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by

Tadashi Minagawa

Shin Kawai

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Equilibrium Price Dispersion in a Model of Discount Competition

Tadashi Minagawa*
and
Shin Kawai†

Abstract

This paper considers the existence of equilibrium price dispersion in a model of discount competition with perfect information and homogeneous agents. The congestion effect is introduced as the scarcity of good sold at low prices. Consumers take into account not only the prices but also the availability of goods. Firms set their bargain prices and limited supplies. There exists a continuum of asymmetric Nash equilibria in which any kinds of price dispersion exist. The game structure coincides with the proportional-share game which is known in the rent-seeking literature.

JEL classification: L11, L13
Keywords: Price dispersion, Congestion effects, Rent seeking

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*Professor, Graduate School of Economics, Nagoya University, Furo-cho, Chikusa-ku, Nagoya, 464-8601, Japan. Tel.: +81-52-789-2384. Fax.: +81-52-789-4924. E-mail: minagawa@soec.nagoya-u.ac.jp

†Researcher, Economic Research Center (ERC), Graduate School of Economics, Nagoya University, Furo-cho, Chikusa-ku, Nagoya, 464-8601, Japan. Tel.: +81-52-789-5456. Fax.: +81-52-789-5456. E-mail: skawai@soec.nagoya-u.ac.jp

1 Introduction

Numerous studies have been made regarding the existence of price dispersion (see Stiglitz 1989 and chapter 16 in Shy 1995 for discussions of various price dispersion theories)[1]. Since the seminal paper of Stigler (1961), most of these models assume a lack of information concerning the prices charged by firms. Consumers learn them either through search (e.g., Salop and Stiglitz 1977; Stiglitz 1979; Carlson and McAfee 1983; Burdett and Judd 1983) or through advertisements (e.g., Butters 1977; Bester and Petrakis 1995). Moreover, the assumption of heterogeneity of either consumers or firms is common in price dispersion models (e.g., Reinganum 1979; Wilde and Schwartz 1979; Rob 1985). In particular, the assumption of heterogeneous consumers is crucial in models that do not entail the lack of information (Luski 1976; Reitman 1991)[2]. Chen and Kong (2004), however, demonstrate that price dispersion is possible even in a world of perfect information and identical consumers and firms. The driving force in their model is the cost of service capacity and congestion cost which is a variation of Luski (1976) and Reitman (1991)[3].

The purpose of this paper is to provide another source of price dispersion. Like Chen and Kong (2004), both firms and consumers are identical and all consumers know the exact prices charged by firms[4]. Unlike their model, we don't assume both the congestion cost and the cost of building capacity. Instead, we consider another congestion effect which is introduced by Barro and Romer (1987). The congestion arises from the scarcity of the good sold at low prices. Imagine the bargain sales at some stores; if there are lots of consumers, some of them cannot purchase the good at a low price. It is natural to suppose that the consumer *ex ante* expects the probability of purchase at a low price. In other words, we assume that consumers take into account not only the prices but also the availability of goods.

We focus on the retail markets in which there are two kinds of prices;

[1]There are several models of price dispersion in monetary economies through the random matching process, see Kamiya and Sato 2003 and the references therein.

[2]For a more detailed discussion regarding these assumptions, see Chen and Kong (2004).

[3]The congestion cost in their model can be interpreted as waiting cost (see Luski 1976; Reitman 1991).

[4]Thus, there are no search costs and no advertisement costs.

regular price and bargain price like Bester and Petrakis (1995). Unlike their model, we assume that all consumers know the bargain prices charged by firms. Firms set bargain price and its limited supply simultaneously. We call it the model of discount competition. In discussing the discount competition, the Statistics Bureau in Japan said:

> The form of transaction of consumer products has been diversified as indicated by the curtailment of distribution or trade channel. In the area of retail trade, new type of outlets such as discount stores have been growing rapidly. And the lead in price decision has moved from manufacturers and trades in distribution to retailers.[5]

In section 2, we present a model of discount competition in which firms have to choose both bargain price and the availability of good simultaneously given a regular price. In section 3, the existence of multiple-price equilibria is proved. The game structure coincides with the proportional-share game. In section 4, we show that the degree of price dispersion varies with the number of firms. In section 5, we modify the model by introducing a cost function. In section 6, we conclude the paper.

2 The Model

Consider an oligopolistic retail market in which there are two identical firms which sells an indivisible good, and N identical consumers ($N > 0$). The consumer has common preferences defined by

$$u(x, y - px) = ax + y - px, \quad a > 0, \tag{1}$$

where $x \in \{0, 1\}$, $y > 0$, $p > 0$, and $a > 0$ denote consumption of the good, income, price of the good, and the reservation utility[6], respectively. The term $(y - px)$ represents the residual income. Each consumer purchases at most one unit of the good if the price is equal to or less than the reservation

[5] The 1997 National Survey of Prices: Bargain Prices

[6] The reservation utility is derived from the following equation: $u(1, y - p) = u(0, y)$.

utility. Each firm i ($i = 1, 2$) sells the good to his customers n_i at zero cost. The good is sold at high price p_h as a **regular price** defined by a manufacturer's suggested retail price. Each firm can sell the good at a low price $p_{li} \in [0, p_h)$ with a limited supply $s_i \in (0, N]$. The low price can be interpreted as a **bargain price** or a **sale price** in order to obtain customers from its rival store. For simplicity, suppose that the high price level equals the reservation utility.

First we must notice that there is a Nash equilibrium in which the bargain price equals the marginal cost like Bertrand competition with identical goods. Thus, we assume that each firm restricts himself to the quantity s_i less than or equal to the number of customers at both firms (i.e., $s_i \leq n_i,\ i = 1, 2$) in order to avoid the Bertrand competition and the outcome with zero profit[7].

ASSUMPTION 1 Each firm takes p_h as given and $p_h = a > 0$.

2.1 The firm i's demand function

Each consumer chooses one of two stores for purchase of the good. Several **unlucky** consumers may purchase the good at a high price since the high-price equals the reservation utility level based on Assumption 1. Taking price and quantity vector $(p_h, p_{l1}, p_{l2}, s_1, s_2)$ as given, the consumers rationally expect the number of the customers in each store. Thus, from (1), the consumer's utility function V from his choice of store i can be written by[8]:

$$V(p_h, p_{li}, s_i) = \frac{s_i}{n_i}(a + y - p_{li}) + \left(1 - \frac{s_i}{n_i}\right)(a + y - p_h), \tag{2}$$

The number of customers at each store changes as long as there is the chance to obtain the larger surplus. Then n_i is determined at which the utility from each store is indifferent, that is,

$$V(p_h, p_{l1}, s_1) = V(p_h, p_{l2}, s_2); \tag{3}$$

hence, from (2), we obtain,

$$\frac{s_1}{n_1}(p_h - p_{l1}) = \frac{s_2}{n_2}(p_h - p_{l2}). \tag{4}$$

[7]See Appendix for a more detailed discussion,

[8]The fraction s_i/n_i means the probability of low price at store i. If $s_i > n_i$, the probability then should be 1. However, we can ignore the case in equilibrium when certain condition is satisfied. Therefore, we focus on the case $s_i \leq n_i, i = 1, 2$. See Appendix.

Using $n_1 + n_2 = N$, we can rewrite (4) as,

$$\begin{aligned} n_i(p_h, p_{li}, p_{lj}, s_i, s_j) &= N\left(\frac{(p_h - p_{li})\, s_i}{(p_h - p_{li})\, s_i + (p_h - p_{lj})\, s_j}\right), \\ &= N\left(\frac{C_i}{C_1 + C_2}\right) \quad i, j = 1, 2,\ i \neq j, \end{aligned} \tag{5}$$

where C_i $(i = 1, 2)$ represents the consumer surplus of firm i's customers; that is,

$$C_i \equiv (p_h - p_{li})s_i = ((a + y - p_{li}) - (a + y - p_h))s_i. \tag{6}$$

Equation (5) is the firm i's demand function. The firm i can attract consumers by decreasing the low price or increasing the limited quantity; that is,

$$\frac{\partial n_i(\cdot)}{\partial p_{li}} < 0, \frac{\partial n_i(\cdot)}{\partial s_i} > 0, \quad \text{and} \quad \frac{\partial n_i(\cdot)}{\partial p_{lj}} > 0, \ \frac{\partial n_i(\cdot)}{\partial s_j} < 0,$$

for every $p_{li} \in [0, p_h)$, $s_i \in (0, N]$. From (5), we find that the condition $s_i \leq n_i(\cdot)$ (i=1,2) can be reduced to

$$C_i + C_j \leq (p_h - p_{li})N \quad i, j = 1, 2,\ i \neq j. \tag{7}$$

This condition is satisfied if and only if

$$C_i + C_j \leq \min\{(p_h - p_{li})N, (p_h - p_{lj})N\}. \tag{8}$$

We assume the following[9]:

ASSUMPTION 2 Each firm takes action within the condition (8).

2.2 Two-seller Game

Here we solve for an oligopoly equilibrium. We first have to define a **dicount competition** as a normal-form game. There are two firms as players of this game. Let each firm's actions be defined as choosing its low price and quantity levels taking high price p_h as given, and assume that both firms

[9] See Appendix regarding the discussion of Assumption 2.

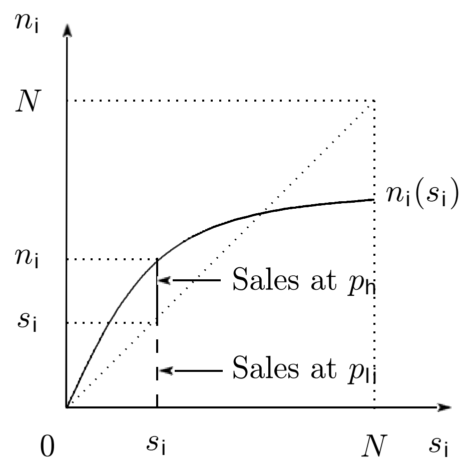

Figure 1: An Example of Strategy of s_i

choose their actions simultaneously. Thus, each firm i chooses $p_{li} \in [0, p_h)$ and $s_i \in (0, N]$, $i = 1, 2$. The payoff function of each firm i can be defined by

$$\pi_i(p_h, p_{li}, p_{lj}, s_i, s_j) = p_{li}s_i + p_h(n_i(p_h, p_{li}, p_{lj}, s_i, s_j) - s_i).$$

The first term of RHS is the revenue from bargain sales and the second term is the revenue from regular sales. Assume, for simplicity, that the costs of production are zero.

Firm i takes (p_{lj}, s_j) as given and chooses (p_{li}, s_i) to

$$\begin{aligned} \max_{p_{li}, s_i} \ \pi_i(\cdot) &= p_{li}s_i + p_h(n_i(\cdot) - s_i) \\ &= p_{li}s_i + p_h\left(\frac{NC_i}{C_i + C_j} - s_i\right) \\ & i, j = 1, 2, \quad i \neq j. \end{aligned} \tag{9}$$

The first-order conditions are given by

$$\frac{\partial \pi_i}{\partial p_{li}} = s_i - p_h\left(\frac{NC_j s_i}{(C_i + C_j)^2}\right) = 0, \tag{10}$$

and

$$\frac{\partial \pi_i}{\partial s_i} = p_{li} + p_h\left(\frac{N(p_h - p_{li})C_j}{(C_i + C_j)^2} - 1\right) = 0. \tag{11}$$

From (10) and (11), we obtain, respectively,

$$p_{li} = p_h - \frac{\sqrt{Np_h(p_h - p_{lj})s_j} - (p_h - p_{lj})s_j}{s_i}, \tag{12}$$

and

$$s_i = \frac{\sqrt{Np_h(p_h - p_{lj})s_j} - (p_h - p_{lj})s_j}{p_h - p_{li}}. \tag{13}$$

Substituting (13) into (12), we find that the solution to this problem is indeterminate[10]. We can, however, derive the condition of symmetric Nash equilibrium by substituting $s_i = s_j = s^*$ and $p_{li} = p_{lj} = p_l^*$ for (12) and (13). In this process, we obtain:

$$p_{li} = p_h + (p_h - p_{lj}) - \sqrt{\frac{p_h N(p_h - p_j)}{s^*}}, \tag{14}$$

and

$$s_i = -s_j + \sqrt{\frac{Np_h s_j}{p_h - p_l^*}}. \tag{15}$$

Eqs.(14) and (15) appear as in Figure 2. Therefore, a set of a price and a

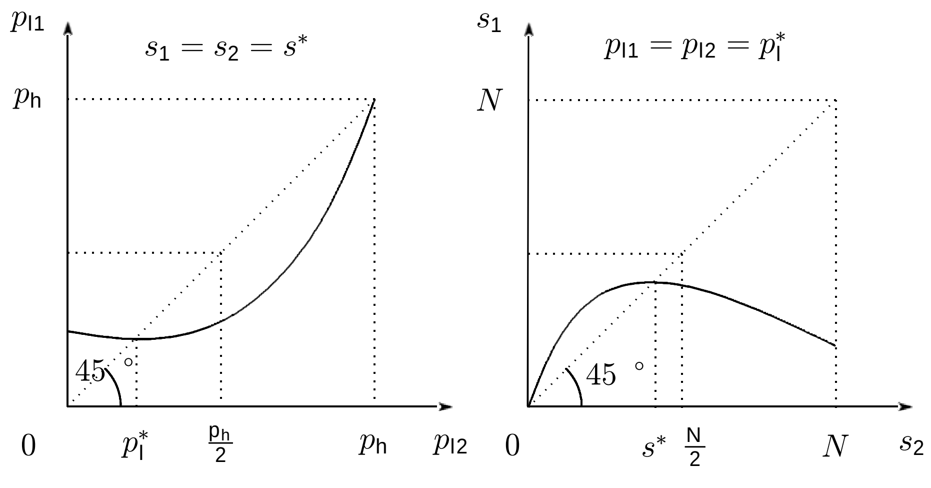

Figure 2: An Example of Symmetric Nash Equilibrium

[10]We will explain the reason for this in the next section.

quantity levels that satisfies (12) or (13) is

$$p_{\mathsf{l}\mathsf{i}} = p_{\mathsf{l}\mathsf{j}} = p_{\mathsf{l}}^* = \left(1 - \frac{N}{4s^*}\right) p_{\mathsf{h}} \quad \text{and} \quad s_{\mathsf{i}} = s_{\mathsf{j}} = s^* = \frac{p_{\mathsf{h}} N}{4(p_{\mathsf{h}} - p_{\mathsf{l}}^*)}. \tag{16}$$

Notice that, in equilibrium, the number of customers in each firm become $n_{\mathsf{i}}^* = n^* = N/2$. Since $s_{\mathsf{i}} \leq n_{\mathsf{i}}(\cdot)$ (i=1,2), s^* must satisfy $s^* \leq N/2$ and hence, from (16), p_{l}^* must satisfy $p_{\mathsf{l}}^* \leq p_{\mathsf{h}}/2$. The low price level, on the other hand, should be nonnegative (i.e., $p_{\mathsf{l}}^* \geq 0$), thus the quantity is bounded below (i.e., $s^* \geq N/4$). These ranges are appeared in Figure 3.

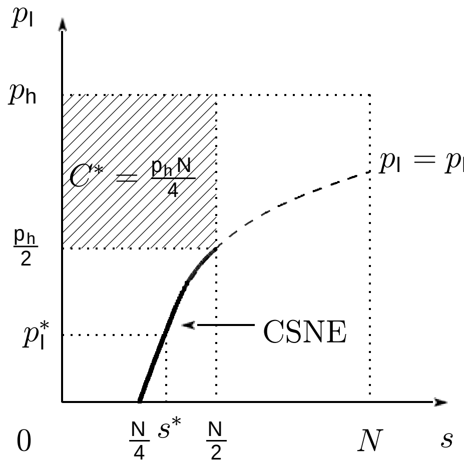

Figure 3: A Continuum of Symmetric Nash Equilibria (CSNE)

From the above discussions, we can establish the following proposition.

PROPOSITION 2.1 There exists a continuum of symmetric Nash equilibria in which the good is sold at high price p_{h} and low price p_{l}^*. Any set of $p_{\mathsf{l}\mathsf{i}} = p_{\mathsf{l}}^* \in [0, p_{\mathsf{h}}/2]$ and $s_{\mathsf{i}} = s^* \in [N/4, N/2]$ for $i = 1, 2$ which satisfies

$$(p_{\mathsf{h}} - p_{\mathsf{l}}^*)s^* = \frac{p_{\mathsf{h}} N}{4}$$

is an equilibrium. The number of customers and the profit of firm i are $N/2$ and $p_{\mathsf{h}} N/4$, respectively, in all equilibria.

The bold line in Figure 3 illustrates a continuum of symmetric Nash equilibria in Proposition 2.1.

3 Existence of Multiple Price Equilibria

3.1 Two-seller Game and Two-price Equilibria

Proposition 2.1 shows that there exists a continuum of **symmetric** Nash equilibria. In this section, we will show that a continuum of **asymmetric** Nash equilibria do exist in which there are any price distributions among low price levels. From (9), the profit maximization problem of firm i can be rewritten as

$$\begin{aligned} \max_{C_i} \ \pi_i(C_i, C_j) &= p_h \left(\frac{NC_i}{C_i + C_j} \right) - (p_h - p_{li}) s_i \\ &= p_h N \left(\frac{C_i}{C_i + C_j} \right) - C_i, \quad i, j = 1, 2, \ i \neq j, \end{aligned} \tag{17}$$

where C_i is the consumers' surplus at firm i, which is defined by (6). This payoff function implies that the firm i gives away the surplus to consumers in order to obtain his customer from the rival store. The set of strategies (p_{li}, s_i) is reduced to the unique strategy variable C_i ($\in (0, p_h N]$). This is the reason for the indeterminacy in (12) and (13)[11].

The first-order condition of this problem is

$$\frac{\partial \pi_i}{\partial C_i} = p_h N \left(\frac{C_j}{(C_i + C_j)^2} \right) - 1 = 0.$$

[11]This profit function is essentially the same as the payoff function of **proportional share** game in the Rent-Seeking literature (see Congleton 1980). The payoff function is

$$P(X) = \frac{X}{X+Y}(R - X - Y),$$

where X is the competitive effort of player X, Y is the level of effort of other players, and R is the resource base of the game. This equation is reduced to

$$P(X) = R\left(\frac{X}{X+Y}\right) - X.$$

Comparing with (17), since the term $p_h N$ is the maximum profit of this game, C_i, C_j, and $p_h N$ can be regarded as X, Y, and R respectively. Therefore, this game can be regarded as a good example of proportional share game. The difference is that the starategic variable C consists of two strategic variables p_l and s in our model.

The second-order condition is satisfied since

$$\frac{\partial^2 \pi_i}{\partial C_i^2} = -2p_h N \left(\frac{C_j}{(C_i + C_j)^3} \right) < 0$$

for every $C_i \in (0, p_h N]$. Hence, the best-response function of firm i as a function of the consumer surplus level of firm j is given by

$$C_i = R_i(C_j) = \sqrt{p_h N C_j} - C_j. \tag{18}$$

The solution of this game is[12]

$$C^* = \frac{p_h N}{4}. \tag{19}$$

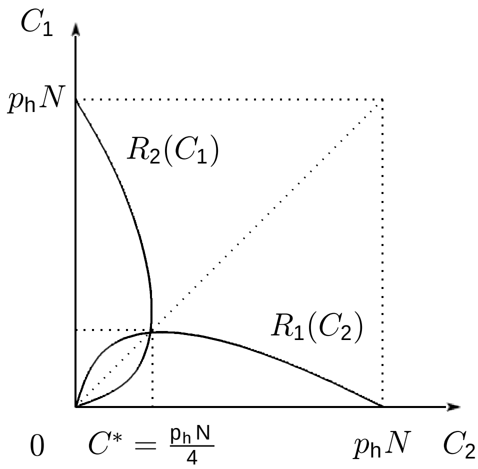

Figure 4: The Best-Response Functions

The original game's strategy is the set of p_{li} and s_i. We find that any set of p_{li} and s_i which satisfy (19) is a Nash equilibrium. In other words, there is a continuum of asymmetric Nash equilibria in the original game.

From the above discussions, we have established the following proposition.

[12]Notice that $C^* = 0$ is another solution of (18). However, it cannot be a Nash equilibrium because the profit of firm i is not continuous at $C_i = 0$ for $C_j = 0$. That is if $C_j = 0$,

$$\pi_i(C_i, 0) = \begin{cases} \frac{p_h N}{2}, & C_i = 0, \\ p_h N - C_i, & C_i > 0. \end{cases}$$

Then, the firm i can obtain larger profit by increasing $C_i (> 0)$.

PROPOSITION 3.1 There exists a continuum of asymmetric Nash equilibria in which the good is sold at one high price p_{h} and two low prices $(p^*_{\mathsf{l}1}, p^*_{\mathsf{l}2})$. Any set of $p_{\mathsf{li}} = p^*_{\mathsf{li}} \in [0, p_{\mathsf{h}}/2]$ and $s_{\mathsf{i}} = s^*_{\mathsf{i}} \in [N/4, N/2],\ i = 1, 2,$ which satisfies

$$C^* = (p_{\mathsf{h}} - p^*_{\mathsf{li}})s^*_{\mathsf{i}} = \frac{p_{\mathsf{h}} N}{4}, \quad i = 1, 2.$$

is an equilibrium. The number of customers and the profit of firm i are $N/2$ and $p_{\mathsf{h}}N/4$, respectively, in all equilibria.

Notice that the symmetric Nash equilibria in Proposition 2.1 is included in the equilibria in Proposition 3.1. Figure 5 illustrates an example of the best response correspondence of firm 1 when firm 2 adopts a set of equilibrium strategies $(p^*_{\mathsf{l}2}, s^*_2)$.

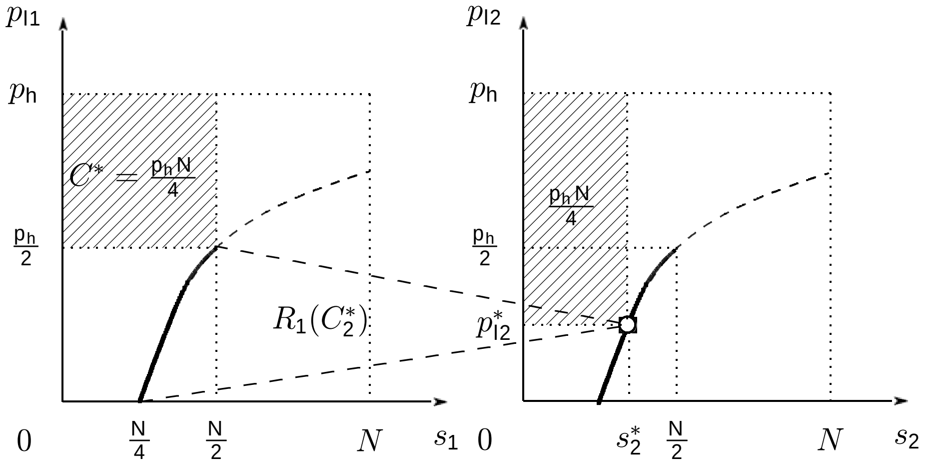

Figure 5: A Continuum of Asymmetric Nash Equilibria

3.2 M-seller Game and M-price Equilibria

Suppose now that the market consists of M (≥ 1) identical firms. We found that, in the two-seller game, the firm's strategy is represented by choosing the consumer surplus level, instead of choosing price and quantity levels independently. In the M-seller game, we need to deduce the firm i's demand function as a function of the consumer surplus levels of all firms. Then (4) can be modified by

$$\frac{C_{\mathsf{i}}}{n_{\mathsf{i}}} = \frac{C_{\mathsf{j}}}{n_{\mathsf{j}}}, \quad i, j = 1, 2, \ldots, M, \ i \neq j. \tag{20}$$

This condition means that the average consumer surplus per capita at the store must be equal among the stores in equilibrium. Although there are M equations in (20), one of them is not independent. Hence, there are $M - 1$ independent equations and $\sum_{\mathsf{i}=1}^{\mathsf{M}} n_{\mathsf{i}} = N$. Solving these $(M-1)+1$ equations with M unknowns, the firm i's demand function can be calculated as

$$n_{\mathsf{i}}(C_{\mathsf{i}}, C_{-\mathsf{i}}) = N\left(\frac{C_{\mathsf{i}}}{C_{\mathsf{i}} + C_{-\mathsf{i}}}\right), \quad \text{where} \quad C_{-\mathsf{i}} = \sum_{\mathsf{j}\neq\mathsf{i}}^{\mathsf{M}-1} C_{\mathsf{j}}. \tag{21}$$

The condition (7) can be rewritten as

$$C_{\mathsf{i}} + C_{-\mathsf{i}} \leq (p_{\mathsf{h}} - p_{\mathsf{li}})N, \text{for } i = 1, 2, \ldots, M,$$

and hence,

$$C_{\mathsf{i}} + C_{-\mathsf{i}} \leq \min\{(p_{\mathsf{h}} - p_{\mathsf{l1}})N, (p_{\mathsf{h}} - p_{\mathsf{l2}})N, \ldots, (p_{\mathsf{h}} - p_{\mathsf{lM}})N\}. \tag{22}$$

ASSUMPTION 3 Each firm takes action within the condition (22).

Using this demand function (21), firm i chooses C_{i} to

$$\max_{C_{\mathsf{i}}} \ \pi_{\mathsf{i}}(C_{\mathsf{i}}, C_{-\mathsf{i}}) = p_{\mathsf{h}}N\left(\frac{C_{\mathsf{i}}}{C_{\mathsf{i}} + C_{-\mathsf{i}}}\right) - C_{\mathsf{i}}.$$

The first order condition is given by

$$\frac{\partial \pi_{\mathsf{i}}}{\partial C_{\mathsf{i}}} = p_{\mathsf{h}}N\left(\frac{C_{-\mathsf{i}}}{(C_{\mathsf{i}} + C_{-\mathsf{i}})^2}\right) - 1 = 0.$$

Hence, the best-response function of firm i as a function of the consumer surplus levels of firm $-i$ is given by

$$C_{\mathsf{i}} = R_{\mathsf{i}}(C_{-\mathsf{i}}) = \sqrt{p_{\mathsf{h}}NC_{-\mathsf{i}}} - C_{-\mathsf{i}}. \tag{23}$$

Since all firms are identical regarding cost structure, we can find that the solution where $C_{\mathsf{i}} = C^*$ for all $i = 1, \ldots, M$. Substituting the common C^* into the already derived best-response functions. We obtain

$$C^* = \sqrt{p_{\mathsf{h}}N(M-1)C^*} - (M-1)C^*.$$

Hence, the solution of this game is

$$C^* = \left(1 - \frac{1}{M}\right)\frac{p_{\mathsf{h}} N}{M}. \tag{24}$$

Similar to the discussion of the two-seller game, $C^* = 0$ could not be a Nash equilibrium since if $C_{-\mathsf{i}} = 0$, the firm i has an incentive to deviate from that state. Notice that, in equilibrium, the number of customers in each firm become $n^*_{\mathsf{i}} = n^* = N/M$. Since $s_{\mathsf{i}} \leq n_{\mathsf{i}}(\cdot)$ (i=1,2, ... ,M), s^*_{i} must satisfy $s^*_{\mathsf{i}} \leq N/M$ and hence, from (24), p^*_{li} must satisfy $p^*_{\mathsf{li}} \leq p_{\mathsf{h}}/M$. The low price level, on the other hand, should be nonnegative (i.e., $p^*_{\mathsf{li}} \geq 0$), thus the quantity is bounded below; i.e., $s^*_{\mathsf{i}} \geq (1 - 1/M)N/M$. We can now establish the following proposition.

PROPOSITION 3.2 There exists a continuum of asymmetric Nash equilibria in which the good is sold at one high price and M low prices $(p^*_{\mathsf{l1}}, \dots, p^*_{\mathsf{lM}})$. Any set of

$$0 \leq p^*_{\mathsf{li}} \leq \frac{p_{\mathsf{h}}}{M}, \quad \text{and} \quad \left(1 - \frac{1}{M}\right)\frac{N}{M} \leq s^*_{\mathsf{i}} \leq \frac{N}{M} \tag{25}$$

which satisfies

$$C^* = (p_{\mathsf{h}} - p^*_{\mathsf{li}})s^*_{\mathsf{i}} = \left(1 - \frac{1}{M}\right)\frac{p_{\mathsf{h}} N}{M}, \quad i = 1, 2, \dots, M.$$

is an equilibrium. The number of customers of firm i is the same in each equilibrium, which is $n^*_{\mathsf{i}} = n^* = N/M$. The profit of the firm i is also the same as

$$\pi^*_{\mathsf{i}} = \pi^* = \frac{p_{\mathsf{h}} N}{M^2}, \quad i = 1, 2, \dots, M.$$

in each equilibrium.

Note that each firm does not necessarily set different prices. Thus, there exists any kind of price distribution in equilibrium.

4 Multiple Price Equilibria and Welfare

4.1 Varying the Number of Sellers

We now investigate the changes in the degree of price dispersion among low price levels as we change the number of firms in the industry. First, note that substituting $M = 1$ into (24) yields $C^* = 0$, that is, the firm does not adopt the discount strategy hence the equilibrium price becomes monopoly price p_{h}. Second, substituting $M = 2$ yields the duopoly solution described in Proposition 3.1.

Now we let the number of firms grow with no bounds. Then, from (25), we obtain

$$\lim_{\mathsf{M}\to\infty} p^*_{\mathsf{li}} = 0 \quad \text{and} \quad \lim_{\mathsf{M}\to\infty} s^*_{\mathsf{i}} = 0.$$

The former equation $\lim p^*_{\mathsf{li}} = 0$ implies that price dispersions disappear in the limit. The latter equation $\lim s^*_{\mathsf{i}} = 0$ occurs as the number of sales itself converges to zero; i.e., $\lim n^* = 0$.

From (25), the range of the total supply of the good at a low price is

$$\left(1 - \frac{1}{M}\right) N \leq S^* < N, \quad \text{where } S^* = \sum s^*_{\mathsf{i}}.$$

Hence, the limit of S^* is

$$\lim_{\mathsf{M}\to\infty} S^* = N.$$

These equations imply that the multiple-price equilibria converge to the unique single-price (i.e., $p^*_{\mathsf{li}} = p^*_{\mathsf{l}} = 0$) equilibrium.

PROPOSITION 4.1 As the number of firms increases,

1. The multiple-price equilibria converge to the unique single-price equilibrium,

2. The variance of price dispersion decreases.

4.2 Welfare Analysis

The consumer surplus from each firm is C^*. Thus, the consumer surplus in this market is

$$CS^*(M) = M \cdot C^* = \left(1 - \frac{1}{M}\right) p_{\mathsf{h}} N.$$

The producer surplus is aggregate profit,

$$PS^*(M) = M\pi^*(M) = \frac{p_h N}{M}.$$

Thus, the social welfare of M-firm equilibrium is

$$\begin{aligned} W^* &= CS^*(M) + PS^*(M) \\ &= \left(1 - \frac{1}{M}\right) p_h N + \frac{p_h N}{M} \\ &= p_h N. \end{aligned}$$

From the above discussions, we can establish the following proposition.

PROPOSITION 4.2 The consumer surplus increases and the producer surplus decreases with respect to M, while total surplus $W^* = p_h N$ is constant.

Notice that if all firms charge the high price, each firm's profit is $p_h N/M$. Therefore, this market has the prisoner's dilemma characteristic as in usual imperfect competition models.

5 Introducing a Cost Function

In multiple-price equilibria, the supremum low price level is at most $p_h/2$. This is not realistic because we observe that, for example, the good is sold at 75% of its regular price, etc. We can, however, explain this by introducing a cost function. The cost function is defined by

$$K(n_i) = kn_i + A,$$

where $k \in [0, p_h)$ and $A > 0$ are the marginal costs and fixed costs, respectively. As in Varian (1980), this function is based on the casual observation that retail stores are characterized by fixed costs of rent and sales force, plus constant variable costs (the wholesale cost) of the good being sold. Since the marginal cost is k, it seems natural that the lower bound of the low price is k (and hence $p_{li} \in [k, p_h)$ and $C_i \in (0, (p_h - k)N])$. Formally, by substituting the cost function into profit, the profit of firm i is

$$\begin{aligned} \pi_i &= p_h n_i - C_i - K(n_i) \\ &= p_h N \left(\frac{C_i}{C_i + C_{-i}}\right) - C_i - k\left(N\left(\frac{C_i}{C_i + C_{-i}}\right)\right) - A. \end{aligned}$$

Then, the firm i chooses $C_i \in (0, (p_h - k)N]$ to

$$\max_{C_i} \ \pi_i(C_i, C_{-i}) = (p_h - k)N \left(\frac{C_i}{C_i + C_{-i}} \right) - C_i - A.$$

Since the marginal cost and the fixed cost are constant, the same arguments can be applied to this problem.

The first order condition is

$$(p_h - k)N \left(\frac{C_{-i}}{(C_i + C_{-i})^2} \right) = 1.$$

The best response function is

$$C_i = R(C_{-i}) \equiv \sqrt{(p_h - k)NC_{-i}} - C_{-i}$$

Therefore, we can establish the following result.

PROPOSITION 5.1 There exists a continuum of asymmetric Nash equilibria in which the good is sold at one high price and M low prices $(p^*_{l1}, \ldots, p^*_{lM})$. Any set of

$$k \leq p^*_{li} \leq \frac{p_h}{M} + \left(1 - \frac{1}{M}\right) k, \quad \text{and} \quad \left(1 - \frac{1}{M}\right) \frac{N}{M} \leq s^*_i \leq \frac{N}{M}. \tag{26}$$

which satisfies

$$C^* = (p_h - p^*_{li})s^*_i = \left(1 - \frac{1}{M}\right) \frac{(p_h - k)N}{M}, \quad i = 1, 2, \ldots, M.$$

is an equilibrium. The number of customers of firm i is the same in each equilibrium, which is $n^*_i = n^* = N/M$. The profit of the firm i is also the same as

$$\pi^*_i = \pi^* = \frac{(p_h - k)N}{M^2} - A, \quad i = 1, 2, \ldots, M.$$

in each equilibrium. Furthermore, the number of firms is determined by $\pi^* = 0$; i.e.,

$$M^* = \sqrt{\frac{(p_h - k)N}{A}}. \tag{27}$$

From (27), the fixed cost A, the price-cost margin $(p_h - k)$, and the number of consumers N determines the number of firms and hence the degree of price dispersion.

6 Concluding Remarks

We found that price dispersion occurs in an oligopolistic retail market with perfect information, homogeneous agents, and without cost functions. The key role of price dispersion is that each firm can choose both price and quantity levels. This generates consumers' expectations of congestion. As a result, the number of customers is determined endogenously in this model. It is worth noticing that, in a multiple-price equilibrium, each different price is paired with each different quantity. In this sense, the good is discriminated.

Appendix

In this Appendix, we consider the case in which the condition (8) doesn't need to be satisfied in equilibrium. We'll show that a continuum of Nash equilibria still exists with that the range of p_{li} narrows from less than $p_{\mathsf{h}}/2$ to less than $p_{\mathsf{h}}/4$ in two seller game. However, we'll also show that the range doesn't change if the game is considered as the each period's outcome of infinitely repeated game with trigger strategy if the discount factor is more than 1/2.

The firm i's demand function

Consider the case of two firms (M=2) for simplicity. Taking into account of the case when $s_{\mathsf{i}} > n_{\mathsf{i}}$, $i = 1$ or 2, the probability of low price at store i is 1. Thus, the utility function (2) is modified to the following:

$$V_{\mathsf{i}}(p_{\mathsf{h}}, p_{\mathsf{li}}, s_{\mathsf{i}}) = \lambda_{\mathsf{i}}(a + y - p_{\mathsf{li}}) + (1 - \lambda_{\mathsf{i}})(a + y - p_{\mathsf{h}}) \tag{28}$$

where $y > 0$ is income, and $a > 0$ is the reservation utility. The term $(y - p)$ represents the residual income when he purchases the good. Furthermore, λ_{i} represents the probability of low price at store i which is defined by

$$\lambda_{\mathsf{i}} = \begin{cases} 1, & \text{if } 0 \leq n_{\mathsf{i}} < s_{\mathsf{i}}, \\ \frac{\mathsf{s_i}}{\mathsf{n_i}}, & \text{if } s_{\mathsf{i}} \leq n_{\mathsf{i}} \leq N. \end{cases} \tag{29}$$

Here, we define a function

$$U_i(n_i) = \begin{cases} (p_h - p_{li}), & \text{if } 0 \leq n_i < s_i, \\ \frac{C_i}{n_i}, & \text{if } s_i \leq n_i \leq N, \end{cases} \tag{30}$$

where C_i denotes consumer surplus at store i;

$$C_i \equiv (p_h - p_{li})s_i = ((a + y - p_{li}) - (a + y - p_h))s_i$$

and $n_i \in [0, N]$ denotes the number of customers who choose the store i (see, Figure 6). Then if $C_1 > 0$ and $C_2 > 0$,

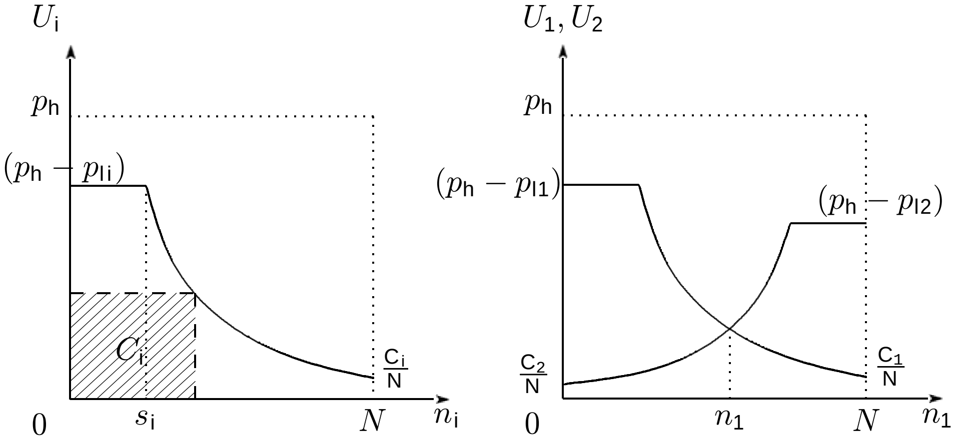

Figure 6: Consumer surplus per customer at firm i

$$V_1(p_h, p_{l1}, s_1) \gtreqless V_2(p_h, p_{l2}, s_2) \quad \Leftrightarrow \quad U_1(n_1) \gtreqless U_2(n_2), \tag{31}$$

and using $n_1 + n_2 = N$,

$$U_1(n_1) \gtreqless U_2(N - n_1). \tag{32}$$

The number of customers at each store changes as long as there is the chance to obtain the larger surplus. Denote $(p_{li}, p_{lj}, s_i, s_j)$ by (p_l, s). From Figure 6, the firm i's demand function $n_i(p_l, s)$ can be defined by (for $i, j = 1, 2,\ i \neq j$);

if $p_{li} < p_{lj}$

$$n_i(p_l, s) = \begin{cases} N\left(\frac{C_i}{C_i+C_j}\right), & \text{if } 0 < C_i \le (p_h - p_{lj})N - C_j, \\ \frac{C_i}{p_h - p_{lj}}, & \text{if } (p_h - p_{lj})N - C_j < C_i \le (p_h - p_{lj})N, \\ N, & \text{if } (p_h - p_{lj})N < C_i, \end{cases} \tag{33}$$

if $p_{li} = p_{lj}$

$$n_i(p_l, s) = N\left(\frac{s_i}{s_i + s_j}\right), \tag{34}$$

if $p_{li} > p_{lj}$

$$n_i(p_l, s) = \begin{cases} N\left(\frac{C_i}{C_i+C_j}\right), & \text{if } 0 < C_j \le (p_h - p_{li})N - C_i, \\ N - \frac{C_j}{p_h - p_{li}}, & \text{if } (p_h - p_{li})N - C_i < C_j \le (p_h - p_{li})N, \\ 0, & \text{if } (p_h - p_{li})N < C_j. \end{cases} \tag{35}$$

Notice that, from the above demand function and figure 6, It can be found that $s_i \le n_i(p_l, s)$, for $i = 1, 2$ if and only if

$$C_i + C_j \le \min\{(p_h - p_{li})N, (p_h - p_{lj})N\},$$

which is the condition (8).

One-shot game

Using the above demand function $n_i(p_l, s)$, firm i maximize his profit function:

$$\pi_i(p_l, s) = \begin{cases} p_{li}s_i + p_h(n_i(p_l, s) - s_i), & \text{if } s_i \le n_i(p_l, s), \\ p_{li}n_i(p_l, s), & \text{if } s_i > n_i(p_l, s). \end{cases}$$

Consider the case $p_{li} < p_{lj}$. From (33), if $(p_h - p_{lj})N - C_j < C_i \le (p_h - p_{lj})N$ (and hence $s_i \le n_i(p_l, s)$), then the profit can be written as:

$$\pi_i(C_i) = \left(\frac{p_h}{p_h - p_{lj}} - 1\right) C_i. \tag{36}$$

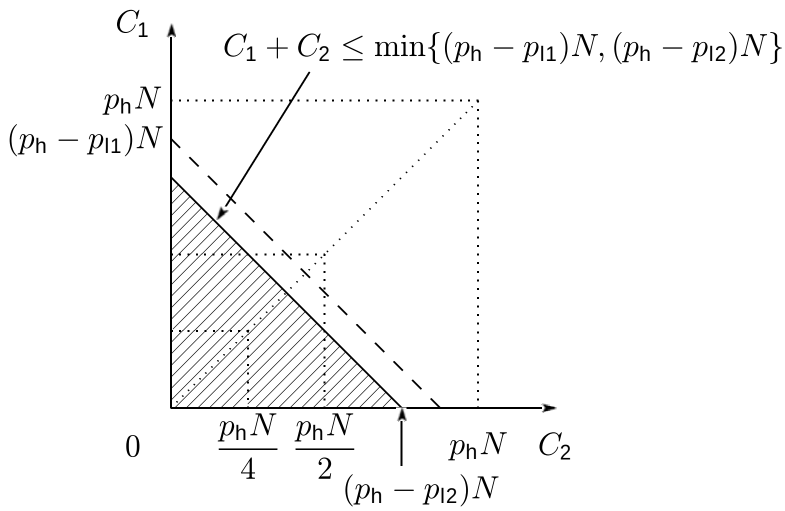

Figure 7: The strategy set under condition (8)

The profit of firm i (36) is maximum at $C_i(p_{lj}) = (p_h - p_{lj})N$;

$$\pi_i(C_i, p_{lj}) = p_{lj}N.$$

Notice that $\pi_j = 0$ since $n_j = 0$ when $C_i = (p_h - p_{lj})N$.

If $C_i > (p_h - p_{lj})N$, the profit can be written as:

$$\pi_i(C_i, p_{lj}) = p_{li}N$$

The supremum of this profit is $p_{lj}N$ since $p_{li} < p_{lj}$. Hence, we can conclude that the optimal strategy and its profit are $\bar{C}_i = (p_h - p_{lj})N$ and $\pi_i = p_{lj}N$ respectively when the condition (8) does not satisfied.

From the above discussion, we have to check whether the Nash equilibria in Proposition 3.1 can be held or not when $\bar{C}_i$ is considered. Since the profit is $p_h N/4$, there is an incentive to change the strategy from $C_i^* = p_h N/4$ to $\bar{C}_i$ if $p_{lj} > p_h/4$. Therefore Proposition 3.1 is held except for changing the range from $p_{li}^* \in [0, p_h/2]$ to $p_{li}^* \in [0, p_h/4]$ and hence the range of s_i^* also changes from $s_i^* \in [N/4, N/2]$ to $s_i^* \in [N/4, N/3]$. There is another Nash equilibrium $p_{li} = p_{lj} = 0$ and $s_i = s_j = N$ which is similar to Bertrand equilibrium.

Repeated game

In one-shot game, the range of Nash equilibria is reduced as in Figure 8. However, it seems natural in real world that the competition is repeated

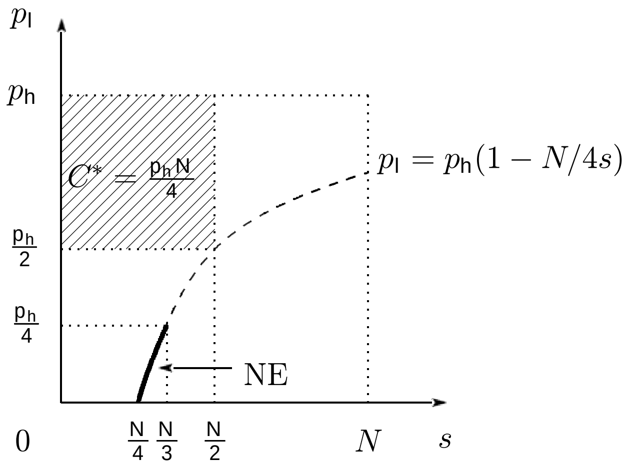

Figure 8: The modified version of Nash equilibria

infinitely and then there is the **implicit collusion**. We'll show that if the time discount factor $\delta \geq 1/2$, then Propositions are held as an outcome of each period's game.

Consider an infinitely repeated game. Suppose that each firm takes the **trigger strategy**; if firm j takes strategy $C_j^* = p_h N/4$, then firm i also takes $C_i^* = p_h N/4$ and if firm j takes the strategy $\bar{C}_j$, then the firm i takes $(p_{li}, s_i) = (0, N)$ after next period. Thus, for firm i, the gain from deviation of C_i^* is;

$$p_{li} N - \frac{p_h N}{4}. \tag{37}$$

On the other hand, the discounted profit of keeping the strategy C_i^* is;

$$\frac{\delta}{(1-\delta)} \cdot \frac{p_h N}{4}.$$

where δ is the time discount factor. Hence, both firms do not deviate the Nash equilibrium $C^* = p_h N/4$ as long as

$$p_{li} N - \frac{p_h N}{4} \leq \frac{\delta}{(1-\delta)} \cdot \frac{p_h N}{4}. \tag{38}$$

The maximum of LHS is $p_h N/4$ since $p_{li}^* \in [0, p_h/2]$. Hence, (38) is satisfied if

$$1 \leq \frac{\delta}{(1-\delta)}$$

Therefore, $\delta \geq 1/2$ is the sufficient condition of Nash equilibrium as the implicit collusion.

The above discussion and the condition $\delta \geq 1/2$ give the microfoundation of Assumption 2.

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