial \tau_i}\{\rho + (\theta-1)\gamma\} + \left(C_i(0)\right)^{1-\theta}\frac{\partial \gamma}{\partial \tau_i}\right], \tag{19a}$$

$$\frac{\partial V_i(0)}{\partial \tau_j} = \frac{1}{[\rho + (\theta-1)\gamma]^2}\left[\left(C_i(0)\right)^{-\theta}\frac{\partial C_i(0)}{\partial \tau_j}\{\rho + (\theta-1)\gamma\} + \left(C_i(0)\right)^{1-\theta}\frac{\partial \gamma}{\partial \tau_j}\right]. \tag{19b}$$

Note that by equation (18), in a general case, we have

$$\frac{\tau_i}{C_i(0)}\frac{\partial C_i(0)}{\partial \tau_i} = \frac{\eta_i}{\left[\eta_i + \left(\frac{\alpha}{1-\alpha}\right)\kappa_i\right]}\frac{\tau_i}{\eta_i}\frac{\partial \eta_i}{\partial \tau_i} + \frac{\left(\frac{\alpha}{1-\alpha}\right)\kappa_i}{\left[\eta_i + \left(\frac{\alpha}{1-\alpha}\right)\kappa_i\right]}\frac{\tau_i}{\kappa_i}\frac{\partial \kappa_i}{\partial \tau_i} + \frac{\tau_i}{r}\frac{\partial r}{\partial \tau_i}, \tag{20a}$$

$$\frac{\tau_j}{C_i(0)}\frac{\partial C_i(0)}{\partial \tau_j} = \frac{\eta_i}{\left[\eta_i + \left(\frac{\alpha}{1-\alpha}\right)\kappa_i\right]}\frac{\tau_j}{\eta_i}\frac{\partial \eta_i}{\partial \tau_j} + \frac{\left(\frac{\alpha}{1-\alpha}\right)\kappa_i}{\left[\eta_i + \left(\frac{\alpha}{1-\alpha}\right)\kappa_i\right]}\frac{\tau_j}{\kappa_i}\frac{\partial \kappa_i}{\partial \tau_j} + \frac{\tau_j}{r}\frac{\partial r}{\partial j}. \tag{20b}$$

3. Local Government Fiscal Policy

In this section, we examine the effects of fiscal policy by local governments. The policy objectives of local governments are to maximize regional welfare growth. We derive the social welfare-maximizing tax rates.

The Growth-Maximizing Local Government. We begin our analysis with a case where the local government has the policy objective of maximizing growth. Then, the first order conditions for growth-maximizing local governments must satisfy

$$\frac{\tau_i}{r}\frac{\partial r}{\partial \tau_i} = -\frac{\tau_i}{1-\tau_i} + \alpha\left[\frac{\tau_i}{g}\frac{\partial g}{\partial \tau_i} - \frac{\tau_i}{\kappa_i}\frac{\partial \kappa_i}{\partial \tau_i}\right] = 0 \Leftrightarrow \frac{\phi_i}{\phi_i + \phi_j}\left[\frac{\alpha}{1-\alpha} - \frac{\tau_i}{1-\tau_i}\right] = 0. \tag{21}$$

Solving equation (21) with respect to τ_i, we obtain the following proposition:

Proposition 2. *Growth-maximizing local governments set the regional income tax rate to*

$$\tau_1^g = \tau_2^g = \alpha, \tag{22}$$

Growth maximization is equivalent to maximizing the interest rate. Equation (21) implies that the local government policy to maximize the interest rate is independent of other local government policies. Therefore, as in a standard Barro model (1990), the local government must set the tax rate to satisfy equation (22)[3].

A Regional Welfare-Maximizing Local Government. The local government is regionally benevolent and therefore has the policy objective of maximizing regional welfare. Equations (19a) and (19b) show that the welfare effect of income taxes is composed of two different effects: one is the growth effect that relates to equation (21), and the other is the effect on initial consumption. By Proposition 2, an increase in welfare through the growth effect is maximized when equation (22) holds. If

$$\frac{\partial \eta_i}{\partial \tau_i} = -\left(\frac{\alpha}{1-\alpha}\right)\frac{\partial \kappa_i}{\partial \tau_i}, \tag{23}$$

[3] The relationship between Proposition 2 and the Barro tax rule is discussed in Section 5.

then we have

$$\frac{\partial \eta_i}{\partial \tau_i} = -\left(\frac{\alpha}{1-\alpha}\right)\frac{\partial \kappa_i}{\partial \tau_i} \Rightarrow \frac{\tau_i}{C_i(0)}\frac{\partial C_i(0)}{\partial \tau_i} = \frac{\tau_i}{r}\frac{\partial r}{\partial \tau_i}.$$

With equation (23), the effect on initial consumption is equivalent to the growth effect. Then, regional welfare maximization is equivalent to growth maximization, and equation (21) holds.

Initially, however, regional wealth and total capital stock are fixed. Therefore, equation (21) is dismissed; $\partial \eta_i / \partial \tau_i = 0$. Then, by equations (19a)-(22), we obtain

$$\left.\frac{\partial V_i(0)}{\partial \tau_i}\right|_{\tau_i=\tau_j=\alpha} = \frac{\left(C_i(0)\right)^{1-\theta}}{\rho + (\theta-1)\gamma}\frac{\left(\frac{\alpha}{1-\alpha}\right)}{\left[\eta_i + \left(\frac{\alpha}{1-\alpha}\right)\kappa_i\right]}\frac{\partial \kappa_i}{\partial \tau_i} < 0.$$

The inequality mentioned above implies that the regional welfare-maximizing tax rate is less than the growth-maximizing tax rate, which is equal to the output elasticity of public inputs. A rise in the income tax rate has a positive effect on output through an increase in public input and negatively affects welfare through a decrease in disposable income and capital outflow. If the local government aims to maximize the welfare of its own region, it must take into account not only the positive welfare effect of economic growth but also the negative welfare effect of capital outflow. Therefore, the local government maximizes regional welfare by setting the tax rate at a level below the growth-maximizing tax rate. This result is summarized as the following proposition:

Proposition 3. *The regional government that maximizes regional welfare sets the regional income tax rate to* τ_i^{**} *such as*

$$\max[\tau_1^{**}, \tau_2^{**}] < \tau_1^g = \tau_2^g = \alpha. \tag{24}$$

4. Central Government Fiscal Policy

In this section, we consider the relationships between the equilibria derived in the previous section with the central government. We assume that $\theta = 1$ throughout the remainder of this paper and also assume that the central government chooses income tax rates for each region to maximize social welfare by keeping tax rates constant. Define the sum of regional welfare functions as the social welfare function: $W = \lambda_1 V_1 + \lambda_2 V_2$. Then, the partial derivatives of W, with respect to τ_i, is given as

$$\frac{\partial W}{\partial \tau_i} = \lambda_1 \frac{\partial V_1}{\partial \tau_i} + \lambda_2 \frac{\partial V_2}{\partial \tau_i}, \tag{27}$$

where (19a)-(20b).

The first-order conditions for maximizing social welfare are $\partial W / \partial \tau_i = 0$ $(i = 1,2)$. Our task focuses on the relationship among tax rates under different government objectives. Thus, we only have to check the sign of $\partial W/\partial \tau_i$ evaluated at the different first-order conditions. At the tax rate that maximizes regional welfare, we obtain

$$\left.\frac{\partial W}{\partial \tau_1}\right|_{\frac{\partial V_1}{\partial \tau_1}=\frac{\partial V_2}{\partial \tau_2}=0} = \frac{\lambda_2}{\tau_1 \rho}\left[\frac{\left(\frac{\alpha}{1-\alpha}\right)\kappa_2}{\eta_2+\left(\frac{\alpha}{1-\alpha}\right)\kappa_2}\frac{\tau_1}{\kappa_2}\frac{\partial \kappa_2}{\partial \tau_1} - \frac{\left(\frac{\alpha}{1-\alpha}\right)\kappa_1}{\eta_1+\left(\frac{\alpha}{1-\alpha}\right)\kappa_1}\frac{\tau_1}{\kappa_1}\frac{\partial \kappa_1}{\partial \tau_1}\right] > 0, \quad (28a)$$

$$\left.\frac{\partial W}{\partial \tau_2}\right|_{\frac{\partial V_1}{\partial \tau_1}=\frac{\partial V_2}{\partial \tau_2}=0} = \frac{\lambda_1}{\tau_2 \rho}\left[\frac{\left(\frac{\alpha}{1-\alpha}\right)\kappa_1}{\eta_1+\left(\frac{\alpha}{1-\alpha}\right)\kappa_1}\frac{\tau_2}{\kappa_1}\frac{\partial \kappa_1}{\partial \tau_2} - \frac{\left(\frac{\alpha}{1-\alpha}\right)\kappa_2}{\eta_2+\left(\frac{\alpha}{1-\alpha}\right)\kappa_2}\frac{\tau_2}{\kappa_2}\frac{\partial \kappa_2}{\partial \tau_2}\right] > 0. \quad (28b)$$

Equations (28a) and (28b) show that the regional welfare-maximizing tax rate by a local government is lower than the social welfare-maximizing tax rate. Each regional government maximizes welfare in its own region by being concerned about the negative effects of capital outflows. However, capital flow of one region is capital inflow of another region. Capital inflow brings about financial benefits and an increase in public expenditures in the destination location. Furthermore, the negative welfare effect of capital outflows is diminished by simultaneous control of tax rates in two regions. Therefore, the social welfare maximizing tax rate is set larger than the tax rate of Proposition 3.

On the other hand, we have the following results for the growth-maximizing tax rate:

$$\left.\frac{\partial W}{\partial \tau_1}\right|_{\tau_1^g=\tau_2^g=\alpha} = \frac{\lambda_1\lambda_2}{(1-\alpha)^2\rho}\frac{\lambda_2\eta_1-\lambda_1\eta_2}{\left[\eta_1+\left(\frac{\alpha}{1-\alpha}\right)\lambda_1\right]\left[\eta_2+\left(\frac{\alpha}{1-\alpha}\right)\lambda_2\right]}, \quad (29a)$$

$$\left.\frac{\partial W}{\partial \tau_2}\right|_{\tau_1^g=\tau_2^g=\alpha} = -\frac{\lambda_1\lambda_2}{(1-\alpha)^2\rho}\frac{\lambda_2\eta_1-\lambda_1\eta_2}{\left[\eta_1+\left(\frac{\alpha}{1-\alpha}\right)\lambda_1\right]\left[\eta_2+\left(\frac{\alpha}{1-\alpha}\right)\lambda_2\right]}. \quad (29b)$$

Note that $\kappa_i = \lambda_i$ holds when $\tau_1 = \tau_2$. Equations (29a) shows that the social welfare-maximizing tax rate is larger (smaller) than the growth-maximizing tax rate if the denominator is positive (negative). Equation (29b) shows the opposite result of (29a). In asymmetric equilibrium, the social welfare tax rate of Region 1 and Region 2 will differ from each other, because endowment of the labor force and of capital differ from each other. However, the growth-maximizing tax rate is common to the two regions. Therefore, a common tax rate under the heterogeneity of regions engenders an additional distortion, according to the misdistribution of endowments.

These results provide the following proposition.

Proposition 4. *(i) The regional welfare-maximizing tax rate is less than the social welfare-maximizing tax rate of region i*:

$$\tau_i^{**} < \tau_i^{*}. \quad (30a)$$

(ii) The relationship between the *social welfare-maximizing tax rate of region i and the growth-maximizing tax rate is determined by the relative per-capita wealth of two regions:*

$$\tau_1^* \gtreqless \alpha, \tau_2^* \lesseqgtr \alpha \Leftrightarrow \frac{\eta_1}{\lambda_1} \gtrless \frac{\eta_2}{\lambda_2}. \tag{30b}$$

By Proposition 2-4, we summarize the relationship between tax rates under different government objectives as follows:

Proposition 5. *The relationship between the regional welfare-maximizing tax rate, the social welfare-maximizing tax rate, and the growth-maximizing tax rate are*

$$\tau_1^{**} < \alpha < \tau_1^*, \tau_2^{**} < \tau_2^* < \alpha \Leftrightarrow \frac{\eta_1}{\lambda_1} > \frac{\eta_2}{\lambda_2}, \tag{31a}$$

$$\tau_1^{**} < \tau_1^* = \alpha, \tau_2^{**} < \tau_2^* = \alpha \Leftrightarrow \frac{\eta_1}{\lambda_1} = \frac{\eta_2}{\lambda_2}, \tag{31b}$$

$$\tau_1^{**} < \tau_1^* < \alpha, \tau_2^{**} < \alpha < \tau_2^* \Leftrightarrow \frac{\eta_1}{\lambda_1} < \frac{\eta_2}{\lambda_2}. \tag{31c}$$

Standard models of endogenous growth with productive government expenditures assume one sector economy with immobile capital. Since no difference of region and capital flow exists, growth maximization is equivalent to welfare maximization if the production technology follows a Cobb-Douglas production function[4]. However, if there are asymmetric regions in the economy with mobile capital, growth-maximizing and regional welfare-maximizing by regional governments are not equivalent to welfare-maximizing. If and only if the endowments of capital and labor satisfy (31b), for both region 1 and region 2, will growth maximization be equivalent to social welfare maximization. Therefore, social welfare maximization is not attainable by means of common income tax rates.

To characterize this result, we now consider the special case that the central government imposes a flat income tax on a country's residents. The income tax rate is common to all residents in the country; $\tau_i = \tau$ for $i = 1,2$. With $\tau_i = \tau$ $(i = 1,2)$, capital does not move across regions, that is, $\partial\kappa_i/\partial\tau = 0$. Furthermore, we have $\partial\eta_i/\partial\tau_i = 0$, because the initial wealth share of each region is fixed. Then, the following equation is true for the central government:

$$\frac{\partial\eta_i}{\partial\tau} = \frac{\partial\kappa_i}{\partial\tau} = 0 \Rightarrow \frac{\tau}{C_i(0)}\frac{\partial C_i(0)}{\partial\tau} = \frac{\tau}{r}\frac{\partial r}{\partial\tau} \Rightarrow \operatorname{sgn}\frac{\partial V_i(0)}{\partial\tau} = \operatorname{sgn}\frac{\partial\gamma}{\partial\tau} = \operatorname{sgn}\frac{\partial r}{\partial\tau}.$$

Equation (13), with $\tau_i = \tau$ $(i = 1,2)$, leads to

$$r = (1-\tau)(1-\alpha)\left[\beta(\lambda_1)^{\frac{\epsilon-1}{\epsilon}} + (1-\beta)(\lambda_2)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon\alpha}{(\epsilon-1)(1-\alpha)}} \xi^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}}.$$

[4] Misch et al. (2013) investigated the relationship between growth maximization and welfare maximization in the extended Barro (1990) model with the CES production function. They showed that the elasticity of substitution between private capital and public input is key to the relationship between maximization of growth and welfare. Tamai (2013) derived results similar to Misch et al. (2013), using a stochastic growth model of Barro (1990) with the CES production technology.

The partial differentiation of r with respect to τ provides

$$\frac{\partial r}{r\partial \tau} = -\frac{1}{1-\tau} + \frac{\alpha}{(1-\alpha)\tau} \gtreqless 0 \Leftrightarrow \tau \lesseqgtr \alpha.$$

We obtain the following tax rule. The welfare-maximizing income tax rate of the central government is

$$\tau^* = \alpha. \tag{32}$$

This result is the well-known Barro tax rule. With an income tax rate that is common to two regions, a provision of public inputs is independent of the characteristics of the region. Therefore, the welfare-maximizing income tax rate of the central government is independent of deep parameters such as η_i and λ_i. From (31b) of Proposition 5 and (32), we arrive at the following result:

Proposition 6. *The Barro rule holds if and only if* $\frac{\eta_1}{\eta_2} = \frac{\lambda_1}{\lambda_2}$.

Finally, we explain the intuition of Proposition 5 and 6. Proposition 5 implies that residents in regions with large initial per capita assets should be levied a heavy tax on income, while taxes in regions with small initial per capita assets should be low. Focusing on symmetric regions, central governments have to choose a common tax rate to maximize social welfare, because an economy with symmetric regions is equivalent to an economy with representative agents; there is no migration of capital. Maximization of social welfare coincides with maximization of economic growth; thus, the Barro rule holds.

On the other hand, this is not true for an asymmetric region. Suppose that region 1 has a higher initial per capita asset than region 2. If income tax rates are the same for these two regions (e.g., the Barro rule is adopted), the rate of return on capital in region 1 is smaller than that of region 2, for a given, initial per-capita asset before capital moves. Then, capital flight occurs from region 1 to region 2, bringing about income redistribution from region 1 to region 2. This negatively affects region 1's welfare and positively affects region 2's welfare. Since initial per capita assets in region 1 are larger than in region 2, a positive welfare effect dominates when evaluated by the Barro rule.

5. Conclusion

In this paper, we examined the relationship between regional fiscal policy, policy tasks of regional government, and spillover effects, using a two-region model of endogenous growth for productive public goods, mobile capital, and immobile labor. We considered two regional governments' policy tasks: 1. that regional governments want to maximize growth, and 2. that regional governments want to maximize welfare.

To maximize growth, regional governments set the income tax rate to the output elasticity of public services. Then, the growth-maximizing tax rate is common for the two regions, and there is no capital flight. However, to maximize welfare, regional governments attract capital into their own regions, because capital inflow increases current regional income. Then, public services are insufficiently provided, compared with an optimal level obtained through spillover effects.

In general, a growth-maximizing tax policy does not coincide with a social welfare-maximizing policy. Welfare maximization by regional governments may not improve the supply of public services as well as a growth-maximizing policy. However, growth maximization does not provide the best results for social welfare. Similarly, a social welfare-maximizing policy, by means of a common tax rate on income, is not suitable for attaining social welfare maximization in an economy with asymmetric regions. Therefore, the Barro tax rule does not hold in a two-region model with spillover effects. It is necessary initially to introduce asset transfers between two regions to find the optimal equilibrium. This, however, is a highly controversial issue of redistributive policy.

Finally, we would like to mention the directions of certain extended studies. In this study, we assume that regional governments have an identical policy task. However, in reality, regional governments have different policy tasks, leading to a Stackelberg Leader-Follower game. Incorporating these issues into our basic model will derive additional policy implications. In such a case, our study will provide an analytical basis for these future studies.

Appendix

A. Proof of Proposition 1 and Derivation of Equation (17)

Let $z_i \equiv C_i/K$ $(i = 1,2)$. Equation (16) and the resource constraint lead to

$$\frac{\dot{z}_i}{z_i} = \frac{\dot{C}_i}{C_i} - \frac{\dot{K}}{K} = \gamma - \left\{ r + \alpha \xi g^{\alpha} \sum_{i=1}^{2} (1 - \tau_i) \kappa_i^{1-\alpha} \lambda_i^{\alpha} \right\} + z_1 + z_2.$$

By solving $\dot{z}_i = 0$ with respect to $z_1 + z_2$, we get a unique solution $z_1 + z_2$ such as $\dot{z}_i = 0$:

$$z_1 + z_2 = \left\{ r + \alpha \xi g^{\alpha} \sum_{i=1}^{2} (1 - \tau_i) \kappa_i^{1-\alpha} \lambda_i^{\alpha} \right\} - \gamma.$$

The value of z_i is determined as follows. By (1), (7), (13), (16), and $\dot{A}_i(0) = \gamma A_i(0)$, we have

$$\begin{aligned} C_i(0) &= (r - \gamma) A_i(0) + w_i(0) L_i = (1 - \tau_i) Y_i(0) + r[A_i(0) - K_i(0)] \\ &= \left[(1 - \tau_i) \mu_i \frac{Y_i(0)}{K_i(0)} + (1 - \mu_i) r \right] A_i(0) \\ &= \left[1 + \left(\frac{\alpha}{1 - \alpha} \right) \mu_i \right] r A_i(0) = \left[\eta_i + \left(\frac{\alpha}{1 - \alpha} \right) \kappa_i \right] r K(0). \end{aligned}$$

Therefore, a unique stationary equilibrium exists. The determinant and trace of Jacobian matrix for the above dynamic system around the stationary equilibrium are $\mathrm{tr}J = z_1 + z_2$ and $\det J = 0$. This is because the dimension of the dynamic system is substantially one. We now define z as $z_1 + z_2$. Then, we have

$$\frac{\dot{z}}{z} = \gamma - \left\{ r + \alpha \xi g^{\alpha} \sum_{i=1}^{2} (1 - \tau_i) \kappa_i^{1-\alpha} \lambda_i^{\alpha} \right\} + z.$$

The characteristic root of this differential equation is positive. Since there is one jumpable variable, the economy immediately reaches the stationary equilibrium at an initial time and remains there afterwards.

B. Derivation of Equation (18)

Using equation (16), the indirect utility function can be calculated as

$$\begin{aligned} V_i(0) &= \int_0^{\infty} \frac{\left(C_i(t)\right)^{1-\theta} - 1}{1 - \theta} e^{-\rho t} dt = \frac{1}{1 - \theta} \left[\int_0^{\infty} \left(C_i(t)\right)^{1-\theta} e^{-\rho t} dt - \int_0^{\infty} e^{-\rho t}\, dt \right] \\ &= \frac{1}{1 - \theta} \left[\int_0^{\infty} (C_i(0) e^{\gamma t})^{1-\theta} e^{-\rho t} dt - \int_0^{\infty} e^{-\rho t}\, dt \right] \\ &= \frac{1}{1 - \theta} \left[\left(C_i(0)\right)^{1-\theta} \int_0^{\infty} e^{\{(1-\theta)\gamma - \rho\} t} dt - \int_0^{\infty} e^{-\rho t}\, dt \right] \end{aligned}$$

$$= \frac{1}{1-\theta}\left[\frac{\left(C_i(0)\right)^{1-\theta}}{\rho + (\theta - 1)\gamma} - \frac{1}{\rho}\right].$$

Acknowledgements

The authors would like to thank Yoshimasa Aoki, Junko Doi, Akira Kamiguchi, Minoru Kitahara, Kautoshi Miyazawa, Tetsuya Nakajima, Hideki Nakamura, Hikaru Ogawa, Ryosuke Okazawa, Akira Yakita, Koichi Yotsuya and the participants of PET2015 (Association for Public Economic Theory), the autumn 2014 meeting of the Japan Association for Applied Economics, the seminar at Osaka City University, and the Nagoya Macroeconomics Workshop for their helpful comments. Tamai also thanks the JSPS KAKENHI (Grant Number 24530377) for its financial support.

References

Bom, P. R., and Ligthart, J. E., 2014. What have we learned from three decades of research on the productivity of public capital? J ECON SURV. 28 (5), 889–916.

Barro, R.J., 1990. Government spending in a simple model of endogenous growth, J Polit Econ . 98 (5), part 2, S103–S125.

Becker, D., and Rauscher, M., 2013. Fiscal competition and growth when capital is imperfectly Mobile. Scan J Econ. 115 (1), 211–233.

Boarnet, M. G., 1998. Spillovers and the locational effects of public infrastructure. J. Regional Sci. 38 (3), 381–400.

Bronzini, R., and Piselli, P., 2009. Determinants of long-run regional productivity with geographical spillovers: the role of R&D, human capital and public infrastructure. REG SCI URBAN ECON. 39 (2), 187–199.

Chu, A. C., and Yang, C. C., 2012. Fiscal centralization versus decentralization: Growth and welfare effects of spillovers, leviathan taxation, and capital mobility. J Urban Econ. 71 (2), 177–188.

Cohen, J.P. and C.J.M. Paul 2004. Public infrastructure investment, interstate spatial spillovers, and manufacturing costs, Rev. Econ. Stat. 86 (2), 551–560.

Figuières, C., F. Prieur and M. Tidball 2013. Public infrastructure, non-cooperative investments, and endogenous growth, Can. J. Econ. 46 (2), 587–610.

Futagami, K., Y. Morita and A. Shibata 1993. Dynamic analysis of an endogenous growth model with public capital, Scan J Econ. 95 (4), 607–625.

Hatfield, J. W., 2015. Federalism, taxation, and economic growth. J Urban Econ, 87, 114–125.

Hirshleifer, J., 1983. From weakest-link to best-shot: the voluntary provision of public goods, Public Choice 41 (3), 371–386.

Hirshleifer, J., 1985. From weakest-link to best-shot: correction, Public Choice 46 (2), 221–223.

Lejour, A. M., and Verbon, H. A., 1997. Tax competition and redistribution in a two-country endogenous-growth model. INT TAX PUBLIC FINANCE. 4 (4), 485–497.

Mikelbank, B. A., and Jackson, R. W., 2000. The role of space in public capital research. Int Reg Sci Rev. 23 (3), 235–258.

Misch, F., N. Gemmell and R. Kenller 2013. Growth and welfare maximization in models of public finance and endogenous growth, JPET. 15 (6), 939–967.

Miller, S. M., and Russek, F. S., 1997. Fiscal structures and economic growth: international evidence. Econ. Inq. 35 (3), 603.

Pereira, A. and J. Andraz 2013. On the economic effects of public infrastructure investment: a survey of the international evidence. J. Dev Econ. 38 (4), 1–37.

Rauscher, M., 2005. Economic growth and tax-competing leviathans. INT TAX PUBLIC FINANCE. 12 (4), 457–474.

Tamai, T., 2013. The macroeconomic effects of fiscal policy in a stochastically growing economy. Econ Model. 35, 464–471.

Thornton, J., 2007. Fiscal decentralization and economic growth reconsidered. J Urban Econ. 61 (1), 64–70.

Turnovsky, S.J. and H. Fisher 1995. The composition of government expenditure and its consequences for macroeconomic performance. J Econ Dyn Control. 19 (4), 747–786.

Wilson, J.D., 1986. A theory of inter-regional tax competition. J Urban Econ 19, 296–315.

Wilson, J.D., and Wildasin, D.E., 2004. Capital tax competition: bane or boon. J Public Econ. 88 (6), 1065–1091.

Zodrow, G.R., and Mieszkowski, P., 1986. Pigou, property taxation and the underprovision of local public goods. J Urban Econ. 19, 356–370.

Zodrow, G.R., 2010. Capital mobility and capital tax competition, National Tax Journal 63 (4), Part 2, 865–902.