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Fiscal and Monetary Policy in an Endogenous Growth Model with Public Capital

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Abstract

This paper describes an endogenous growth model in which the government finances its public investment using both income taxation and seigniorage. Arguments presented in this paper show that monetary policy always has positive impacts on economic growth and the inflation rate. Furthermore, we show that the growth-maximizing tax rate on income is less than the output elasticity of public capital on the BGP, a suitable result compared to those of empirical studies.

Keywords: Fiscal policy; monetary policy; economic growth; welfare

JEL classification: H54; O11; O23

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1 Introduction

An extensive body of literature examines endogenous growth models with government expenditure. Barro (1990) presents a pioneering approach in which productive government expenditure engenders sustained per-capita growth in the long run. In that model, productive government expenditure positively affects the production of final goods and generates endogenous growth in the long run.¹ Futagami et al. (1993) extend the Barro model by assuming that public capital has a positive effect on aggregate production. They subsequently examine whether the results derived by Barro (1990) are achieved in their model.

In particular, one strand of recent literature compares the different means of financing public investment in the growth process. For example, within an endogenous growth model with public capital, Greiner and Semmler (2000) examine the effects on growth of an increase in public investment that is financed by tax and government debt. On the other hand, few studies have addressed the effect of money-financed public investment in public capital. Public investment financing involves the use of both seigniorage and income taxation. Indeed, Basu (2001) shows that public investment positively correlates to seigniorage. This empirical evidence might indicate that governments finance public investment through seigniorage. Therefore, investigation of fiscal and monetary policy interactions might provide some pertinent insights.

In a framework of endogenous growth, some studies have examined the effects of tax-financed and money-financed government expenditures. Espinosa-Vega

¹Aschauer (1989) shows that public capital positively affects the output of final goods.

and Yip (1999) introduce financial factors to compare the effects of money-financed and tax-financed increases in non-productive government expenditure on inflation and the economic growth rate. In contrast to non-productive government expenditures, another strand of recent literature centers upon analyses of productive government expenditure (e.g., Ferreira (1999) and Hung (2005)). Ferreira (1999) develops a model in which government investment in public education is financed entirely by an inflation tax (seigniorage). He concludes that inflation is desirable, provided that seigniorage-financed investment in public education facilitates accumulation of private capital. Hung (2005) develops endogenous growth models with productive government expenditure financed using an income tax and seigniorage.² He derives the composition of productive government expenditure financing, showing that the demand for money plays an important role in reducing the optimal income tax rate.

The purposes of this paper are to investigate the effects of public investment financed by an income tax and seigniorage on economic growth rate, inflation rate, and welfare. The spirit of the arguments presented here is common to existing studies on fiscal and monetary policy interactions. One departure from the existing literature on fiscal and monetary policy interactions is the assumption that the stock of public investment contributes to the production of final goods. Productive public inputs are stock variables in reality. Public capital such as roads, highways, ports, and airports are representative examples. In

²In his paper, that productive government expenditure is called *public capital*. However, it resembles the setting of Barro (1990) rather than public capital in the sense defined by Arrow and Kurz (1970) and Futagami et al. (1993)

examining public capital, we are envisioning particular economic experiences in which public capital is essential for growth promotion.

Our main results can be summarized as follows: First, we show that the growth-maximizing tax rate on income is less than the output elasticity of public capital when public investment is financed by income taxation and seigniorage, and that a rise in the money creation rate has always had a positive impact on the economic growth rate. Seigniorage plays an important role in reducing the growth-maximizing tax rate on income when public investment is financed by income taxation and seigniorage. Second, we demonstrate that a raised income tax rate reduces (raises) the inflation rate if the income tax is lower (higher) than the growth-maximizing one; in addition, an increase in the money creation rate raises the inflation rate under a plausible assumption.

This paper is organized as follows: Section 2 presents a description of our model. Section 3 solves the model and characterizes the transitional dynamics. Section 4 describes an investigation of the effects of fiscal and monetary policy on economic growth and inflation. Section 5 presents discussion of additional issues. Finally, Section 6 concludes this paper.

2 The model

Consider a closed, single-good economy with identical and fully rational individuals and perfectly competitive firms. Final goods are producible using private capital and public capital. The production function for each firm is formulated

as

$$y = F(k, g) = \xi k^{1-\alpha} g^\alpha, \quad (1)$$

where y is the real output, g is the real public capital, k is the real private capital, and $\xi > 0$ is the productivity parameter.³ A representative firm maximizes its profit $\pi = y - rk$ for given g . Then, the factor price of private capital is given as

$$r = f'(x) = (1 - \alpha)\xi x^\alpha, \quad (2)$$

where $x \equiv g/k$.

The representative households in this economy live infinitely. The number of households is normalized to unity; there is no population growth. The objective function of the household is

$$U = \int_0^\infty u(c, m) \exp(-\rho t) dt, \quad (3)$$

where c is the per-household consumption, m is the real money holding, ρ is the subjective discount rate of $u_i > 0$, and $u_{ii} < 0$ ($i = c, m$).⁴ The household maximizes Eq. (3) subject to

$$\dot{a} = (1 - \tau)(ra + \pi) - c - [(1 - \tau)r + \gamma_p]m + h. \quad (4)$$

³Throughout the discussion in this paper, capital letters are used for nominal variables; small letters are used to denote real variables.

⁴This type of utility function, presented by Sidrauski (1967), is functionally equivalent to the assumption of liquidity costs (see Feensra (1986)).

In the above, $a \equiv k + m$ is the total available financial wealth, τ is the income tax rate, $\gamma_p \equiv \dot{p}/p$ is the inflation rate, and h is the amount of a lump-sum transfer. Thereby, the Hamiltonian associated with the optimization problem for the household can be set as

$$\max_{c,m} \mathcal{H} = u(c, m) + q [(1 - \tau)(ra + \pi) - c - \{(1 - \tau)r + \gamma_p\}m + h],$$

where q is the costate variable. Solving the optimization problem, the optimum rule for real money holding and the dynamic path of consumption are given as

$$\begin{aligned} \frac{u_m}{u_c} &= (1 - \tau)r + \gamma_p - \delta, \\ \frac{\dot{c}}{c} &= -\frac{u_c}{u_{cc} \cdot c} \left[(1 - \tau)r - \rho + \frac{u_{cm}}{u_c} \frac{\dot{m}}{m} \right]. \end{aligned}$$

We now assume that $u(c, m)$ is logarithmic:

$$u(c, m) = \log c + \eta \log m.$$

In the above, $\eta > 0$ represents the taste for real money holding. Therefore, we obtain the money demand function and the evolution of consumption:

$$m = \frac{\eta}{(1 - \tau)r + \gamma_p} c, \tag{5}$$

$$\frac{\dot{c}}{c} = (1 - \tau)r - \rho. \tag{6}$$

This government apportions tax revenues and seigniorage between public

investment and a lump-sum transfer (if it is available); it maintains the money creation rate and the rates of income tax at constant rates θ and τ , respectively (i.e., $\dot{M}/M \equiv \theta$). Therefore, the flow budget constraint for the government can be written as

$$\dot{g} = \theta m + \tau y - h = (\theta - z)m + \tau y. \quad (7)$$

We assume that government also maintains h/m at a constant rate, z , and that $\theta \geq z$ for analytical simplicity.

Finally, we obtain, using $m \equiv M/p$,

$$\gamma_m = \theta - \gamma_p. \quad (8)$$

Indeed, we can find the resource constraint, $y = c + \dot{g} + \dot{k}$, using Eq. (2), Eq. (4), Eq. (7), and Eq. (8).

3 Dynamic system

This section specifically examines the long-run general equilibrium and investigates the determinacy of BGP. First, we construct a dynamic system. Herein, λ and μ are defined respectively as c/m and k/m . Using (1), (2), and (4)-(8),

$$\dot{\lambda} = [\eta\lambda - (\rho + \theta)]\lambda, \quad (9)$$

$$\dot{\mu} = [\alpha(1 - \tau)\xi x^\alpha - \theta + \eta\lambda]\mu - (\lambda + \theta - z), \quad (10)$$

$$\dot{x} = [\tau - (1 - \tau)x]\xi x^\alpha + [(1 + x)(\theta - z) + \lambda x]/\mu. \quad (11)$$

Equations (9)-(11) constitute a system of three equations in λ , μ , and x . We define a stationary equilibrium as a solution of the system constituted by Eqs. (9)-(11). In a stationary equilibrium, all of c , k , m , and g grow at the same rate γ^* , i.e., $\dot{c}/c = \dot{k}/k = \dot{m}/m = \dot{g}/g = \gamma^*$, so-called BGP.

We now establish the existence, uniqueness, and stability of the stationary equilibrium as follows.

Proposition 1 *There exists a unique stationary equilibrium; a convergent path to such an equilibrium is locally determinate if $\rho + \gamma_p^* > 0$.*

(Proof) In a stationary equilibrium, we have $\dot{\lambda} = \dot{\mu} = \dot{x} = 0$. From Eq. (9), Eq. (10), and Eq. (11),

$$\lambda^* = (\rho + \theta)/\eta, \quad (12)$$

$$\frac{1}{\mu} = \frac{\alpha(1-\tau)\xi x^\alpha - \theta + \eta\lambda}{\lambda + \theta - z}, \quad (13)$$

$$\frac{1}{\mu} = \frac{(1-\tau)\xi x^{\alpha+1} - \tau\xi x^\alpha}{\theta - z + (\lambda + \theta - z)x}. \quad (14)$$

Using Eq. (12), Eq. (13), and Eq. (14), $P(x)$ is defined as

$$P(x) = \tau\xi x^{\alpha-1} + \frac{\alpha(\theta - z)(1-\tau)\xi x^{\alpha-1}}{\lambda^* + \theta - z} - (1-\alpha)(1-\tau)\xi x^\alpha + \frac{\rho(\theta - z)}{(\lambda^* + \theta - z)x^*} + \rho. \quad (15)$$

The function, P , has the following properties: $P(0) = \infty$, $P(\infty) = -\infty$, and

$$P'(x) = -(1-\alpha)\tau\xi x^{\alpha-2} - \alpha(1-\alpha)(1-\tau)\xi x^{\alpha-1} - \frac{\alpha(1-\alpha)(\theta-z)(1-\tau)\xi(x^*)^{\alpha-2}}{\lambda^* + \theta - z} - \frac{\rho(\theta-z)}{(\lambda^* + \theta - z)(x^*)^2} < 0. \quad (16)$$

Therefore, there exists x^* such that $P(x^*) = 0$. Substituting x^* for x in Eq. (13), we obtain μ^* . Linearizing the dynamic system of Eqs. (9)–(11) around the stationary equilibrium, the linearized system is given as

$$\begin{pmatrix} \dot{\lambda} \\ \dot{\mu} \\ \dot{x} \end{pmatrix} = \begin{pmatrix} J_{11} & 0 & 0 \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix} \begin{pmatrix} \lambda - \lambda^* \\ \mu - \mu^* \\ x - x^* \end{pmatrix}, \quad (17)$$

where

$$J_{11} = \eta\lambda^* = \rho + \theta > 0,$$

$$J_{21} = \eta\mu^* - 1,$$

$$J_{22} = \alpha(1-\tau)\xi(x^*)^\alpha - \theta + \eta\lambda^* = \alpha(1-\tau)\xi(x^*)^\alpha + \rho > 0,$$

$$J_{23} = \alpha^2(1-\tau)\xi(x^*)^{\alpha-1}\mu^* > 0,$$

$$J_{31} = x^*/\mu^* > 0,$$

$$J_{32} = -[(\theta-z)(1+x^*) + \lambda^*x^*]/(\mu^*)^2 = [\tau\xi(x^*)^{\alpha-1} - (1-\tau)\xi(x^*)^\alpha]x^*/\mu^*,$$

$$\begin{aligned} J_{33} &= -(1-\alpha)\tau\xi(x^*)^{\alpha-1} - \alpha(1-\tau)\xi(x^*)^\alpha - (\theta-z)/(\mu^*x^*) \\ &= \alpha\tau\xi(x^*)^{\alpha-1} - (1-\tau)\xi(x^*)^\alpha + \rho. \end{aligned}$$

The characteristic polynomial of the above linearized system Eq. (17) is

$$\chi^3 - \text{tr}J\chi^2 + \Delta\chi - \det J = 0, \quad (18)$$

where

$$\text{tr}J \equiv J_{11} + J_{22} + J_{33} = \alpha\tau\xi(x^*)^{\alpha-1} + 2\rho + \theta - \gamma^* = \alpha\tau\xi(x^*)^{\alpha-1} + 2\rho + \gamma_p^*,$$

$$\Delta = J_{11}J_{22} + J_{11}J_{33} + J_{22}J_{33} - J_{23}J_{32},$$

and

$$\det J \equiv |J| = J_{11}(J_{22}J_{33} - J_{23}J_{32}).$$

Because $J_{11} > 0$, the sign of $\det J$ depends on $J_{22}J_{33} - J_{23}J_{32}$:

$$\begin{aligned} J_{22}J_{33} - J_{23}J_{32} &= -\rho[(1-\alpha)(1-\tau)\xi(x^*)^\alpha - \rho] \\ &\quad - \alpha(1-\tau)\xi(x^*)^\alpha[(1-\alpha)(1-\tau)\xi(x^*)^\alpha] + \rho\alpha\tau\xi(x^*)^{\alpha-1}. \end{aligned} \quad (19)$$

Using Eq. (7) and $\dot{g}/g = \gamma^*$,

$$\alpha\tau\xi(x^*)^{\alpha-1} = \alpha\gamma^* - \frac{\alpha(\theta - z)}{\mu^*x^*}. \quad (20)$$

Combining Eq. (20) with Eq. (19),

$$J_{22}J_{33} - J_{23}J_{32} = -\rho(1-\alpha)\gamma^* - \alpha(1-\tau)\xi(x^*)^\alpha(\gamma^* + \rho) - \frac{\alpha(\theta - z)}{\mu^*x^*} < 0.$$

Then, we obtain $\det J < 0$. These conditions show that the characteristic polynomial Eq. (18) has two unstable roots and one stable root. Because both λ and μ are jumpable variables and x is a state variable, the converging path to the stationary equilibrium is uniquely determined. Q.E.D.

Under Proposition 1, the economic growth rate is given as

$$\gamma^* = (1 - \alpha)(1 - \tau)\xi(x^*)^\alpha - \rho. \quad (21)$$

Thus, the economic growth rate depends on the policy variables: τ , θ , and z .

In a stationary equilibrium, Eq. (8) is rewritten as

$$\gamma_p^* = \theta - \gamma^*. \quad (22)$$

The inflation rate, γ_p^* , is positive ($\gamma_p^* > 0$) if the money creation rate, θ , is higher than the economic growth rate, γ^* ($\theta > \gamma^*$). On the other hand, the inflation rate is negative ($\gamma_p^* < 0$) if the money creation rate is lower than the economic growth rate ($\theta < \gamma^*$).

4 Policy analysis

In this section, we investigate the macroeconomic effects of fiscal and monetary policy. First, we examine the effects of government policy on the economic growth rate. Second, we analyze the effects of government policy on the inflation

rate.

To analyze the growth effects of government policy, it is sufficient to derive the effects of changes in the income tax rate and money creation rate on ratios of public capital to private capital, x . Differentiating Eq. (15) yields

$$\frac{\tau}{x^*} \frac{\partial x^*}{\partial \tau} \equiv \epsilon_1 = -\frac{1 + (1 - \alpha)x^* - \alpha(\theta - z)/(\lambda^* + \theta - z)}{P'(x^*)} \tau \xi(x^*)^{\alpha-2} > 0, \quad (23)$$

$$\frac{\theta}{x^*} \frac{\partial x^*}{\partial \theta} \equiv \epsilon_2 = -\frac{(\rho + \alpha(1 - \tau)\xi(x^*)^\alpha)\theta\lambda^*}{(\lambda^* + \theta - z)^2 P'(x^*)(x^*)^2} > 0. \quad (24)$$

Using Eq. (21), Eq. (23), and Eq. (24), we obtain

$$\frac{\partial \gamma^*}{\partial \tau} = (1 - \alpha)\xi(x^*)^\alpha \left[(1 - \tau) \frac{\alpha}{x^*} \frac{\partial x^*}{\partial \tau} - 1 \right] = \frac{\alpha(\gamma^* + \rho)}{\tau} \left(\epsilon_1 - \frac{\tau}{\alpha(1 - \tau)} \right) \geq 0, \quad (25)$$

$$\frac{\partial \gamma^*}{\partial \theta} = \alpha(1 - \alpha)(1 - \tau)\xi(x^*)^{\alpha-1} \frac{\partial x^*}{\partial \theta} = \frac{\alpha(\gamma^* + \rho)}{\theta} \epsilon_2 > 0. \quad (26)$$

From Eq. (25) and Eq. (26), we have the following proposition.

Proposition 2 *The growth-maximizing tax rate on income is given as*

$$\tau = \frac{\alpha(\gamma^* + \rho) - (1 - \alpha)(\theta - z)\rho}{\gamma^* + \rho + (\theta - z)(1 - \alpha)\gamma^*}.$$

Furthermore, a rise in money creation rate, θ , has a positive effect on the economic growth rate when $\theta > z$.

(Proof) From Eq. (25) and $\partial\gamma^*/\partial\tau = 0$,

$$-1 + (1 - \tau)\frac{\alpha}{x^*}\frac{\partial x^*}{\partial\tau} = 0. \quad (27)$$

Using Eq. (23) and Eq. (25),

$$(\alpha - \tau)\xi(x^*)^{\alpha-1} - \frac{(\alpha(1 - \tau)\xi(x^*)^\alpha + \rho)(\theta - z)}{(\lambda^* + \theta - z)x^*} = 0. \quad (28)$$

After some manipulations, Eq. (28) yields

$$\tau = \frac{\alpha(\gamma^* + \rho) - (1 - \alpha)(\theta - z)\rho}{\gamma^* + \rho + (\theta - z)(1 - \alpha)\gamma^*}. \quad (29)$$

Q.E.D.

Equation (29) implies that the growth-maximizing tax rate on income is less than α if $\theta > z \geq 0$. This equation indicates that seigniorage plays a key role in reducing the growth-maximizing tax rate on income when the government finances its public investment through income taxation and seigniorage. If $\theta = z = 0$, the growth-maximizing tax rate on income is $\tau = \alpha$, which is derived by Barro (1990) and Futagami et al. (1993).

Next we investigate the effects of changes in the income tax rate and money creation rate on the inflation rate. Using Eq. (22), Eq. (25), and Eq. (26), we

have

$$\frac{\partial \gamma_p^*}{\partial \tau} = -\frac{\partial \gamma^*}{\partial \tau} \stackrel{\geq}{\leq} 0, \quad (30)$$

$$\frac{\partial \gamma_p^*}{\partial \theta} = 1 - \frac{\partial \gamma^*}{\partial \theta}. \quad (31)$$

From Eq. (30) and Eq. (31), we obtain the following proposition.

Proposition 3 *First, $\theta > z$ is assumed. A rise in the income tax rate, τ , reduces (raises) the inflation rate when the income tax is lower (higher) than the growth-maximizing one. On the other hand, a rise in the money creation rate, θ , raises the inflation rate if $\epsilon_2 < \theta/[\alpha(\gamma^* + \rho)]$.*

(Proof) From Eq. (30) and Proposition 2, the former half of Proposition 3 is derived. Using Eq. (26) and Eq. (31),

$$\frac{\partial \gamma_p^*}{\partial \theta} = 1 - \frac{\alpha(\gamma^* + \rho)}{\theta} \epsilon_2. \quad (32)$$

If the second term of Eq. (32) is less than unity, we have $\partial \gamma_p^* / \partial \theta > 0$:

$$1 > \frac{\alpha(\gamma^* + \rho)}{\theta} \epsilon_2 \Leftrightarrow \epsilon_2 < \frac{\theta}{\alpha(\gamma^* + \rho)} \Leftrightarrow \frac{\partial \gamma_p^*}{\partial \theta} > 0.$$

Q.E.D.

In a stationary equilibrium, the inflation rate negatively depends on the economic growth rate. When the income tax rate is less than the growth-

maximizing rate, a rise in the income tax rate raises the economic growth rate. Then, such a change reduces the inflation rate. However, a rise in the income tax rate raises the economic growth rate if the income tax rate is less than the growth-maximizing rate. Consequently, such a policy raises the inflation rate.

The effect of an increase in the money creation rate on the inflation rate has been broken down into two sets of effects. First is the positive inflation effect given by the first term of Eq. (31). Second is the negative inflation effect given by the second term of Eq. (31). If $\epsilon_2 < \theta/[\alpha(\gamma^* + \rho)]$, the growth effect of a change in money-supply growth rate is less than unity.⁵ Consequently, the first effect always dominates the second effect, and a rise in money creation rate raises the inflation rate.

5 Discussion

In this section, we discuss additional issues: welfare effects of fiscal and monetary policy. We restrict our analysis of welfare effects of government policy to a stationary equilibrium. Following Greiner and Hanusch (1998), we assume that the economy immediately attains a new stationary equilibrium after a change in the income tax and money creation rate. A main objective of the present study is to investigate whether growth maximization and welfare maximization involve identical implications.

⁵The assumption, $\epsilon_2 < \theta/[\alpha(\gamma^* + \rho)]$, implies that one unit of increase in the economic growth rate is less than one unit of the money creation rate.

At the stationary equilibrium, the indirect utility function is given as⁶

$$U = \frac{\ln(\rho + \theta) + (1 + \eta)[\ln(\alpha\gamma^* + \rho) - \ln(\rho + \theta + \eta(\theta - z))]}{\rho} + \frac{(1 + \eta)\gamma^*}{\rho^2} + \frac{(1 + \eta)[\ln k(0) - \ln(1 - \alpha)]}{\rho}. \quad (33)$$

Differentiating Eq. (33) leads to

$$\frac{\partial U}{\partial \tau} = \frac{1 + \eta}{\rho} \left[\frac{\alpha}{\alpha\gamma^* + \rho} + \frac{1}{\rho} \right] \frac{\partial \gamma^*}{\partial \tau}, \quad (34)$$

$$\frac{\partial U}{\partial \theta} = \frac{1}{\rho} \left[-\frac{\eta(2 + \eta)(\rho + \theta) - \eta(\theta - z)}{(\rho + \theta)(\rho + \theta + \eta(\theta - z))} + (1 + \eta) \left(\frac{\alpha}{\alpha\gamma^* + \rho} + \frac{1}{\rho} \right) \frac{\partial \gamma^*}{\partial \theta} \right]. \quad (35)$$

From Eq. (34), the welfare-maximizing tax rate on income tax is equal to the growth-maximizing one on the BGP.

When the government finances its public investment through seigniorage, public investment always has a positive effect on economic growth (Proposition 2) and also has positive effect on the inflation rate (Proposition 3). A positive growth effect engenders a positive welfare effect on the BGP. On the other hand, a positive inflation effect tends to lower welfare on the BGP because a rise in the inflation rate reduces real money holding. Then, a welfare-maximizing growth rate of money creation exists, as denoted by θ^* . Because $\text{sign} \partial U / \partial \tau = \text{sign} \partial \gamma / \partial \tau$, the welfare-maximizing tax rate on income is given as the growth-maximizing one for given θ . When the government sets the money creation rate

⁶See the Appendix for a derivation of Eq. (33).

at θ^* , the welfare-maximizing tax rate on income is given as

$$\tau^* = \frac{\alpha(\gamma^* + \rho) - (1 - \alpha)\theta^*\rho}{\gamma^* + \rho + (1 - \alpha)\theta^*\gamma^*}.$$

Consequently, the welfare-maximizing tax rate on income is also less than the output elasticity of public capital on the BGP.

6 Conclusion

This paper developed an endogenous growth model in which the government finances its public investment using both income taxation and seigniorage. The main novelty of this model compared with those found in the literature on endogenous growth with public capital is that it incorporates monetary policy.

Analyzing the model, we characterized the transitional dynamics and derive the growth and welfare effects of fiscal and monetary policy. We showed that monetary policy always has positive effects on both economic growth and the inflation rate. In addition, we demonstrated that the growth-maximizing tax rate on income is less than the output elasticity of public capital on the BGP.

Finally, we consider the direction of future research. Based on our model, the composition of government public investment financing plays an important role in determination of the optimal income tax rate. Therefore, it will be interesting to investigate whether public investment that is financed by public debt engenders a higher or lower growth rate than that financed by income taxation and seigniorage. Furthermore, it might be fruitful to analyze the effects

of fiscal policy on growth rates of an economy on the transition path. These topics will be important avenues for future investigations.

Appendix

On the BGP, Eq. (5), Eq. (6), Eq. (8), Eq. (21), and Eq. (22) yield

$$c(t) = c(0) \exp(\gamma^* t), \quad (36)$$

$$m(t) = \frac{\eta}{(1-\tau)r^* + \gamma_p^*} c(t) = \frac{\eta}{\gamma^* + \rho + \gamma_p^*} c(t) = \frac{\eta}{\theta + \rho} c(t). \quad (37)$$

Furthermore, Eq. (4) at time 0 is written as

$$\begin{aligned} \gamma^*(k(0) + m(0)) &= (1-\tau)y(0) - c(0) - (\gamma_p^* - z)m(0) \\ &= \frac{\gamma^* + \rho}{1-\alpha} k(0) - c(0) - (\gamma_p^* - z)m(0). \end{aligned} \quad (38)$$

After some manipulations, Eq. (38) leads to

$$c(0) = \frac{(\rho + \theta)(\alpha\gamma^* + \rho)}{(1-\alpha)(\rho + \theta + \eta(\theta - z))} k(0). \quad (39)$$

Using Eq. (3), Eq. (36), and Eq. (37),

$$U = \int_0^\infty [(1+\eta) \ln c(0) - \eta \ln(\rho + \theta) + (1+\eta)\gamma^* t] \exp(-\rho t) dt. \quad (40)$$

Substituting Eq. (39) for $c(0)$ in Eq. (40) and integrating Eq. (40) with respect to t from 0 to infinity, we obtain

$$U = \frac{\ln(\rho + \theta) + (1 + \eta)[\ln(\alpha\gamma^* + \rho) - \ln(\rho + \theta + \eta(\theta - z))]}{\rho} + \frac{(1 + \eta)\gamma^*}{\rho^2} + \frac{(1 + \eta)[\ln k(0) - \ln(1 - \alpha)]}{\rho}.$$

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