The Impacts of Agricultural Public Infrastructure on Urban Unemployment and Social Welfare: A General Equilibrium Analysis

by

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Abstract: We establish four-sector general equilibrium models to investigate the impacts of increased provision of agricultural public infrastructure on urban unemployment and welfare. The welfare measurement is based on Sen (1974)’s welfare index. We consider three types of agricultural public infrastructure, the neutral one, the land-augmenting one, and the labor-augmenting one. The impacts of increased provision of agricultural public infrastructure on urban unemployment and welfare are determined by the interactions among three effects, the “agricultural productivity effect,” “rural labor employment effect,” and “urban expected wage effect.” We also identify the impacts on the skilled-unskilled wage inequality.

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1. Introduction

The agricultural (rural) public infrastructure provision is crucial for facilitating rural development in developing countries. Using the data from China, India, the Philippines and some South American countries, Zhang and Fan (2004), Fan and Zhang (2004), Teruel and Kuroda (2005), and Pinnstrup-Andersen and Shinokawa (2006) have empirically shown that agricultural public infrastructure, which stimulates agricultural productivity, is essential to regional development and poverty alleviation.

Despite the numerous empirical works that demonstrate a positive link between the provision of agricultural public infrastructure and agricultural productivity in different developing countries, few works have examined the impacts of rural agricultural public infrastructure on urban unemployment and social welfare, which have always been the two big concerns for the governments of developing countries. Is the increasing provision of agricultural public infrastructure always beneficial to the developing economy in terms of urban unemployment and social welfare? This issue motivates us to construct a unified theoretical framework to investigate how increased provision of agricultural public infrastructure affects the interactions among different economic sectors and what impacts it brings to urban unemployment and social welfare.

To fill the current research gap, this paper builds four-sector general equilibrium models to study how the increased provision of agricultural public infrastructure influences urban unemployment and welfare. The economy consists of an urban high-skill sector, an urban low-skill sector, an agricultural sector, and a sector to produce agricultural public infrastructure. Different from most of existing literature\(^2\), which use national income as the welfare measurement, we adopt Sen (1974)’s welfare index to measure welfare. Sen’s index contains two components, per capita labor income and Gini coefficient, thus balancing efficiency and fairness in the welfare measurement. In addition, different types of agricultural public infrastructure display diversified functions. The construction of roads, irrigation systems and provision of electricity benefit both rural labor and land; the public provision of advanced fertilizers and high-tech hybridization seeds typically improves the productivity of rural land; and public training and machines mainly make rural labor more efficient. To fully investigate the impacts of the agricultural public infrastructure provision, therefore, we divide the agricultural public infrastructure into three types: the neutral one, the land-augmenting one, and the labor-augmenting one. The neutral agricultural public infrastructure indicates that it improves the efficiency of all the production factors; the land-augmenting agricultural public infrastructure benefits rural land only; and the labor-augmenting agricultural public

\(^2\) Represented works include Grinols (1991), Beladi and Marjit (1992), Marjit and Beladi (1996), Gupta (1997) and Sen et al. (1997).
infrastructure only increases the productivity of rural labor. In the basic model, we assume that the agricultural public infrastructure is produced by skilled labor, but we relax this assumption in our extension.

We find that increasing the provision of agricultural public infrastructure does not necessarily reduce urban unemployment or improve social welfare. The impact of increased provision of agricultural public infrastructure on urban unemployment is determined by three effects, which are the “agricultural productivity effect,” the “rural labor employment effect,” and the “urban expected wage effect.” The “agricultural productivity effect” arises from the increase in the productivity of agricultural production factors; the “rural labor employment effect” is generated by the change in the labor employment in the agricultural sector; and the “urban expected wage effect” stems from the change in the expected wage rate of the urban low-skill sector. When the agricultural public infrastructure is neutral or land-augmenting, the “agricultural productivity effect” and the “rural labor employment effect” help reduce urban unemployment by sticking unskilled labor to the rural area, while the “urban expected wage effect” may potentially increase urban unemployment by attracting unskilled labor to move to the urban area. Therefore, urban unemployment falls if the “agricultural productivity effect” and the “rural labor employment effect” dominate the “urban expected wage effect,” and rises otherwise. When the agricultural public infrastructure is labor-augmenting, the “agricultural productivity effect” may reduce the unskilled labor’s willingness to stay in the rural area, depending on the agricultural wage elasticity of an efficient unit labor employment. The impact on welfare is also determined by the interactions of these three effects, and the intuition is similar. We also examine the impact on skilled-unskilled wage inequality and conduct several extensions to testify the generality of our findings. In the extended models, we find that if the agricultural public infrastructure is produced by unskilled labor or is provided in alternative ways (e.g., instead of constructing public infrastructure, the government procures the public services from the private sectors), the main findings of the basic models are still valid.

The present work contributes to the current literature in the following aspects. The first and main related strand of literature is the study on public infrastructure. Representative literature can be referred to Gupta (1997), Anwar (2001, 2006, 2009a, 2009b) and Pi and Zhou (2012, 2014). These studies either consider the relations between urban public infrastructure and international trade (Anwar, 2001), or demonstrate the relation among urban public infrastructure provision, social welfare and wage inequality (Anwar, 2006, 2009a, 2009b; Pi and Zhou, 2012, 2014). However, none of these studies considers the impact on urban unemployment and welfare, especially the welfare measured with wage inequality. The present paper complements this strand of literature by providing a theoretical framework to analyze these neglected but important issues. The second strand of literature focuses on evaluating the
policy impacts on urban unemployment and welfare. Exemplified studies include Beladi and Naqvi (1988), Grinols (1991), Chao and Yu (1992), Gupta (1993, 1994, 1995), Chaudhuri (2000, 2006, 2007), Beladi and Chao (2006), Zenou (2011) and Pi and Zhou (2015a). These papers mainly focus on how the development policies (e.g., subsidy policies, institutional changes, trade policies and environment policies) and endowment growth (e.g., the increased labor and capital endowments) affect urban unemployment and welfare. Nevertheless, none of them looks into the impact of agricultural public infrastructure. We contribute to this strand of literature by providing a new policy evaluation on urban unemployment and welfare. Third, the present paper also studies the relation between the provision of agricultural public infrastructure and skilled-unskilled wage inequality in developing countries, which has not been examined by the theoretical studies on the skilled-unskilled wage inequality in dual economies, such as Beladi et al. (2008), Chaudhuri and Yabuuchi (2007), Yabuuchi and Chaudhuri (2007), Chaudhuri (2008), Beladi et al. (2010) Pi and Zhou (2012, 2014) and Pan (2014). We fill such a research gap and find that increasing the provision of neutral or land-augmenting agricultural public infrastructure would mitigate the growing wage inequality in developing countries, while the increased provision of labor-augmenting agricultural public infrastructure may widen the wage gap.

The rest of this paper is organized as follows. In Section 2, we construct the basic model and consider the case where only skilled labor is employed in the agricultural public infrastructure production. In Section 3, we do the comparative static analysis on different types of agricultural public infrastructure. In Section 4, we extend the basic model by considering the employment of unskilled labor in the public sector, an alternative way to provide the agricultural public infrastructure and a different taxation scheme to finance the agricultural public infrastructure. Section 5 provides concluding remarks and policy implications.

2. The model

Consider a small open economy consisting of four sectors, an urban high-skill sector, an urban low-skill sector, a public sector and an agricultural sector. The urban high-skill sector employs skilled labor and capital as factors of production to produce the import-competing product. The urban low-skill sector uses unskilled labor and capital to produce the exportable good. For analytical simplicity, we assume that the public sector only employs skilled labor to supply the public infrastructure that enhances agricultural production. Being non-tradeable, the public infrastructure typically includes the high-tech public infrastructure and advanced

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3 One may argue that the public sector can also produce the public infrastructure for urban production. However, since our methodology is the comparative static analysis, such a situation can be ignored or alternatively said, we just assume that the public sector provides a fixed amount of public good for urban production. Pi and Zhou (2012, 2014) assume that the public sector only provides the urban public infrastructure, while ignoring that it can also produce the agricultural public infrastructure.
agricultural production technology introduced by the employed skilled labor\textsuperscript{4}. The cost of providing agricultural public infrastructure is financed by taxation on capital via per-capital tax (Alternative ways to finance the agricultural public infrastructure can be referred to the section of Extension and Discussion.). The agricultural sector employs unskilled labor and land to produce the exportable product. Besides, the agricultural production also depends on the provision of agricultural public infrastructure. More agricultural public infrastructure raises the productivity of the agricultural sector. Capital can freely flow between the urban low-skill and high-skill sectors. Skilled labor can freely move between the urban high-skill and public sectors. The wage rate of the agricultural sector is fully flexible, whereas the unskilled labor employed in the urban area is usually protected by the minimum wage act. Since the minimum wage is generally higher than the competitive wage, there exists unemployment of unskilled labor in the urban area. Similar to Beladi et al. (2008), Chaudhuri (2008), Beladi et al. (2010) and Pi and Zhou (2012, 2014), we assume that urban unemployment only exists among unskilled labor. All factor and good markets are perfectly competitive. Throughout the paper, we treat the product of the urban low-skill sector as the numeraire.

The production functions of the urban low-skill and high-skill sectors are given by:

\begin{equation}
X = F^X (L_{UX}, K_X),
\end{equation}

\begin{equation}
Y = F^Y (L_{UY}, K_Y),
\end{equation}

where \(X\) and \(Y\) are the outputs of the urban low-skill and high-skill sectors, respectively; \(L_{UX}\) and \(L_{UY}\) are the unskilled and skilled labor employed in the urban low-skill and high-skill sectors, respectively; \(K_X\) and \(K_Y\) are the capital utilized by the urban low-skill and high-skill sectors, respectively; \(F^X\) and \(F^Y\) represent the production functions of urban low-skill and high-skill sectors, both of which are strictly quasi-concave and linearly homogenous with respect to labor and capital.

The profit maximization conditions yield

\begin{equation}
F^X_L (L_{UX}, K_X) = \overline{w},
\end{equation}

\begin{equation}
F^X_K (L_{UX}, K_X) = r + t,
\end{equation}

\begin{equation}
p_Y F^Y_L (L_{UY}, K_Y) = w_S,
\end{equation}

\begin{equation}
p_Y F^Y_K (L_{UY}, K_Y) = r + t,
\end{equation}

where \(p_Y\) represents the relative price of the urban high-skill sector’s output; \(\overline{w}\) and \(w_S\) are the urban minimum wage rate and the wage rate of the skilled labor employed by the urban

\textsuperscript{4} The Extension and Discussion part provides more discussions about alternative ways to provide agricultural public infrastructure.
high-skill sector, respectively; \( r \) is the interest rate of capital; and \( t \) is the per-capital tax charged by the government. Here \( F_i^j = \partial F^i / \partial j \) (\( i = X, Y \), and \( j = L, K \)).

The public sector’s production function is as follows:

\[
\bar{R} = F^p (L_{SR}) .
\] (7)

where \( \bar{R} \) is the amount of agricultural public infrastructure; \( L_{SR} \) is the skilled labor employed by the public sector; \( F^p \) is the production function, with the property of \( F_L^p > 0 \) and \( F_{LL}^p \leq 0 \). Each skilled labor employed in this sector earns the wage of \( w_S \).

The budget balance of public infrastructure provision implies

\[
w_s L_{SR} = t \bar{R} .
\] (8)

Now we introduce the provision of the agricultural public infrastructure to the agricultural production function. In particular, the agricultural public infrastructure may be neutral, land-augmenting or labor augmenting to the agricultural production.

First, we consider the case when the agricultural public infrastructure is neutral, that is, the public infrastructure increases the productivity of all production factors. Specifically, the agricultural production function takes the following form:

\[
A = g(\bar{R}) F^A(L_{UA}, T) ,
\] (9a)

where \( A \) is the output of the agricultural sector; \( L_{UA} \) and \( T \) are the unskilled labor and land employed in the agricultural sector. The relation between the neutral agricultural public infrastructure \( \bar{R} \) and the agricultural productivity is described by \( g \), which satisfies \( g(0) = 1 \), \( g' > 0 \) and \( g'' < 0 \). The agricultural production function is \( F^A \), satisfying strict quasi-concavity and linear homogeneity with respect to labor and land. Equation (9a) is typically used to introduce the neutral public infrastructure into the production function (see Pi and Zhou, 2012, 2014, 2015a).

The profit maximization condition yields

\[
p_A g(\bar{R}) F^A_L(L_{UA}, T) = w_A ,
\] (10a)

\[
p_A g(\bar{R}) F^A_T(L_{UA}, T) = \tau ,
\] (11a)

where \( p_A \) represents the relative price of the agricultural sector’s output; \( w_A \) and \( \tau \) are the wage rate of the unskilled labor employed by the agricultural sector and the return rate of land, respectively; \( F^A_j = \partial F^A / \partial j \) (\( j = L, T \)).

Apart from being neutral, the agricultural public infrastructure can be factor-augmenting, that is, only some specific factor benefits from the public infrastructure. For instance, if the agricultural public infrastructure is a kind of advanced fertilizer or some high-tech hybridization
seeds, it only increases the productivity of land. In some other circumstances, the agricultural public infrastructure may be advanced agricultural machines or some professional training, which only increases the productivity of rural unskilled labor. When the agricultural public infrastructure is land-augmenting, equations (9a) to (11a) will be restated as

\[ A = F_A(L_{UA}, g(\bar{R})T), \]  
\[ p_A F_A(L_{UA}, g(\bar{R})T) = w_A, \]  
\[ p_A g(\bar{R}) F_T(L_{UA}, g(\bar{R})T) = \tau. \]  

When the agricultural public infrastructure is labor-augmenting, equations (9a) to (11a) will be

\[ A = F_A(g(\bar{R})I_{UA}, T), \]  
\[ p_A g(\bar{R}) F_A(L_{UA}, g(\bar{R}I_{UA}, T)) = w_A, \]  
\[ p_A F_T(g(\bar{R})I_{UA}, T) = \tau. \]  

The factor market clearing conditions are given by

\[ L_{3y} + L_{sr} = \bar{L}_S, \]  
\[ L_{ux} + L_{ux} + L_{ua} = \bar{L}_U, \]  
\[ K_x + K_y = \bar{K}, \]  
\[ T = \bar{T}. \]

where \( L_{ux} \) is the urban unemployment; \( \bar{L}_S \), \( \bar{L}_U \), \( \bar{K} \) and \( \bar{T} \) are the endowments of skilled labor, unskilled labor, capital and land, respectively.

In this paper, we assume that the migration of unskilled labor from the rural to the urban area satisfies the Harris-Todaro rural-urban migration equilibrium condition (Harris and Todaro, 1970):

\[ w_A = \frac{L_{ux}}{L_{ux} + L_{ux}} \bar{w}. \]

which indicates that the unskilled rural migrants will continue to transfer to the urban area until the agricultural wage rate \( (w_A) \) is equal to the expected wage rate of unskilled labor in the urban area \( (\frac{L_{ux}}{L_{ux} + L_{ux}} \bar{w}) \).

We adopt the welfare index from Sen (1974), which is also employed by Gupta (1993) and Gupta (1995). This index balances the average wage level and the wage distribution among

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5 Such an equilibrium condition is widely used to describe the unskilled rural-urban migration, such as Beladi et al. (2008), Chaudhuri(2008), Beladi et al. (2010) and Pi and Zhou (2012, 2014).
different groups of labor. Sen’s index is positively related to the per capita labor income, but negatively related to the Gini coefficient, implying that welfare may decrease due to the worsened income dispersion even if per capita labor income increases. Specifically, welfare is measured by:

\[ W = E(1 - G). \]  

where \( W \) is the welfare level, \( E = \frac{L_{UX} \bar{w} + L_{SR} w_A + L_{SR} w_s}{\bar{L}_Y + \bar{L}_S} \) is the per capita wage level, and \( G \) is the Gini coefficient measured by wage income.

Consumers consume the goods of the urban high-skill sector, the urban low-skill sector and the agricultural sector. This completes the description of the demand side.

Equations (1)-(8), (9a)-(11a) (9b-11b or 9c-11c) and (12)-(16) complete the description of the supply side, which determines the equilibrium values of \( X, Y, A, L_{SY}, L_{SR}, L_{UX}, L_{UA}, L_{UN}, K_x, K_y, T, w_s, w_A, r, t \) and \( \tau \), given the value of the policy variable (exogenous variable) \( \bar{R} \) and other parameters. Then, by equation (17), we can derive the equilibrium value of \( W \).

3. Comparative static analysis
3.1 Neutral agricultural public infrastructure

In this subsection, our model is composed of (1)-(8), (9a)-(11a) and (12)-(16), which determine sixteen endogenous variables, namely, \( X, Y, A, L_{SY}, L_{SR}, L_{UX}, L_{UA}, L_{UN}, K_x, K_y, T, w_s, w_A, r, t \) and \( \tau \). The policy variable (exogenous variable) is \( \bar{R} \), and others are all parameters.

The following lemma is useful to establish our main results.

**Lemma 1:** When the government increases the provision of neutral agricultural public infrastructure:

(i) The labor employment in the urban low-skill sector increases;
(ii) The wage rate of the agricultural sector increases;
(iii) The labor employment of the agricultural sector increases if \( \left( k_y \bar{w} \right) > \frac{k_y \bar{w}}{k_x \left( \bar{L}_Y - \bar{L}_{UA} \right)} \), and decreases otherwise.
Here \( k_x = \frac{K_x}{L_{UX}} \) and \( k_y = \frac{K_y}{L_{SY}} \).

**Proof:** See Appendix A-1.

The intuition is as follows. By equation (7), an increase in the neutral agricultural public infrastructure raises the employment of skilled labor in the public sector, hence reducing the skilled labor in the urban high-skill sector. The decreased employment of skilled labor reduces the marginal productivity of capital in the urban high-skill sector, so capital flows from the high-skill sector to the low-skill sector. This leads to an increase in the marginal productivity of unskilled labor in the urban low-skill sector. Owing to the fixed minimum wage, eventually, the increased capital employment raises the employment of unskilled labor in the urban low-skill sector. This explains Lemma 1-(i). Besides, the increased neutral agricultural public infrastructure raises the marginal productivity of unskilled labor in the agricultural sector, leading to an increase in the agricultural wage rate. This demonstrates Lemma 1-(ii).

From Lemma 1-(i) and 1-(ii), we can identify two opposing effects on the agricultural labor employment. First, Lemma 1-(ii) indicates a positive effect on the rural labor employment owing to the increase in the productivity of the rural labor. We name this effect the “agricultural productivity effect,” which is captured by \( p_A g F_A^L \). Second, the rise in the urban unskilled employment, which raises the expected income of unskilled labor in the urban area, encourages the transfer of the rural unskilled labor to the urban area. This generates a negative effect on rural labor employment. We call this the “urban expected wage effect”, which is described by

\[
\frac{k_y \bar{w}}{k_x (L_U - L_{UA})}.
\]

If the “urban expected wage effect” dominates (is dominated by) the “agricultural productivity effect,” the urban area is more (less) attractive and fewer (more) unskilled labor will work in the agricultural sector. This explains Lemma 1-(iii).

In line with existing literature, such as Beladi et al. (2008), Chaudhuri and Yabuuchi (2007), Yabuuchi and Chaudhuri (2007), Chaudhuri (2008), Beladi et al. (2010), Pan (2014) and Pi and Zhou (2012, 2014, 2015b), throughout the whole paper, we use the difference between the wage rate of skilled labor and the average wage rate of unskilled labor to describe the skilled-unskilled wage inequality. By equation (16), the average wage rate of unskilled labor is \( w_A \).

Then, we have:

**Corollary 1:** The increasing provision of neutral agricultural public infrastructure reduces the skilled-unskilled wage inequality.

**Proof:** See Appendix A-2.
According to Lemma 1, the increased provision of neutral agricultural public infrastructure raises the average wage rate of unskilled labor (the agricultural sector’s wage rate). On the other hand, given the good price of the urban low-skill sector (alternatively, the unit production cost) and the minimum wage rate, the sum of the interest rate and per-capital tax is determined. Since the price of the urban high-skill sector’s product is also given and capital can freely move between the two sectors, the wage rate of skilled labor is independent of the agricultural public infrastructure provision. Hence, the skilled-unskilled wage inequality is reduced. The fixed skilled labor wage is in line with Marjit et al. (2013), who consider a similar economic structure, i.e., two different sectors (the urban high-skill one and low-skill one) in the urban area with three production factors (skilled labor, unskilled labor and capital).

Based on Lemma 1, we use Proposition 1 to show how the increased provision of neutral agricultural public infrastructure affects urban unemployment.

**Proposition 1:** When the government raises the provision of the neutral agricultural public infrastructure, urban unemployment falls if

\[ p_A g' F_L^A - p_A g F_{LL}^A \cdot \frac{k_Y}{k_X} > \frac{k_Y (\bar{w} - w_A)}{k_X (L_U - L_{UA})}, \]

and rises otherwise.

**Proof:** See Appendix A-3.

The change in urban unemployment depends on the change in the employment of urban unskilled labor and that in the labor employment in the agricultural sector. From Lemma 1-(i), the employment of unskilled labor rises with the increased provision of agricultural public infrastructure. In addition, according to Lemma 1-(iii), the labor employment in the agricultural sector rises if the “urban expected wage effect” is dominated by the “agricultural productivity effect,” i.e. \( \frac{k_Y \bar{w}}{k_X (L_U - L_{UA})} < p_A g' F_L^A \). In this case, urban unemployment must fall because both the unskilled labor employed in the agricultural sector and that in the urban low-skill sector increase.

If the “urban expected wage effect” dominates the “agricultural productivity effect”, i.e.

\[ \frac{k_Y \bar{w}}{k_X (L_U - L_{UA})} > p_A g' F_L^A, \]

the unskilled labor employment in the agricultural sector decreases. Then urban unemployment may rise when the decrease in the rural unskilled labor employment outweighs the increase in the unskilled labor employed in the urban low-skill sector. Here we identify one additional effect, which stems from the change in the agricultural labor employment. When a unskilled labor decides whether to migrate from the rural area to the urban
area, (s)he needs to take into account the opportunity cost of the rise in the agricultural wage due to the decrease in the rural labor caused by his (her) migration. We call this effect the “rural labor employment effect,” which is represented by \( \frac{k_y}{k_x} p_A g F_{LL}^A \). If the “agricultural productivity effect” and the “rural labor employment effect” are sufficiently weak such that the “urban expected wage effect” dominates, i.e., \( p_A g' F_{L}^A - \frac{k_y}{k_x} p_A g F_{LL}^A < \frac{k_y \bar{W} - w_A}{k_x (\bar{L}_U - L_{UA})} \), then the increased job vacancies in the urban low-skill sector are insufficient to absorb the increase in the labor transferred from the rural to the urban area, causing a rise in urban unemployment. If the opposite takes place, urban unemployment is increased because jobs created in the urban low-skill sector exceed the increase in rural-urban migrants.

Now, let us come to the welfare analysis. Substituting equation (16) into (17), the per capita wage level \( E \), which represents efficiency, can be expressed by \( \frac{\bar{L}_U w_A + \bar{L}_S w_S}{\bar{L}_U + \bar{L}_S} \). As shown in Corollary 1, the average unskilled wage \( w_A \) rises and the skilled wage \( w_S \) does not change. Consequently, \( E \) always rises with an increase in the provision of agricultural public infrastructure. However, the Gini coefficient \( G \) may be worsened by the rise in unemployment. To investigate the impact on welfare, we expand equation (17) as

\[
W = \frac{w_A}{(\bar{L})^2} [2\bar{L}_S \bar{L}_U + (\bar{L}_U)^2 - L_{UA} (\bar{L}_U + L_{UA})] + \frac{(\bar{L}_S)^2}{(\bar{L})^2} w_S
\]

(17')

where \( \bar{L} = \bar{L}_s + \bar{L}_u \). We summarize the welfare effects of the increased provision of the neutral agricultural public infrastructure in Proposition 2.

**Proposition 2:** An increase in the provision of the neutral agricultural public infrastructure would improve welfare if

\[
(1 + \frac{\zeta_1}{\bar{L}_U - L_{UA}} w_A) p_A g' F_{L}^A - \frac{\zeta_1 k_y \bar{W}}{k_x (\bar{L}_U - L_{UA})} p_A g F_{LL}^A > \frac{k_y \bar{W}}{k_x (\bar{L}_U - L_{UA})},
\]

and worsen welfare otherwise.

Here \( \zeta_1 = \frac{[2\bar{L}_U (\bar{L}_s + L_{UX}) + 2L_{UX} L_{UA} + (L_{UA})^2]L_{UX} \bar{W}}{2(w_A)^2 L_{UA} L_{UA}} > 0 \).

**Proof:** See Appendix A-4.

By equation (17'), we know that the impact on welfare depends on the changes in the agricultural wage rate, the agricultural employment and urban unemployment. Lemma 1
implies that the agricultural wage rate rises, and urban unemployment falls as long as the agricultural employment rises. Therefore, welfare may deteriorate only when urban unemployment rises. As shown in Proposition 1, urban unemployment rises if the “urban expected wage effect” dominates the neutral “agricultural productivity effect” and the “rural labor employment effect”. Here we identify similar effects with adjustments on the condition in Proposition 2. Welfare improves if the neutral “agricultural productivity effect” and the “rural labor employment effect” (LHS) are larger than the “urban expected wage effect” (RHS), and deteriorates otherwise.

At the end of this subsection, two points are worth stressing. First, increasing the provision of agricultural public infrastructure may not necessarily improve welfare if we take both efficiency (per-capita income) and fairness (the Gini coefficient) into account. As discussed before, per-capita income always rises with an increase in the provision of agricultural public infrastructure, but the Gini coefficient may be worsened with the rise of urban unemployment. Second, the reduced skilled-unskilled wage inequality does not imply the increase in welfare because the income dispersion between unskilled and skilled labor, as reflected by the Gini coefficient, may be worsened.

3.2 Land-augmenting agricultural public infrastructure

When the agricultural public infrastructure is land-augmenting, the model also consists of sixteen equations, namely, equations (1)-(8), equations (9b)-(11b) and equations (12)-(16). The mechanism of this case is almost the same as that in the previous case. To save the space, we omit the detailed calculation which is available upon request. The results are qualitatively the same as those in subsection 3.1. We only need to replace $p_A g F_{Ll}^A$ with $p_A F_{Ll}^A$, and $p_A g' F_{LT}^A$ with $p_A g' T F_{LT}^A$ in the conditions of Lemma 1, Corollary 1, Proposition 1 and Proposition 2. In other words, the neutral agricultural productivity effect is replaced by the land-augmenting agricultural productivity effect.

We use Proposition 3 to summarize the findings in this case.

**Proposition 3**: An increase in the land-augmenting agricultural public infrastructure provision will exert almost the same effects as those in Lemma 1, Corollary 1, Propositions 1 and 2, with slight modification of the conditions. The results depend on the interactions among the land-augmenting “agricultural productivity effect,” the “rural labor employment effect” and the “urban expected wage effect.”

**Proof**: See the above discussion. ■

3.3 Labor-augmenting agricultural public infrastructure
When the agricultural public infrastructure is labor-augmenting, the model is constituted by equations (1)-(8), equations (9c)-(11c) and equations (12)-(16). Lemma 2 provides the basis for our analysis.

**Lemma 2:** When the government raises the provision of labor-augmenting agricultural public infrastructure,

(i) The labor employment of the urban low-skill sector increases;
(ii) The wage rate of the agricultural sector increases if
\[ p_A g' F^A_L g_A (1 + \sigma_{t_{ea}}) - \frac{k_y}{k_x} p_A \bar{w} g F^A_{L L} > 0, \]
and decreases otherwise;
(iii) The labor employment of the agricultural sector increases if
\[ p_A g' F^A_L (1 + \sigma_{t_{ea}}) > \frac{\bar{w} k_y}{(L_U - L_{UA}) k_X}, \]
and decreases otherwise.

Here \( \sigma_{t_{ea}} \equiv \frac{\partial w_A}{\partial g_{L_{UA}}} g_{L_{UA}} w_A \) < 0 represents the agricultural wage elasticity of the efficient unit labor employment.

**Proof:** See Appendix A-5.

When the agricultural public infrastructure is labor-augmenting, the “agricultural productivity effect” is described by \( p_A g' F^A_L (1 + \sigma_{t_{ea}}) \), while the “urban expected wage effect” and the “rural labor employment effect” are described by \( \frac{\bar{w} k_y}{(L_U - L_{UA}) k_X} \) and \( \frac{k_y}{k_x} p_A g F^A_{LL} \) as before. Different from subsections 3.1 and 3.2, the labor-augmenting “agricultural productivity effect” is not always positive. The rise in the public infrastructure provision not only generates a positive impact on the agricultural wage rate by increasing agricultural productivity, but also has a negative effect on the agricultural wage rate due to the increase in the efficient unit labor employment \( (g(R) L_{UA}) \). Whether the labor-augmenting “agricultural productivity effect” raises the agricultural wage depends on the agricultural wage elasticity of the efficient unit labor employment \( \sigma_{t_{ea}} \). If the agricultural wage is inelastic of the efficient unit labor employment, i.e. \(-1 < \sigma_{t_{ea}} < 0\), the labor-augmenting “agricultural productivity effect” leads to an increase in the agricultural wage rate, and the analysis is similar to Lemma 1. If the agricultural wage is elastic of the efficient unit labor employment, i.e. \( \sigma_{t_{ea}} < -1 \), however, the labor-augmenting
“agricultural productivity effect” is negative. This effect may be compensated by the “rural labor employment effect” that stimulates the rise of the agricultural wage rate. If the (negative) labor-augmenting agricultural productivity effect is dominated by the “rural labor employment effect,” i.e. $p_A g F^A_L (1 + \sigma_{t/u}) - \frac{k_Y}{k_X} p_A \bar{w} g F^A_{LL} > 0$, the agricultural wage increases. The same logic applies in the opposite case.

The intuition of Lemma 2-(i) and Lemma 2-(iii) is similar to that of Lemma 1-(i) and Lemma 1-(iii).

The impact on the skilled-unskilled wage gap is straightforward from Lemma 2. We state it in Corollary 2.

**Corollary 2**: An increase in the provision of labor-augmenting agricultural public infrastructure provision would narrow down the wage gap if

$$p_A g F^A_L (1 + \sigma_{t/u}) - \frac{k_Y}{k_X} p_A \bar{w} g F^A_{LL} > 0,$$

and widen the wage gap otherwise.

**Proof**: See Appendix A-6.

Similar to the analysis in Corollary 1, we can see that the increased provision of labor-augmenting agricultural public infrastructure has no impact on the skilled wage rate. Then, based on the analysis of Lemma 2, we can immediately obtain Corollary 2.

Now we analyze the impact on urban unemployment. Proposition 4 establishes the results.

**Proposition 4**: When the government raises the provision of labor-augmenting agricultural public infrastructure, urban unemployment falls if

$$p_A g F^A_L (1 + \sigma_{t/u}) - \frac{k_Y}{k_X} p_A \bar{w} L_{3/N} > \frac{k_Y \bar{w} L_{3/N}}{k_X (L_U - L_{3/u})},$$

and rises otherwise.

**Proof**: See Appendix A-7.

The impact on urban unemployment also depends on the three effects identified before. However, as discussed below Lemma 2, the labor-augmenting “agricultural productivity effect” can be positive or negative, depending on the agricultural wage elasticity of the efficient unit labor employment $\sigma_{t/u}$. When $-1 < \sigma_{t/u} < 0$, the labor-augmenting “agricultural productivity effect” is positive, then the intuition is the same as in Proposition 1. When
the labor-augmenting “agricultural productivity effect” is negative, and the “rural labor employment effect” \( p_A g'F_L^A \frac{k_Y}{k_X} \) is the only incentive for unskilled labor to work in the rural area. If the sum of these two opposing forces in the agricultural sector dominates the “urban expected wage effect,” i.e., \( p_A g'F_L^A (1 + \sigma_{UA}) \frac{k_Y}{k_X} > \frac{k_Y \bar{w} L_{UN}}{k_X (L_U - L_{UA})^2} \), urban unemployment falls.

The welfare effect is as follows.

**Proposition 5:** With an increase in the provision of labor-augmenting agricultural public infrastructure, welfare improves if

\[
p_A g'F_L^A (1 + \zeta_3 w_A) (1 + \sigma_{UA}) \frac{k_Y}{k_X} > \frac{k_Y \bar{w} L_{UN}}{k_X (L_U - L_{UA})^2},
\]

and deteriorates otherwise.

Here \( \zeta_3 \equiv \frac{[2 T_U (L_S + L_{UX}) + 2 L_{UX} L_{UA} + (L_{UA})^2] L_{UA} \bar{w}}{2 (w_A)^2 L_{UN} L_{UA}} > 0. \)

**Proof:** See Appendix A-8. ■

The intution of Proposition 5 is similar to that of Proposition 2, except for that the labor augmenting “agricultural productivity effect” \( p_A g'F_L^A (1 + \zeta_3 w_A) (1 + \sigma_{UA}) \) can be positive or negative, as discussed in Lemma 2 and Proposition 4.

Summing up the impacts of the three types of agricultural infrastructure, we find that the changes in urban unemployment and welfare are determined by three effects, i.e. the “agricultural productivity effect,” the “rural labor employment effect,” and the “urban expected wage effect.” We summarize the specific results in table 1.

From the above analysis, we can conclude that increasing the provision of the three types of agricultural infrastructure does not necessarily reduce urban unemployment or improve welfare. Furthermore, Propositions 1, 3 and 4 imply that the growing provision of the three types of agricultural public infrastructure may indirectly exert the “urban job creation puzzle” raised by Harris and Todaro (1970). That is, the number of rural-urban migrants exceeds that of the job vacancies created in urban areas, leading to a rise in urban unemployment. In addition, when the agricultural public infrastructure is neutral or land-augmenting, raising its provision would reduce the skilled-unskilled wage inequality; whereas increasing the provision of labor-augmenting agricultural public infrastructure may widen or narrow down the wage gap.
## Table 1 Impacts of Increasing Agricultural Infrastructure Provision

<table>
<thead>
<tr>
<th>Type</th>
<th>Urban Unemployment</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutral</td>
<td>rise (fall) if</td>
<td>increase (decrease) if</td>
</tr>
<tr>
<td></td>
<td>( p_A g'F^A_L - \frac{k_y}{k_x} p_A g'F^A_{LL} )</td>
<td>( (1 + \frac{\zeta_1}{L_U - L_{UA}}) p_A g'F^A_L )</td>
</tr>
<tr>
<td></td>
<td>( \langle &gt; \frac{k_y \bar{w} L_{UN}}{k_x (L_U - L_{UA})^2} )</td>
<td>( -\frac{\zeta_1 k_y \bar{w} p_A g'F^A_{LL}}{k_x (L_U - L_{UA})} &gt; \langle \frac{k_y \bar{w}}{k_x (L_U - L_{UA})} )</td>
</tr>
</tbody>
</table>

| Land-augmenting  | rise (fall) if     | increase (decrease) if |
|                  | \( p_A g'TF^A_{LT} - \frac{k_y}{k_x} p_A g'F^A_{LL} \) | \( (1 + \frac{\zeta_1}{L_U - L_{UA}}) p_A g'TF^A_{LT} \) |
|                  | \( \langle > \frac{k_y \bar{w} L_{UN}}{k_x (L_U - L_{UA})^2} \) | \( -\frac{\zeta_1 k_y \bar{w} p_A g'F^A_{LL}}{k_x (L_U - L_{UA})} > \langle \frac{k_y \bar{w}}{k_x (L_U - L_{UA})} \) |

| Labor-augmenting | rise (fall) if     | increase (decrease) if |
|                  | \( p_A g'F^A_L (1 + \sigma_{t_{UA}}) - \frac{k_y}{k_x} p_A g'F^A_{LL} \) | \( p_A g'F^A_L (1 + \zeta_3) w_A (1 + \sigma_{t_{UA}}) \) |
|                  | \( \langle > \frac{k_y \bar{w} L_{UN}}{k_x (L_U - L_{UA})^2} \) | \( -\frac{k_y \bar{w}}{k_x (L_U - L_{UA})} > \langle \frac{\zeta_3 k_y \bar{w} p_A g'F^A_{LL}}{k_x (L_U - L_{UA})} \) |

### 4. Extension and discussion

In this section, we extend the model in Section 3 by considering the following three cases. In the first case, unskilled labor is employed to produce agricultural public infrastructure. Second, we consider an alternative way of providing agricultural public infrastructure. Third, we discuss an alternative taxation to finance the provision of agricultural public infrastructure.

#### 4.1 Agricultural public infrastructure by the employment of unskilled labor

Unskilled labor is employed in the production of some types of agricultural public infrastructure, such as new roads, drainage facilities and irrigation systems. We modify the basic model to characterize this scenario.

The production function of the public sector is changed to

\[ \bar{R} = F^F (L_{UR}) . \]  \hspace{1cm} (18)

where \( L_{UR} \) is the unskilled labor employed by the public sector. Here the unskilled labor in this sector is also protected by the urban minimum wage act (Djankov and Ramalho, 2009).

The budget balance for the public infrastructure provision is...
\[ \bar{w} L_{UR} = \bar{t} \bar{K} \].

The market clearing conditions for the skilled and unskilled labor are rewritten as

\[ L_{SY} = \bar{L}_x, \]
\[ L_{UN} + L_{UX} + L_{UR} + L_{UA} = \bar{L}_u. \]

For rural migrants, there are job opportunities both in the urban low-skill sector and in the public sector. Accordingly, the rural-urban migration equilibrium condition is

\[ W_A = \frac{L_{UX} + L_{UR}}{L_{UX} + L_{UR} + L_{UN}} \bar{w}. \]

For the neutral agricultural public infrastructure provision, the extended model is composed of sixteen equations, namely, equations (1)-(6), equations (9a)-(11a), equations (14)-(15), and equations (18)-(22). Then, we can solve sixteen endogenous variables, which are \( X, Y, A \), \( L_{SY}, L_{UX}, L_{UR}, L_{UA}, L_{UN}, K_X, K_Y, T, w_S, w_A, t, r \) and \( \tau \). The policy variable is also \( \bar{R} \), and others are parameters. Similar to the analysis in Section 3, we can obtain the equation systems in the cases when the agricultural public infrastructure is land-augmenting or labor-augmenting. We summarize our findings in Proposition 6.

**Proposition 6**: When the agricultural public infrastructure is produced by the employment of unskilled labor, for each type of agricultural public infrastructure, the results in Section 3 hold qualitatively.

4.2 Procuring agricultural public infrastructure from private sectors

In Section 3, we assume that the agricultural public infrastructure is provided by the public sector and financed by via per-capital tax. Now we consider an alternative way to provide the agricultural public infrastructure.

We treat the products of urban low-skill and high-skill sectors as composite goods, which can be either privately consumed or used as the agricultural public infrastructure. In this case, without setting the public sector, the government procures from the urban low-skill and high-skill sectors to supply the agricultural public infrastructure. Such a way to provide public infrastructure is widely used in the literature on public economics, such as Zodrow and Mieszkowski (1986), Hoyt (2001) and Keen and Kotsogiannis (2002). In this case, equation (7) in the basic model is deleted and equation (12) becomes \( L_{SY} = \bar{L}_s \).

We find that the results in this case are consistent with the results in Section 3 where the employment of skilled labor in the public sector is very small compared to the endowment of

\[ ^6 \text{To save the space, we omit the computation which is available upon request.} \]
skilled labor, i.e., \( \frac{L_{sk}}{L_s} \rightarrow 0 \). In other words, this is a special case of Section 3 where the “urban expected wage effect” is eliminated.

4.3 Non-distortionary tax

Instead of per-capita tax, the agricultural public infrastructure may be financed by non-distortionary tax on consumers. In this case, the government’s objective is to choose an appropriate \( \bar{\Delta} \) to maximize national income, which is a common goal shared by the governments in developing countries.

As argued by Anwar (2006), the initial provision of agricultural public infrastructure needs not be optimal and hence, no attempt should be made to consider the optimal agricultural public infrastructure provision. The government just behaves as a Stackelberg leader who chooses \( \bar{\Delta} \) first, and then given \( \bar{\Delta} \), the sectors in the supply side organize their production behavior. Here we only consider how the marginal increase in \( \bar{\Delta} \) influences the supply side of the whole economy and social welfare, and the results in the basic model still hold.

5. Concluding remarks

Existing literature shows that the provision of agricultural public infrastructure is of great importance for agricultural production and economic development. However, the impact of increased agricultural public infrastructure provision on urban unemployment and social welfare lacks a complete theoretical analysis. The present paper aims at filling this research gap by establishing a theoretical framework to investigate this issue.

Our results have several policy implications. First, it is almost equivalent for the government to provide neutral and land-augmenting agricultural public infrastructure. If governments can efficiently provide these two types of public infrastructures to encourage rural people to work in rural areas, such a policy can reduce urban unemployment and improve social welfare. However, if the increased provision stimulates too much rural-urban migration of unskilled labor, the negative impacts on urban unemployment and welfare dominate. In this case, other actions should be taken to reduce urban unemployment and prevent the deterioration of welfare. Nevertheless, in both cases, the skilled-unskilled wage inequality is reduced. Second, if the government provides the labor-augmenting public infrastructure, the impact on skilled-unskilled wage inequality is ambiguous with more complex economic mechanism. In particular, the wage gap may be widened when the agricultural wage is elastic of the efficient unit labor employment.

All in all, the present paper is just an attempt to theoretically explore the impacts on urban
unemployment, wage inequality and social welfare of one important rural development policy, the agricultural public infrastructure provision. It may provide a theoretical benchmark for empirical studies on the impacts of agricultural public infrastructure. The consideration of environmental protection may also be a direction for future research.
Appendices:

(A-1) Proof of Lemma 1

Equation (7) implies \( \frac{dL_{SR}}{dR} = 1 \). Then, by equations (4), (6), (13) and (14), we have:

\[
F^X_k (L_{UX}, K_x) = p_y F^y_k (\bar{L} - L_{SR}, \bar{R} - K_x).
\]

Total differentiation of the above equation and equation (3) yields:

\[
\begin{pmatrix}
F^X_{KL} & p_y F^y_{KK} + F^X_{KK} \\
F^X_{LL} & F^X_{LK}
\end{pmatrix}
\begin{pmatrix}
dL_{UX} \\
dK_x
\end{pmatrix}
= \begin{pmatrix}
-p_y F^y_{KL} \\
0
\end{pmatrix}
d\bar{R}.
\]

Then, homogeneity of degree zero of \( F^X_L \) and \( F^Y_L \) implies:

\[
\frac{dL_{UX}}{d\bar{R}} = \frac{k_y}{k_x} > 0.
\]

where \( k_x = \frac{K_x}{L_{UX}} \) and \( k_y = \frac{K_y}{L_{SY}} \).

Total differentiation of equations (10a), (11a), (15) and (16) yields:

\[
\begin{pmatrix}
1 & -p_A g F^A_{LL} \\
\bar{L}_U - L_{UA} & -w_A
\end{pmatrix}
\begin{pmatrix}
d\bar{w}_A \\
dL_{UA}
\end{pmatrix}
= \begin{pmatrix}
p_A g' F^A_L \\
\bar{w} k_y
\end{pmatrix}
d\bar{R}.
\]

The determinant of the coefficient matrix of the above equations system is denoted as \( \Delta_1 \), where \( \Delta_1 = -w_A + p_A g F^A_{LL} (\bar{L}_U - L_{UA}) < 0 \). Then:

\[
\frac{d\bar{w}_A}{d\bar{R}} = \frac{1}{\Delta_1} p_A (-g' F^A_L w_A + \bar{w} g F^A_{LL} \frac{k_y}{k_x}) > 0, \quad \frac{dL_{UA}}{d\bar{R}} = \frac{1}{\Delta_1} \left[ -p_A g' F^A_L (\bar{L}_U - L_{UA}) + \bar{w} k_y \right].
\]

If \( \frac{\bar{w} k_y}{(\bar{L}_U - L_{UA}) k_x} < (>) p_A g' F^A_L \), then \( \frac{dL_{UA}}{d\bar{R}} > (<)0 \). ■

(A-2) Proof of Corollary 1

By equation (5),

\[
\frac{dw_s}{d\bar{R}} = p_y F^y_{LL} \frac{dL_{SY}}{d\bar{R}} + p_y F^y_{LK} \frac{dK_y}{d\bar{R}}.
\]

By equations (7) and (12), \( \frac{dL_{SY}}{d\bar{R}} = -\frac{dL_{SR}}{d\bar{R}} = -1 \). In addition, according to equation (14),
\[
\frac{dK_Y}{dR} = -\frac{dK_X}{dR}. \quad \text{By equation (3),} \quad \frac{dK_X}{dR} = -\frac{F_{XX}^X}{F_{XX}^K} \frac{dL_{UX}}{dR}, \quad \text{and} \quad \frac{dL_{UX}}{dR} = \frac{k_Y}{k_X} \quad \text{according to Appendix (A-1). Hence} \quad \frac{dK_Y}{dR} = \frac{F_{XX}^X}{F_{XX}^K} \frac{dK_X}{dR} < 0. \quad \text{Because} \ F_{XX}^X \text{ and } F_{XX}^Y \text{ are homogeneous of degree zero,} \quad \frac{dw_S}{dR} = p_Y F_{XX}^Y(-1+\frac{F_{XX}^X}{F_{XX}^K} \frac{dK_X}{dR} \frac{k_Y}{k_X}) = 0.
\]

Finally, as shown by Lemma 1, \( \frac{dw_A}{dR} > 0 \). Therefore, \( \frac{dw_S}{dR} - \frac{dw_A}{dR} < 0. \) \hfill \blacksquare

(A-3) Proof of Proposition 1

By equation (13), we have:
\[
\frac{dL_{UN}}{dR} = -(\frac{dL_{UX}}{dR} + \frac{dL_{UA}}{dR}).
\]

By Appendix (A-1), we have:
\[
\frac{dL_{UN}}{dR} = \frac{1}{-\Delta_1} \left\{ -p_A g' A F_A^{A}(\bar{L}_U - L_{UA}) + [\bar{w} - w_A + p_A g F_A^{A}(\bar{L}_U - L_{UA})] \frac{k_Y}{k_X} \right\}.
\]

If \( p_A g' A F_A^{A} > \frac{k_Y}{k_X} p_A g F_A^{A} \frac{\bar{w} - w_A}{k_X (\bar{L}_U - L_{UA})} \), then, \( \frac{dL_{UN}}{dR} < 0 \);

If \( p_A g' A F_A^{A} < \frac{k_Y}{k_X} p_A g F_A^{A} \frac{\bar{w} - w_A}{k_X (\bar{L}_U - L_{UA})} \), then, \( \frac{dL_{UN}}{dR} > 0 \). \hfill \blacksquare

(A-4) Proof of Proposition 2

Equation (17) can be rewritten as,
\[
W = \frac{w_A}{(\bar{L})^2} \left[ 2\bar{L}_S \bar{L}_U + (\bar{L}_Y)^2 - L_{UN}(\bar{L}_U + L_{UA}) \right] + \frac{(\bar{L}_Y)^2}{(\bar{L})} w_S.
\]

where \( \bar{L} = \bar{L}_S + \bar{L}_Y \). By totally differentiating the above equation and substituting equations (13) and (16), we have:
\[
(\bar{L})^2 \frac{dW}{dR} = \frac{2(w_A)^2 L_{UN} L_{UA}}{(\bar{L}_U)^2} \frac{dL_{UA}}{dR} + [2\bar{L}_U (\bar{L}_S + L_{UA}) + 2\bar{L}_U L_{UA} + (L_{UA})^2] \frac{dw_A}{dR}.
\]

Substituting \( \frac{dL_{UA}}{dR} \) and \( \frac{dw_A}{dR} \) into the above expression yields:
(L)^2 \frac{dW}{d\bar{R}} = \frac{1}{\Delta_1} \frac{2(w_A)^2}{L_{UX}} \frac{L_{UX}}{\bar{W}} \left[-p_A g' F_L^A (\bar{L}_U - L_{UA}) + \bar{w} k_y \right] \frac{k_x}{k_y} + \frac{1}{\Delta_1} \left[2 \bar{L}_U (\bar{L}_S + L_{UX}) + 2 L_{UX} (\bar{L}_U - L_{UA})^2 \right] \left[-p_A g' F_L^A w_A + p_A \bar{w} g F_{LL}^A \right] \frac{k_y}{k_x}.

Let \( \zeta_1 = \frac{2 \bar{L}_U (\bar{L}_S + L_{UX}) + 2 L_{UX} (\bar{L}_U - L_{UA})^2}{2(w_A)^2 L_{UX} \bar{W}} \). Then,

If \( 1 + \frac{\zeta_1}{L_U - L_{UA}} - w_A \) \( p_A g' F_L^A - \frac{\zeta_1 k_y \bar{w}}{k_x (L_U - L_{UA})} p_A g F_{LL}^A > \frac{k_y \bar{w}}{k_x (L_U - L_{UA})} \), then \( \frac{dU}{d\bar{R}} > 0 \);

If \( 1 + \frac{\zeta_1}{L_U - L_{UA}} - w_A \) \( p_A g' F_L^A - \frac{\zeta_1 k_y \bar{w}}{k_x (L_U - L_{UA})} p_A g F_{LL}^A < \frac{k_y \bar{w}}{k_x (L_U - L_{UA})} \), then \( \frac{dU}{d\bar{R}} < 0 \).

(A-5) Proof of Lemma 2

Similar to Appendix (A-1), we can get \( \frac{dL_{UX}}{d\bar{R}} = \frac{k_{L}}{k_{X}} > 0 \). Total differentiation of equations (10c), (11c), (15) and (16) yields:

\[
\begin{bmatrix}
1 & -p_A g F_{LL}^A \\
L_U - L_{UA} & -w_A
\end{bmatrix}
\begin{bmatrix}
dw_A \\
dL_{UA}
\end{bmatrix}
= \begin{bmatrix}
g' w_A (1 + \sigma_{tUA}) \\
g \bar{w} k_y \\
\frac{k_x}{k_y}
\end{bmatrix}
\begin{bmatrix}
d\bar{R}
\end{bmatrix}.
\]

The determinant of the coefficient matrix of the above equations system is denoted as \( \Delta_2 \), where \( \Delta_2 = -w_A + p_A g F_{LL}^A (L_U - L_{UA}) < 0 \) and \( \sigma_{tUA} = \frac{\partial w_A}{\partial g L_{UA}} g L_{UA} < 0 \). Then:

\[
\frac{dL_{UA}}{d\bar{R}} = \frac{1}{\Delta_2} \left[-p_A g' F_L^A w_A (1 + \sigma_{tUA}) + p_A \bar{w} g F_{LL}^A \right] \frac{k_y}{k_x}.
\]

The impact on \( w_A \) is ambiguous. If \( p_A g' F_L^A (1 + \sigma_{tUA}) - p_A \bar{w} g F_{LL}^A \frac{k_y}{k_x} > (0) \), then

\[
\frac{dL_{UA}}{d\bar{R}} > (0) \]

The impact on \( L_{UA} \) is also ambiguous. If \( p_A g' F_L^A (1 + \sigma_{tUA}) > (0) \) \( \frac{\bar{w} k_y}{(L_U - L_{UA}) k_x} \), then

\[
\frac{dL_{UA}}{d\bar{R}} < (0) \]

22 / 27
(A-6) Proof of Corollary 2

Similar to Appendix (A-2), \( \frac{dw_s}{dR} = 0 \). The impact on \( w_A \) has been derived in Appendix (A-5). Thus, the impact on skilled-unskilled wage inequality is straightforward.

In the situation where \(-1 < \sigma_{tua} < 0\), \( \frac{dw_s}{dR} - \frac{dw_A}{dR} < 0 \).

In the situation where \( \sigma_{tua} < -1 \), if \( p_A g'F_L^A (1 + \sigma_{tua}) > (\frac{k_y}{k_x}L_U - L_{UA})p_A gF_{Ll}^A \), then \( \frac{dw_s}{dR} - \frac{dw_A}{dR} < (>)0 \).

(A-7) Proof of Proposition 4

Equation (13) implies:

\[
\frac{dL_{UN}}{dR} = -\left( \frac{dL_{UX}}{dR} + \frac{dL_{UA}}{dR} \right).
\]

From Appendix (A-5), we have:

\[
\frac{dL_{UN}}{dR} = \frac{1}{-\Lambda_2} \left[ -p_A g'F_L^A (1 + \sigma_{tua}) L_U^{UA} + \left[ \tilde{w} - w_A + p_A gF_{Ll}^A (L_U - L_{UA}) \right] \frac{k_y}{k_x} \right].
\]

In the situation where \(-1 < \sigma_{tua} < 0\), \( \frac{dL_{UN}}{dR} < (>)0 \) if:

\[
p_A g'F_L^A (1 + \sigma_{tua}) - \frac{k_y}{k_x} p_A gF_{Ll}^A > (\frac{k_y \tilde{w} L_{UN}}{k_x (L_U - L_{UA})^2});
\]

In the situation where \( \sigma_{tua} < -1 \), \( \frac{dL_{UN}}{dR} < (>)0 \) if:

\[
-p_A g'F_L^A (1 + \sigma_{tua}) + \frac{k_y \tilde{w} L_{UN}}{k_x (L_U - L_{UA})^2} < (>) - p_A gF_{Ll}^A \frac{k_y}{k_x}.
\]

(A-8) Proof of Proposition 5

Total differentiation of equation (21') yields:

\[
(\bar{L})^x \frac{dW}{dR} = \frac{2(w_A)^2 L_{UN} L_{UA}}{L_{UX} \bar{w}} \frac{dL_{UA}}{dR} + [2 \bar{L}_U (\bar{L}_U + L_{UX}) + 2L_{UX} L_{UA} + (L_{UA})^2] \frac{dw_A}{dR}.
\]

Substituting \( \frac{dL_{UA}}{dR} \) and \( \frac{dw_A}{dR} \) derived in Appendix (A-5), we have:
\[
(\bar{L})^2 \frac{dW}{dR} = \frac{1}{\Delta_1} \frac{2(w'_A)^2 L_{UX} L_{UA}}{L_{UX} w} [-p_A g^L A (1 + \sigma_{x} t_e)(\bar{L}_U - L_{UA}) + \bar{w} k_y] \\
+ \frac{1}{\Delta_1} [2 \bar{L}_U (\bar{L}_U + L_{UX}) + 2 L_{UX} L_{UA} + (L_{UA})^2] [-p_A g^L A w_A (1 + \sigma_{x} t_e) + p_A \bar{w} g^L A k_y k_x] .
\]

Denote \( \zeta_3 = \frac{[2 \bar{L}_U (\bar{L}_U + L_{UX}) + 2 L_{UX} L_{UA} + (L_{UA})^2] L_{UX} \bar{w}}{2(w'_A)^2 L_{UX} L_{UA}} \). Then,

In the situation where \(-1 < \sigma_{x} t_e < 0\), \( \frac{dU}{dR} > 0 \) if :

\[
p_A g^L A (1 + \zeta_3 w_A)(1 + \sigma_{x} t_e) - \frac{k_y \bar{w}}{k_x (\bar{L}_U - L_{UA})} > 0 \quad \frac{\zeta_3 k_y \bar{w} p_A g^L A}{k_x (\bar{L}_U - L_{UA})}.
\]

In the situation where \( \sigma_{x} t_e < -1\), \( \frac{dU}{dR} > 0 \) if :

\[
-p_A g^L A (1 + \zeta_3 w_A)(1 + \sigma_{x} t_e) + \frac{k_y \bar{w}}{k_x (\bar{L}_U - L_{UA})} > 0 \quad -\frac{\zeta_3 k_y \bar{w} p_A g^L A}{k_x (\bar{L}_U - L_{UA})}.
\]

\[\blacksquare\]
References:


