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Public Capital Accumulation, Income Transfers, and Fiscal Sustainability*

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Abstract

In this study, we present an endogenous growth model comprising public investment and income transfer, with focus on fiscal sustainability. The main finding of this paper is that the effects of an increase in the public expenditure/GDP ratio on fiscal sustainability evidently depend on how expenditures function. An increase in the income transfer/GDP ratio may create a more sustainable economy, whereas, as previous studies have shown, an increase in the public investment/GDP ratio creates a less sustainable economy. The growth effects of changes in the two ratios are also examined.

Keywords: Public capital accumulation; income transfers; debt sustainability
JEL classification: E62, H40, H63, O41

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1 Introduction

The effect of government policy on fiscal sustainability is an age-old topic; it not only fascinates academic curiosity, but also attracts the attention of policymakers who cope with serious fiscal sustainability problems. The classic analysis of Domar (1944) presents the importance of economic growth for public debt accumulation sustainability, and Bohn (1998) proposes a new condition for fiscal sustainability associated with the primary surplus and debt-GDP ratios. Recent theoretical studies have analyzed the effects of government policy on fiscal sustainability by introducing practical policy rules. Using the model of exogenous growth, Chalk (2000) and Rankin and Roffia (2003) derive a finite maximum sustainable level of debt. Departing from the exogenous growth model and using an endogenous growth model, Bräuninger (2005), Yakita (2008), and Arai (2011) also investigate the conditions for fiscal sustainability, in which the government is supposed to spend a given pro! portion of GDP and to borrow a fixed fraction of GDP. [1] The sustainability analysis under some exogenous policy rules works for finding out the effects of practical policies, such as Maastricht treaty in EU and the roof constraint on public expenditure and flotation of government bond introduced in countries being troubled over excessive public debt.

Our paper also concerns with the relationship between government policy and fiscal sustainability in the endogenous growth model. Our paper, however, differs from precedent studies in breakdown of public spending and aims to open up the option for government's policy instruments. While Bräuninger (2005) assumes that government expenditures virtually play no role in the economy, Yakita (2008) and Arai (2011) study the fiscal sustainability in a model with productivity-enhancing public expenditure. A critical difference between Yakita (2008) and Arai (2011) is that the former assumes the investment in public capital formation based on a model of Futagami et al. (1993), and the latter assumes the public expenditure based on Barro (1990)'s model in which the public expenditure is treated as a flow variable. Departing from the simple setting of productivity-enhancing public expenditure, we incorporate the spending for income transfer policy. To figure out the essence of our main argument, we simply assume that the government allocates its financial resources between productivity-enhancing investment and income transfer. The income transfer

[1] Some other studies on the effects of debt policy on growth and welfare should be mentioned. For instance, Futagami et al. (2008) use an endogenous growth model with productive public spending to prove that two balanced growth paths exist, one saddle-point stable, and the other saddle-point stable, or asymptotically stable. They then show that a deficit-financed increase in public spending impacts the two balanced growth rates oppositely. Greiner (2008) also uses an endogenous growth model with public capital to study the growth and welfare effects of debt policies. The critical difference of the two papers is the policy rule. Futagami et al. assume that government debt converges to a certain exogenously given debt ratio, whereas Greiner (2008) follows Bohn's (1998) rule: debt policy is managed under the rule whereby the primary surplus moves positively on public debt, which is justified by some empirical studies [Bohn (1998) and Greiner et al. (2007)]. Ghosh and Mourmouras (2004) analyze an economy in which the government provides productive government spending in a debt-financed economy from a welfare perspective. They show that the government's less strict budgetary stance reduces the welfare level.

policies, in the form of social aid, child and social benefit, and social security paycheck, account for a measurable share of government budget in modern welfare states. What the precedent studies pass over is the effects on the long-run fiscal sustainability of increase in the share of social welfare and the effects of allocation between public investment and direct income transfers to households. The aim of this paper is thus to redeem and generalize the work of Bräuninger (2005), Yakita (2008), and Arai (2011) by incorporating an alternative policy option of governments.

This extension can be evaluated from several standpoints. First, empirical researchers wishing to test the fiscal sustainability desire to use model which accounts not only productivity-enhancing policies but the policies with a high regard for income support. The results would be biased more than a bit if we count out the effects of income redistribution policy in the modern welfare states. This paper offers a simple but tractable model to distinguish the role of public investment and income transfers. Second, researchers in applied work need a model that clearly distinguishes two types of public expenditures since the model with income transfers would be easily applied to various issues on income redistribution. Although we do not explicitly formulate public pension, welfare payments, and health-care expenditures, the income transfers policy modeled in this paper explores an aspect of these practical policies.

The main argument of our paper is that the effects of increase in public expenditure/GDP ratio on the fiscal sustainability evidently depend on how expenditures work. Our analysis follows the capital accumulation model of Yakita (2008), which shows that the increase in expenditure/GDP ratio impacts negatively on the fiscal sustainability since it increases the marginal productivity of private capital and thus induces interest-rate runup. An increase in interest rates is conductive to further dependence on government bond for interest payment. From this standpoint, the decrease in public expenditure/GDP ratio is required for fiscal sustainability. In contrast, in our model, the public resources are spent not only for public capital accumulation but for income transfers, which increases the private saving through income effect, and therefore it reduces the interest rate eventually. An interest-rate reduction works positively on maintaining fiscal sustainability. In fact, our analysis reveals that the decrease in public expenditure/GDP ratio has the negative effects on fiscal sustainability if public expenditures are used for something that increases the interest rate, but it works positively on fiscal sustainability if the expenditures are made available for decreasing the interest rate.

Although the policy of income transfers to young generation is incorporated in our study, we should not take income transfers to young in a restricted sense. The point is to spare a thought of public spending that leads to encourage younger generation's willingness to save, which results in a reduction in interest rate. For instance, various policies for savings promotion, such as government-led advantageous interest rate offers and preferred charge for service for young, fall into our argument. The employment-generating projects for young generation are other possible case, which will contribute enhancing savings of young in a broad sense. Suppose that there are employed and unemployed workers in young generation. If unemployed succeeds in getting away from unemployment pool, they are now able to save, which increase the total saving in the young generation.[2] After all, we can qualify public spending that enhance young generation to save as expenditure item which we add in our model.

The paper is organized as follows. Section 2 presents the model, and section 3 derives the sustainability condition. Section 4 is devoted to examining how the changes in policy rules effect on the fiscal sustainability. After confirming the result of Yakita (2008), i.e., the increase in the ratio between public expenditure (investment) and GDP has negative effects on fiscal sustainability, this section shows that the increase in the ratio between public expenditure as a income transfer and GDP might have positive impacts on fiscal sustainability if households have a low discount factor and/or the debt financing ratio is sufficiently small. The effects of changes in policy rule on the long-run growth rate are discussed in section 5. Section 6 concludes the paper.

2 The Model

We follow Yakita (2008) and use a standard overlapping generations model, which consists of firms, a government, and households. Public capital acts as the growth engine of this economy.

2.1 Households

Individuals live for two periods in our overlapping generations economy. Each individual supplies one unit of labor in the first period, and retires in the second period. We assume that the population of each generation is constant over time. The utility function of each individual belonging to generation t is given by $u_t = (1-\delta)\ln c_t^y + \delta \ln c_{t+1}^o$, where c_t^y and c_{t+1}^o represent the consumption in the young and old periods, respectively, and $\delta \in (0,1)$ denotes the time preference rate. An individual in the young period allocates the sum of his/her disposable income and a lump-sum transfer to consumption, c_t^y, and saving s_t. In the retirement period, the individual consumes using the net returns of savings. The budget constraints of the individual in the first and second periods are given by $c_t^y + s_t = (1-\tau_t)w_t + p_t$ and $c_{t+1}^o = [1 + (1-\tau_{t+1})r_{t+1}]s_t$, respectively, where p_t denotes the lump-sum income tran! sfer from the government. From these equations, we derive the lifetime budget constraint:

$$c_t^y + \frac{c_{t+1}^o}{1 + (1-\tau_{t+1})r_{t+1}} = (1-\tau_t)w_t + p_t. \tag{1}$$

[2] This argument conflict with Yakita (2014, p.89)'s argument. In his model, all public expenditures are financ! ed by debt issuance only. In this case, to finance the lump-sum transfers, the government must increase newly issued public bonds, which crowds out private capital. This results in an increase the return rate to capital.

From the first-order conditions for utility maximization, the demand for consumption in the young and old periods and for savings are respectively derived as

$$c_t^y = (1-\delta)[(1-\tau_t)w_t + p_t], \tag{2}$$
$$c_{t+1}^o = \delta[1+(1-\tau_{t+1})r_{t+1}][(1-\tau_t)w_t + p_t], \tag{3}$$
$$s_t = \delta[(1-\tau_t)w_t + p_t]. \tag{4}$$

2.2 Firms

A representative firm uses physical capital K_t and labor L_t to produce homogeneous goods. The production function is assumed as $Y_t = AK_t^\alpha(E_tL_t)^{1-\alpha}$, where E_t denotes labor efficiency. To simplify the notation, we assume $A = 1$ in the following analysis. From the profit-maximizing conditions of the firm in competitive markets, the interest rate and wage rate are respectively given as $r_t = \partial Y_t/\partial K_t = \alpha Y_t/K_t$ and $w_t = \partial Y_t/\partial L_t = (1-\alpha)Y_t/L_t$. We assume that labor efficiency equals public capital stock per unit of labor, $E_t \equiv G_t/L_t$. Using this assumption, we yield the aggregate production function as $Y_t = K_t^\alpha G_t^{1-\alpha}$. Assuming that factor markets are competitive, the interest rate, r_t, and wage rate for labor, w_t, equal the marginal productivity:

$$r_t = \alpha\left(\frac{G_t}{K_t}\right)^{1-\alpha} = \alpha\frac{Y_t}{K_t}, \tag{5}$$
$$w_t = (1-\alpha)\left(\frac{G_t}{K_t}\right)^{-\alpha}\frac{G_t}{L_t} = (1-\alpha)\frac{Y_t}{L_t}. \tag{6}$$

2.3 Government

The government levies a tax on income. We assume that the government spends a given proportion $\theta \in (0,1)$ of GDP, θY_t, on public investment, which contributes to public capital accumulation. The evolution of public capital is, thus, specified as

$$G_{t+1} = G_t + \theta Y_t. \tag{7}$$

In addition to public investment, the government provides a lump-sum income transfer to households. The income transfer amount, P_t, is constrained by $P_t = \eta Y_t$, implying that the government transfers income at a constant fraction of GDP, $\eta \in (0,1)$. We assume that $\theta + \eta < 1$.

The government finances a proportion of the expenditure, $\lambda \in (0,1)$, by issuing bonds, $\lambda(\theta+\eta)Y_t$. Thus, the dynamic equation of public debt, D_t, is given by

$$D_{t+1} = D_t + \lambda(\theta + \eta)Y_t. \tag{8}$$

The government finances its budget through public debt and tax revenues, and spends on interest payments of public debt, public capital investment, and lump-sum transfers to households:

$$(D_{t+1} - D_t) + \tau_t(w_t + r_t s_{t-1})N = r_t D_t + (G_{t+1} - G_t) + \eta Y_t, \tag{9}$$

where N denotes the (constant) total population, τ_t is the income tax rate in period t. Substituting (7) and (8) into (9), the budget constraint of the government is rewritten as

$$\lambda(\theta + \eta)Y_t + \tau_t(w_t + r_t s_{t-1})N = r_t D_t + (\theta + \eta)Y_t. \tag{10}$$

Here, using the individuals' budget constraints for the young and old periods and the government budget constraint (10), we obtain the resource constraint at period t as

$$Y_t = c_t^y N + c_t^o N + G_{t+1} - G_t + K_{t+1} - K_t,$$

where c_t^y and c_t^o represent the consumption of young and old individuals at period t, respectively.

2.4 Dynamics

The capital market in equilibrium satisfies

$$D_{t+1} + K_{t+1} = s_t L_t. \tag{11}$$

Using (4) and (6), (11) is rewritten as

$$K_{t+1} = \delta[(1 - \tau_t)(1 - \alpha)Y_t + P_t] - D_{t+1}. \tag{12}$$

Dividing both sides of (12) by K_t and using (8) and $P_t = \eta Y_t$, we obtain the evolution of private capital as

$$\frac{K_{t+1}}{K_t} = [\delta(1 - \tau_t)(1 - \alpha) + \delta\eta - \lambda(\theta + \eta)] \left(\frac{G_t}{K_t}\right)^{1-\alpha} - \frac{D_t}{K_t}. \tag{13}$$

The evolution of public capital is derived by dividing both sides of (7) by G_t:

$$\frac{G_{t+1}}{G_t} = 1 + \theta\frac{Y_t}{G_t}. \tag{14}$$

The evolution of public debt is also obtained by dividing both sides of (8) by D_t:

$$\frac{D_{t+1}}{D_t} = 1 + \lambda(\theta+\eta)\frac{Y_t}{D_t}. \tag{15}$$

Using (5), (6), and (10), we obtain the relationship between the tax rate and the public debt/private capital ratio as

$$1-\tau_t = \frac{1-(1-\lambda)(\theta+\eta)}{1+\alpha(D_t/K_t)}. \tag{16}$$

Defining $g_t \equiv G_t/K_t$ and $z_t \equiv D_t/K_t$, and using (13)-(16), we depict the dynamics of the economy using two dynamic equations:

$$\frac{g_{t+1}}{g_t} = \frac{1+\theta g_t^{-\alpha}}{\zeta_t(z_t)g_t^{1-\alpha} - z_t}, \tag{17}$$

$$\frac{z_{t+1}}{z_t} = \frac{1+\lambda(\theta+\eta)\frac{g_t^{1-\alpha}}{z_t}}{\zeta_t(z_t)g_t^{1-\alpha} - z_t}, \tag{18}$$

where

$$\zeta_t(z_t) \equiv \frac{\delta(1-\alpha)[1-(1-\lambda)(\theta+\eta)]}{1+\alpha z_t} + \eta(\delta-\lambda) - \lambda\theta.$$

In the following analysis, to consider reasonable equilibria, we assume that $\zeta_t(z_t) > 0$.

On the balanced growth path, we have $g_{t+1} = g_t$ and $z_{t+1} = z_t$. From (17), (18), $g_{t+1} = g_t$ and $z_{t+1} = z_t$, at the steady state, we have

$$g_t = \frac{\theta}{\lambda(\theta+\eta)} z_t. \tag{19}$$

3 Debt Sustainability Condition

The steady-state equilibrium satisfies (17), (18), $g_{t+1} = g_t$ and $z_{t+1} = z_t$. For a graphical representation of the steady-state equilibrium, we first use (17) and $g_{t+1} = g_t$ to obtain

$$1 + \theta g_t^{-\alpha} + z_t = \zeta_t(z_t) g_t^{1-\alpha}. \tag{20}$$

We define the left-hand side of (20) as $\beta(g_t, z_t)$, and the right-hand side as $\varepsilon(g_t, z_t)$. Fig.1 depicts $\beta(g_t, z_t)$ and $\varepsilon(g_t, z_t)$ for a given size of the public capital/private capital ratio, g_t, which shows that the two curves labeled as $\beta(g_t, z_t)$

and $\epsilon(g_t, z_t)$, intersect each other, and thus (20) is satisfied at $z_t = z_{1t}$. As the public capital/private capital ratio increases to g_t', the $\varepsilon(\cdot)$ curve shifts up and the $\beta(\cdot)$ curve shifts down, indicating that the debt/private capital ratio that satisfies (20) increases from z_{1t} to z_{1t}'. Hence, equation (20) can be depicted using the upward-sloping curve represented by GG in the $g_t - z_t$ plane in Fig.3. This is formally confirmed using (20) to derive $(dg_t/dz_t)|_{GG}$ as follows.[3]

$$\frac{dg_t}{dz_t}|_{GG} = \frac{1 - \zeta_t'(z_t) g_t^{1-\alpha}}{[(1-\alpha) g_t \zeta_t(z_t) + \alpha\theta] g_t^{-(1+\alpha)}} > 0, \tag{21}$$

where $\zeta_t'(z_t) \equiv -\alpha\delta(1-\alpha)[1-(1-\lambda)(\theta+\eta)]/(1+\alpha z_t)^2 < 0$.

In a similar manner, using (18) and $z_{t+1} = z_t$, we have

$$1 + \lambda(\theta+\eta) g_t^{1-\alpha} z_t^{-1} + z_t = \zeta_t(z_t) g_t^{1-\alpha}. \tag{22}$$

Defining the left-hand side of (22) as $\chi(g_t, z_t)$, and because the right-hand side of (22) is identical to $\varepsilon(g_t, z_t)$ defined in (20), we can depict the two functions as in Fig.2, which shows that there are two intersections. We denote z_{1t} and z_{2t} as the public debt/private capital ratio levels that satisfy (22).[4]

An increase in the public capital/private capital ratio to g_t' shifts the $\chi(\cdot)$ and $\varepsilon(\cdot)$ curves upward, with the shift of the $\varepsilon(\cdot)$ curve greater than that of the $\chi(\cdot)$ curve.[5] Thus, an increase in g_t causes a reduction in the level of z_{1t} and an increase in the level of z_{2t}. Therefore, we can depict the ZZ curve, which represents (22), on the $g_t - z_t$ plane in Fig.3 as a U-shaped ZZ curve.

Next, we confirm the slope of the curve ZZ. From (22), we have

$$\frac{dg_t}{dz_t}|_{ZZ} = \frac{\left[\frac{\alpha\delta(1-\alpha)(1-(1-\lambda)(\theta+\eta))}{(1+\alpha z_t)^2} - \frac{\lambda(\theta+\eta)}{z_t^2}\right] g_t^{1-\alpha} + 1}{(1-\alpha)\left[\zeta_t(z_t) - \frac{\lambda(\theta+\eta)}{z_t}\right] g_t^{-\alpha}} \gtrless 0. \tag{23}$$

The sign of (23) depends on the sign of the numerator, because the sign of the denominator is positive.[6] The numerator in (23) tends to have a negative sign when z_t is small, but it is likely to have a positive sign when z_t is large, implying that the ZZ curve has features of a U-shaped curve.

The dynamic equations on g_t and z_t are depicted in a phase diagram of Fig.3. The intersections of the GG curve and ZZ curve in these figures depict the steady-state values of g_t and z_t. In Fig.3, there are two intersections, in which

[3] z_t and g_t have an upper and lower limit, respectively. See Appendix A.

[4] To ensure at least one intersection, we must define the lower limit of g_t. $\chi_t \to 1 + z_t$ as $g_t \to 0$, and the interception of $\chi(g_t, z_t)$ as 1 when $z_t = 0$. Hence, an interception of $\varepsilon(\cdot)$ must be greater than 1 to ensure an intersection of the $\chi(\cdot)$ and $\varepsilon(\cdot)$ curves, indicating that the lower limit $\underline{\underline{g_t}}$ must satisfy $1 < [\delta(1-\alpha)(1-(1-\lambda)(\theta+\eta)) + \eta(\delta-\lambda) - \lambda\theta]\, \underline{\underline{g_t}}^{1-\alpha}$.

[5] See Appendix B.

[6] Equation (22) can be written as $g_t^{1-\alpha} = z_t(1+z_t)[z_t\zeta_t(z_t) - \lambda(\theta+\eta)]^{-1} > 0$. Hence, the sign of the denominator in (23) is positive.

point A is a sink and point B is a saddle point (see Appendix C).[7] The economy with the initial public capital/private capital ratio g_0 and public debt/private capital ratio z_0 starting from the point below the level of the stable arm, the economy converges to the equilibrium A. The economy with (g_0, z_0) starting from the point on the stable arm, the economy converges to the equilibrium B. In contrast, the economy with (g_0, z_0) starting from the point higher than the level of the stable arm which converges to the saddle-point steady state, the economy cannot converge to the steady states and the economy is unsustainable relative to public debt. Thus, some policy rule that sifts z_B to the leftward and narrows the range between z_A and z_B, such policies create an unsustainable economy.

4 Policy Effects

We analyze the interaction between policy variable and debt sustainability. First, we clarify how the GG and ZZ curves shift when the policy captured by the variable λ changes. This affirms the results presented by Yakita (2008). From (20) and (22), we have

$$\frac{dg_t}{d\lambda}|_{GG} = \frac{\left[1 - \frac{\delta(1-\alpha)}{1+\alpha z_t}\right](\theta+\eta)g_t^2}{(1-\alpha)g_t\zeta_t(z_t) + \alpha\theta} > 0, \tag{24}$$

$$\frac{dg_t}{d\lambda}|_{ZZ} = \frac{\left[1 - \frac{\delta(1-\alpha)}{1+\alpha z_t} + \frac{1}{z_t}\right](\theta+\eta)g_t}{(1-\alpha)\left[\zeta_t(z_t) - \frac{\lambda(\theta+\eta)}{z_t}\right]} > 0, \tag{25}$$

indicating that the GG and ZZ curves in Fig.3 shift upwards. Then, from (24) and (25), we find that the magnitude of the shift of the ZZ curve is greater than that of the GG curve;

$$\frac{dg_t}{d\lambda}|_{ZZ} - \frac{dg_t}{d\lambda}|_{GG} = \frac{\theta(\theta+\eta)\left(1 - \frac{\delta(1-\alpha)}{1+\alpha z_t}\right) + \frac{\theta+\eta}{z_t}[(1-\alpha)g_t\zeta_t(z_t) + \alpha\theta]}{\frac{(1-\alpha)}{g_t}\left[\zeta_t(z_t) - \frac{\lambda(\theta+\eta)}{z_t}\right][(1-\alpha)g_t\zeta_t(z_t) + \alpha\theta]} > 0.$$

Hence, an increase in λ is described by the shift of the GG and ZZ curves to $G'G'$ and $Z'Z'$, which shifts the upward-sloping stable arm which depicted as the dotted curve leftward, as in Fig.3, implying that the economy is becoming unsustainable. This is because if the economy with (g_0, z_0) starts from the point larger than the level of the stable arm that converges to the saddle-point equilibrium B in Fig.3, the economy cannot converge to the steady states, and hence, the economy is not sustainable relative to public debt.

[7] There is an exceptional case whereby only one equilibrium exists in the economy even if both (18) and $z_t = z_{t+1}$ are satisfied.

We now focus on the effects of a change in the ratio of spending for income transfers, η. The shifts of the GG and ZZ curves with a change in η can be checked with the following equations:

$$\frac{dg_t}{d\eta}|_{GG} = \frac{-\left[\delta - \lambda - \frac{\delta(1-\alpha)(1-\lambda)}{1+\alpha z_t}\right] g_t^2}{(1-\alpha)g_t\zeta_t(z_t) + \alpha\theta} \lesseqgtr 0, \tag{26}$$

$$\frac{dg_t}{d\eta}|_{ZZ} = \frac{-\left[\delta - \lambda - \frac{\delta(1-\alpha)(1-\lambda)}{1+\alpha z_t} - \frac{\lambda}{z_t}\right] g_t}{(1-\alpha)\left[\zeta_t(z_t) - \frac{\lambda(\theta+\eta)}{z_t}\right]} \lesseqgtr 0. \tag{27}$$

Furthermore, from (26) and (27), we have

$$\frac{dg_t}{d\eta}|_{ZZ} - \frac{dg_t}{d\eta}|_{GG} = \frac{-\left[\delta - \lambda - \frac{\delta(1-\alpha)(1-\lambda)}{1+\alpha z_t}\right]\left[\alpha\theta + \frac{\lambda(\theta+\eta)(1-\alpha)}{z_t}\right]}{\frac{(1-\alpha)}{g_t}\left[\zeta_t(z_t) - \frac{\lambda(\theta+\eta)}{z_t}\right][(1-\alpha)g_t\zeta_t(z_t) + \alpha\theta]} \lesseqgtr 0.$$

The levels of δ and λ are critical for determining how the GG and ZZ curves shift with an increase in η. We use three cases to study the effects of an increase in η on fiscal sustainability.[8]

Case 1; Figure 4(a). $\delta < \frac{\lambda}{1-\frac{(1-\alpha)(1-\lambda)}{1+\alpha z_t}}$. In this case, the GG and ZZ curves shift upward and the magnitude of the shift in the ZZ curve is larger than that in the GG curve, indicating that an increase in η renders the economy more unsustainable.

Case 2; Figure 4(b). $\frac{\lambda}{1-\frac{(1-\alpha)(1-\lambda)}{1+\alpha z_t}} < \delta < \frac{\lambda\left(1+\frac{1}{z_t}\right)}{1-\frac{(1-\alpha)(1-\lambda)}{1+\alpha z_t}}$. In this case, the GG curve shifts downward and the ZZ curve shifts upward, directly making the economy more unsustainable.

Case 3; Figure 4(c). $\frac{\lambda\left(1+\frac{1}{z_t}\right)}{1-\frac{(1-\alpha)(1-\lambda)}{1+\alpha z_t}} < \delta$. In this case, GG curve and ZZ curve shift downward, and the magnitude of the shift in the ZZ curve is larger than that in the GG curve, indicating that an increase in η renders the economy more sustainable.

In Cases 1 and 2, the increase in income transfer narrows the range between z_{1t} and z_{2t}, and thus, such an income transfer policy renders the fiscal budget unsustainable. In a direction opposite to these cases, in Case 3, an increase in spending for income transfers renders the fiscal budgets sustainable. Based on this arrangement, we conclude that if δ is sufficiently large and λ is sufficiently low to satisfy $\frac{\lambda\left(1+\frac{1}{z_t}\right)}{1-\frac{(1-\alpha)(1-\lambda)}{1+\alpha z_t}} < \delta$, an increase in η leads to a more sustainable economy.

[8] We exclude the case where the GG curve shifts upward and the ZZ curve shifts downward because this never happens as $\lambda < \lambda(1 + z_t^{-1})$ holds for any given value of z_t.

The intuition behind this result is simple. An increase in the income transfer ratio η promotes further debt accumulation, which directly leads to an unsustainable economy. In contrast, an increase in η increases private savings through the income effect. When δ is sufficiently large, households put a high weight on the consumption in the second period, and hence, from (4), private savings increase. Thus, an increase in η stimulates private savings in Case 3. The increase in savings lowers the interest rate in the capital market, which contributes to a reduction in interest payments. In Case 3, the latter effect exceeds the former, and thus the economy becomes more sustainable.

5 Effects on the Growth Rate

To study the effects of changes in λ and η on the economic growth rate, we first use (13) to derive the growth rate in period t, $1+\gamma = \zeta_t(z_t)g_t^{1-\alpha} - z_t$. The comparative statistics on λ are provided by

$$\frac{d\gamma}{d\lambda} = \frac{\alpha(\theta+\eta)g_t^{-\alpha}}{H}\left\{1-\zeta_t'(z_t)+\theta g_t^{-\alpha}\left[1-\frac{\delta(1-\alpha)}{1+\alpha z_t}\right]\right\}, \tag{28}$$

where

$$H \equiv \lambda(\theta+\eta)\theta^{-1}[1-\zeta_t'(z_t)g_t^{1-\alpha}] - g_t^{-(1+\alpha)}[(1-\alpha)g_t\zeta_t(z_t)+\alpha\theta]. \tag{29}$$

Since the numerator in (28) is positive, the sign of (28) is determined by the sign of H. Because the slope of the GG curve at equilibrium B in Figs.4(a)-(c) is steeper than $\lambda(\theta+\eta)/\theta$ and gradual at equilibrium A, the sign of H is positive at equilibrium B and negative at equilibrium A.[9] We further find that $d\gamma/d\lambda > 0$ holds at equilibrium B and $d\gamma/d\lambda < 0$ holds at equilibrium A, which replicates the result of Yakita (2008).

The effects of an increase in income transfer ratio on the growth rate can also be obtained as follows.

$$\frac{d\gamma}{d\eta} = \frac{\alpha g_t^{-\alpha}}{H}\left\{\lambda[1-\zeta_t'(z_t)g_t^{1-\alpha}] - \frac{\theta}{g_t^{\alpha}}\left[\delta - \frac{\delta(1-\alpha)(1-\lambda)}{1+\alpha z_t} - \lambda\right]\right\}.$$

H is positive at equilibrium B and negative at equilibrium A. The first term in the angle bracket is positive. Hence, an increase in η increases the growth rate over the range of

$$\frac{(1+z_t)\alpha\delta}{1+\alpha z_t+\delta(1-\alpha)} \le \lambda, \tag{30}$$

at point B and reduces that at point A in Figures 4(a)-4(c).

[9] See Appendix D.

The policy effects on the growth rate can be explained as follows. Let us focus on the case of point B in Figures 4(a)-4(c), and thus on $H > 0$. A small increase in the income transfer, represented by η, increases an individual's incentive to save through the income effects. In contrast, an increase in η raises the tax rate, given other variables [see (16)], which constitutes a limiting factor of individual savings. The former effect tends to exceed the latter when the government operates a bond-dependent budget, in other words, when λ is large. This is because the negative effects of tax increases on individual savings are relatively small when the government heavily relies on debt-financing and, thereby, the tax rate is low. Thus, when λ is sufficiently large to satisfy (30), the positive effect of an increase in η on savings exceeds the negative effect, and an increase in the transfer policy increases the savings and theref! ore accelerates the capital accumulation and growth rate. In contrast, when the savings are contained through a tax increase associated with an increase in spending for an income transfer, an increase in η reduces the growth rate.

6 Conclusion

In this paper, we formulated an endogenous growth model comprising public capital investment and income transfer, with focus on fiscal sustainability. While public expenditure has had limited use on productivity-enhancing investment in previous studies, the main focus of this paper is the effect of an expansion of public expenditures for income transfers on fiscal sustainability. We find that an increase in public expenditure for income transfers does not necessarily create an unsustainable environment because income transfers increase private savings and thereby reduce the interest rate, which mitigates interest payments. A noteworthy feature of this paper is that in describing public expenditure, we formally incorporate two types of public expenditures, enabling us to examine the various effects of expansion of public expenditure on fiscal sustainability and long-run economic growth.

The key to our argument is the public expenditure that promotes savings in young generation. Although we take income transfers to young generation for simple example to derive our main finding, we should not restrict the policy instrument in a narrow sense. Any public expenditure that enhances young generation's savings fall into our model, and thus, an increase in the public expenditure/GDP ratio may create a more sustainable economy relative to public debt.

Before concluding this paper, we discuss some problems that remain to be solved. First, because our main interest is to present a simple model examining the effects of public expenditure on fiscal sustainability, we assume that all policy variables, θ, η, and λ are exogenous: the current model considers neither objective of the government. Incorporation of endogenous policy choices could provide insightful information related to the implications of the optimal policies adopted. Second, a thorough empirical examination can be conducted on the basis of this model. An important research topic from both academic and practical perspectives would be to test how changes in public expenditure level affect the sustainability and growth paths. These are beyond of the scope of this study and remain unresolved issues.

Appendices

Appendix A. *Upper and Lower Limits of z_t and g_t.*

Because the right-hand side of (21) is positive, z_t has an upper limit

$$\overline{z} = \frac{\delta(1-\alpha)[1-(1-\lambda)(\theta+\eta)] + \eta(\delta-\lambda) - \lambda\theta}{\alpha[\lambda\theta - \eta(\delta-\lambda)]},$$

whereas g_t has a lower limit $\underline{g}$ when $z_t = 0$, which satisfies

$$1 + \theta\underline{g}^{-\alpha} = [\delta(1-\alpha)(1-(1-\lambda)(\theta+\eta)) + \eta(\delta-\lambda) - \lambda\theta]\underline{g}^{1-\alpha}.$$

Appendix B. *Proof of $d\chi_t/dg_t < d\varepsilon_t/dg_t$.*

The left- and right-hand sides of (22) are denoted by χ_t and ε_t, respectively. Differentiation of χ_t and ε_t with respect to g_t provides

$$\begin{aligned} \frac{d\chi_t}{dg_t}\frac{g_t}{1-\alpha} &= \lambda(\theta+\eta)\frac{g_t^{1-\alpha}}{z_t} = \chi_t - (1+z_t), \\ \frac{d\varepsilon_t}{dg_t}\frac{g_t}{1-\alpha} &= \zeta_t(z_t)g_t^{1-\alpha} = \varepsilon_t. \end{aligned}$$

Using these equations, and evaluating at $\chi_t = \varepsilon_t$ which holds under (22), we have

$$\frac{g_t}{1-\alpha}\left(\frac{d\chi_t}{dg_t} - \frac{d\varepsilon_t}{dg_t}\right) = -(1+z_t) < 0.$$

Appendix C. *Stability.*

The linearized system of (17) and (18) around the stationary equilibrium is

$$\begin{pmatrix} g_{t+1} - \bar{g} \\ z_{t+1} - \bar{z} \end{pmatrix} = \begin{pmatrix} \partial g_{t+1}/\partial g_t & \partial g_{t+1}/\partial z_t \\ \partial z_{t+1}/\partial g_t & \partial z_{t+1}/\partial z_t \end{pmatrix} \begin{pmatrix} g_t - \bar{g} \\ z_t - \bar{z} \end{pmatrix},$$

where

$$\frac{\partial g_{t+1}}{\partial g_t} = \frac{g_t \partial \frac{g_{t+1}}{g_t}}{\partial g_t} + 1, \quad \frac{\partial g_{t+1}}{\partial z_t} = \frac{g_t \partial \frac{g_{t+1}}{g_t}}{\partial z_t}, \quad \frac{\partial z_{t+1}}{\partial g_t} = \frac{z_t \partial \frac{z_{t+1}}{z_t}}{\partial g_t},$$
$$\text{and } \frac{\partial z_{t+1}}{\partial z_t} = \frac{z_t \partial \frac{z_{t+1}}{z_t}}{\partial z_t} + 1.$$

These values are evaluated at the steady state. The characteristic polynomial is

$$P(\mu) = \mu^2 - \left(\frac{\partial g_{t+1}}{\partial g_t} + \frac{\partial z_{t+1}}{\partial z_t}\right)\mu + \left(\frac{\partial g_{t+1}}{\partial g_t}\frac{\partial z_{t+1}}{\partial z_t} - \frac{\partial g_{t+1}}{\partial z_t}\frac{\partial z_{t+1}}{\partial g_t}\right). \tag{A.1}$$

To analyze the local stability of each equilibrium, we confirm the sign of $P(\mu)$ when $\mu = 0, 1$. The values of $P(0)$ and $P(1)$ are

$$P(0) = \frac{\partial g_{t+1}}{\partial g_t}\frac{\partial z_{t+1}}{\partial z_t} - \frac{\partial g_{t+1}}{\partial z_t}\frac{\partial z_{t+1}}{\partial g_t}, \tag{A.2}$$

$$= \frac{\alpha\delta(1-\alpha)\frac{\alpha\delta(1-\alpha)(1-(1-\lambda)(\theta+\eta))}{(1+\alpha z_t)^2} g_t^{1-\alpha} z_t + (\alpha + (1-\alpha)z_t)(1+z_t)}{(1+\theta g_t^{-\alpha})^2} > 0,$$

$$P(1) = 1 - \left(\frac{\partial g_{t+1}}{\partial g_t} + \frac{\partial z_{t+1}}{\partial z_t}\right) + \left(\frac{\partial g_{t+1}}{\partial g_t}\frac{\partial z_{t+1}}{\partial z_t} - \frac{\partial g_{t+1}}{\partial z_t}\frac{\partial z_{t+1}}{\partial g_t}\right),$$

$$= g_t z_t \left(\frac{\partial \frac{g_{t+1}}{g_t}}{\partial g_t}\frac{\partial \frac{z_{t+1}}{z_t}}{\partial z_t} - \frac{\partial \frac{g_{t+1}}{g_t}}{\partial z_t}\frac{\partial \frac{z_{t+1}}{z_t}}{\partial g_t}\right) \gtrless 0, \tag{A.3}$$

where

$$\frac{\partial \frac{g_{t+1}}{g_t}}{\partial g_t} = -\frac{\alpha\theta g_t^{-\alpha-1} + (1-\alpha)g_t^{-\alpha}\zeta_t(z_t)}{1+\theta g_t^{-\alpha}} < 0,$$

$$\frac{\partial \frac{g_{t+1}}{g_t}}{\partial z_t} = \frac{1 + g_t^{1-\alpha}\frac{\alpha\delta(1-\alpha)(1-(1-\lambda)(\theta+\eta))}{(1+\alpha z_t)^2}}{1+\theta g_t^{-\alpha}} > 0,$$

$$\frac{\partial \frac{z_{t+1}}{z_t}}{\partial g_t} = -\frac{(1-\alpha)g_t^{-\alpha}\left[\zeta_t(z_t) - \frac{\lambda(\theta+\eta)}{z_t}\right]}{1+\theta g_t^{-\alpha}} < 0,$$

$$\frac{\partial \frac{z_{t+1}}{z_t}}{\partial z_t} = \frac{g_t^{1-\alpha}\left[\frac{\alpha\delta(1-\alpha)(1-(1-\lambda)(\theta+\eta))}{(1+\alpha z_t)^2} - \frac{\lambda(\theta+\eta)}{z_t^2}\right] + 1}{1+\theta g_t^{-\alpha}} \gtrless 0.$$

The sign of the last equation is positive (negative) when the sign of $P(1)$ is negative (positive), implying that both an unstable and a stable equilibrium exist. Since z_t is large, $g_t^{1-\alpha}\left[\frac{\alpha\delta(1-\alpha)(1-(1-\lambda)(\theta+\eta))}{(1+\alpha z_t)^2} - \frac{\lambda(\theta+\eta)}{z_t^2}\right] + 1 > 0$ holds at equilibrium B. Further, as the slope of the ZZ curve is steeper than that of the GG curve, $\frac{dg_t}{dz_t}|_{ZZ} > \frac{dg_t}{dz_t}|_{GG}$ holds at equilibrium B. We rewrite the slope of the ZZ and the GG curves as

$$\frac{dg_t}{dz_t}|_{ZZ} = -\frac{\frac{\partial \frac{z_{t+1}}{z_t}}{\partial z_t}}{\frac{\partial \frac{z_{t+1}}{z_t}}{\partial g_t}}, \quad \frac{dg_t}{dz_t}|_{GG} = -\frac{\frac{\partial \frac{g_{t+1}}{g_t}}{\partial z_t}}{\frac{\partial \frac{g_{t+1}}{g_t}}{\partial g_t}}.$$

Since $\frac{dg_t}{dz_t}|_{ZZ} > \frac{dg_t}{dz_t}|_{GG}$ holds at equilibrium B, we derive

$$\frac{\partial \frac{g_{t+1}}{g_t}}{\partial g_t}\frac{\partial \frac{z_{t+1}}{z_t}}{\partial z_t} > \frac{\partial \frac{g_{t+1}}{g_t}}{\partial z_t}\frac{\partial \frac{z_{t+1}}{z_t}}{\partial g_t}.$$

Now, at equilibrium B, $P(1) < 0$ holds from (A.3). Thus, the values of the eigenvalues (μ_i) are $0 < \mu_1 < 1 < \mu_2$. Therefore, equilibrium B is a saddle point.

By contrast, $g_t^{1-\alpha}\left[\frac{\alpha\delta(1-\alpha)(1-(1-\lambda)(\theta+\eta))}{(1+\alpha z_t)^2} - \frac{\lambda(\theta+\eta)}{z_t^2}\right] + 1 < 0$ holds at equilibrium A, since z_t is small at equilibrium A. Thus, $P(1)$ is positive and the discriminant of (A.1) is also positive, and $P'(0) < 0$ and $P'(1) > 0$ hold at equilibrium A. Therefore, we have real value eigenvalues $0 < \mu_1, \mu_2 < 1$ and then, equilibrium A is a sink. However, we have imaginary value solutions when the discriminant of (A.1) is negative.

Appendix D. *Proof of sign of H.*

The slope of the GG curve is given by (21). At point B in Figures 4(a)-4(c), the slope of the GG curve is steeper than $\theta\lambda^{-1}(\theta+\eta)^{-1}$;

$$\frac{dg_t}{dz_t}|_{GG} = \frac{1-\zeta_t'(z_t)g_t^{1-\alpha}}{[(1-\alpha)g_t\zeta_t(z_t)+\alpha\theta]g_t^{-(1+\alpha)}} > \frac{\theta}{\lambda(\theta+\eta)}.$$

Rewriting this condition, we have

$$\lambda(\theta+\eta)\theta^{-1}[1-\zeta_t'(z_t)g_t^{1-\alpha}] - g_t^{-(1+\alpha)}[(1-\alpha)g_t\zeta_t(z_t)+\alpha\theta] > 0.$$

Because the left-hand side is H [see (29)], $H > 0$ at point B in Figures 4(a)-4(c). The reverse is also true, that is, $H < 0$, at point A.

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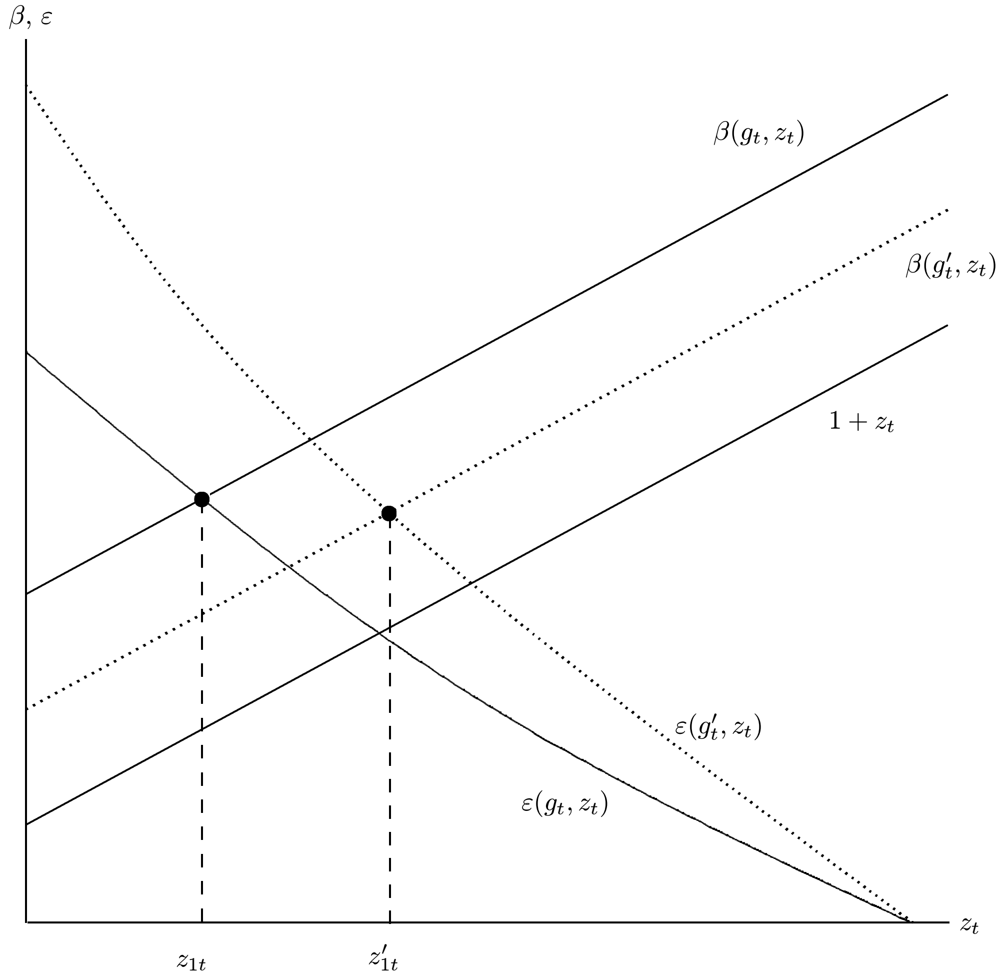

Figure 1. Diagramatic representation of (20).

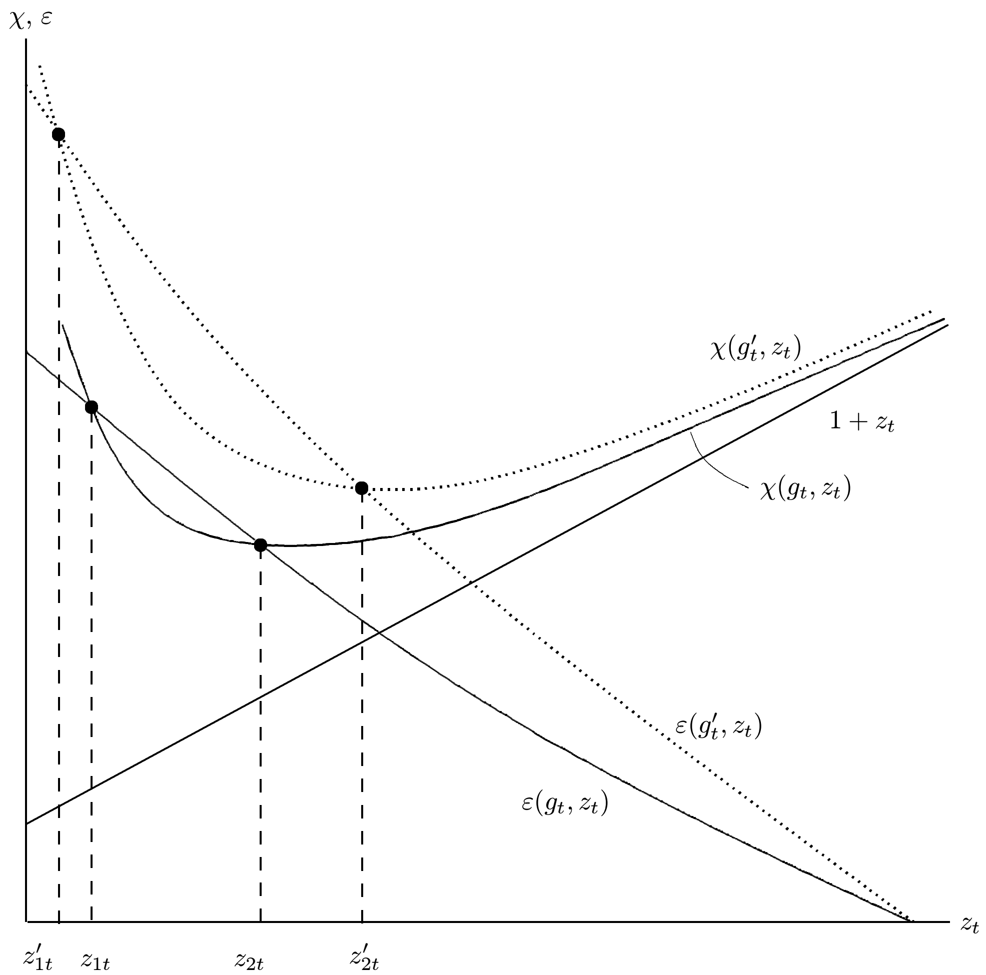

Figure 2. Diagramatic representation of (22).

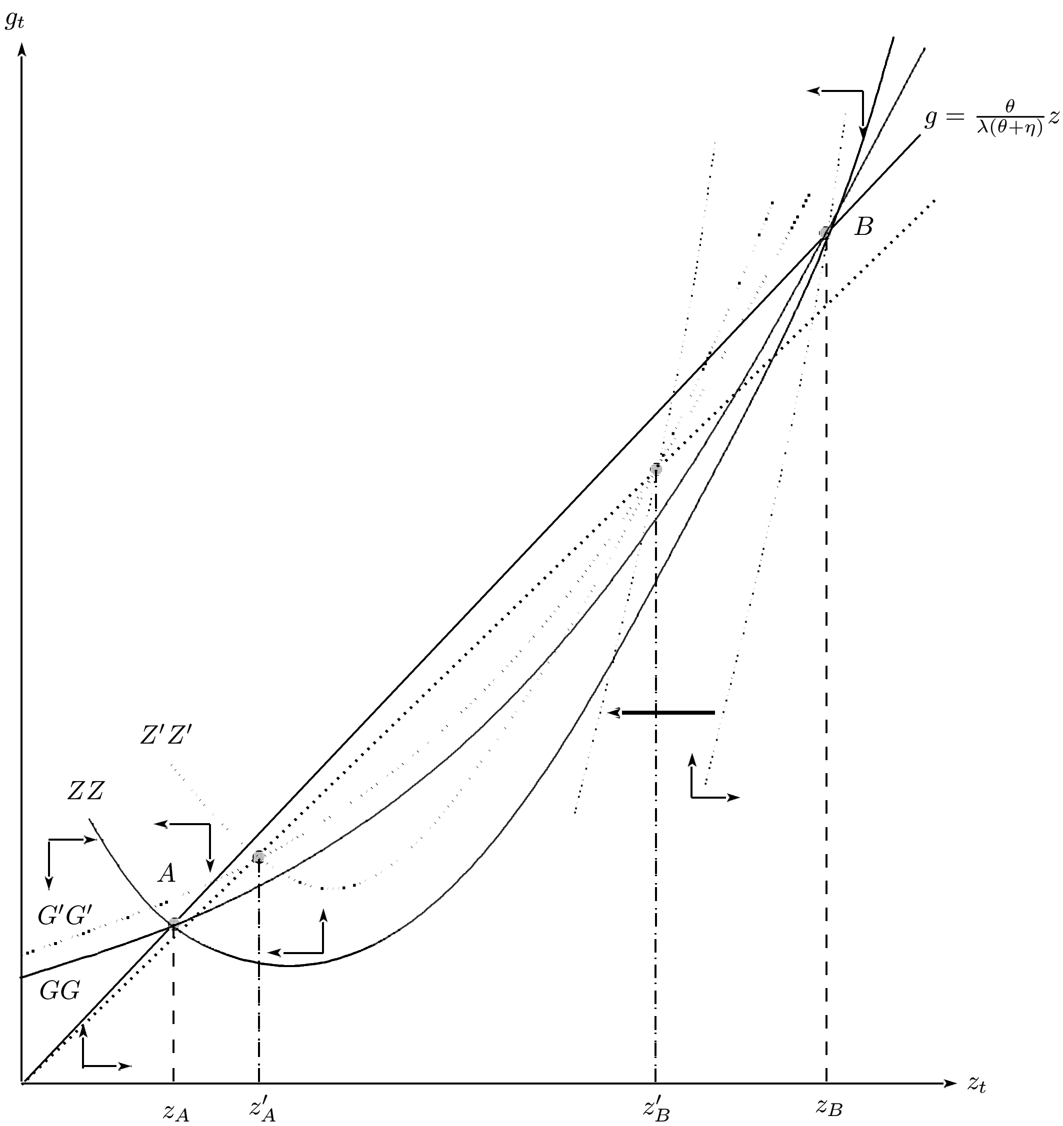

Figure 3. Phase diagram and effects of an increase in λ on the steady state.

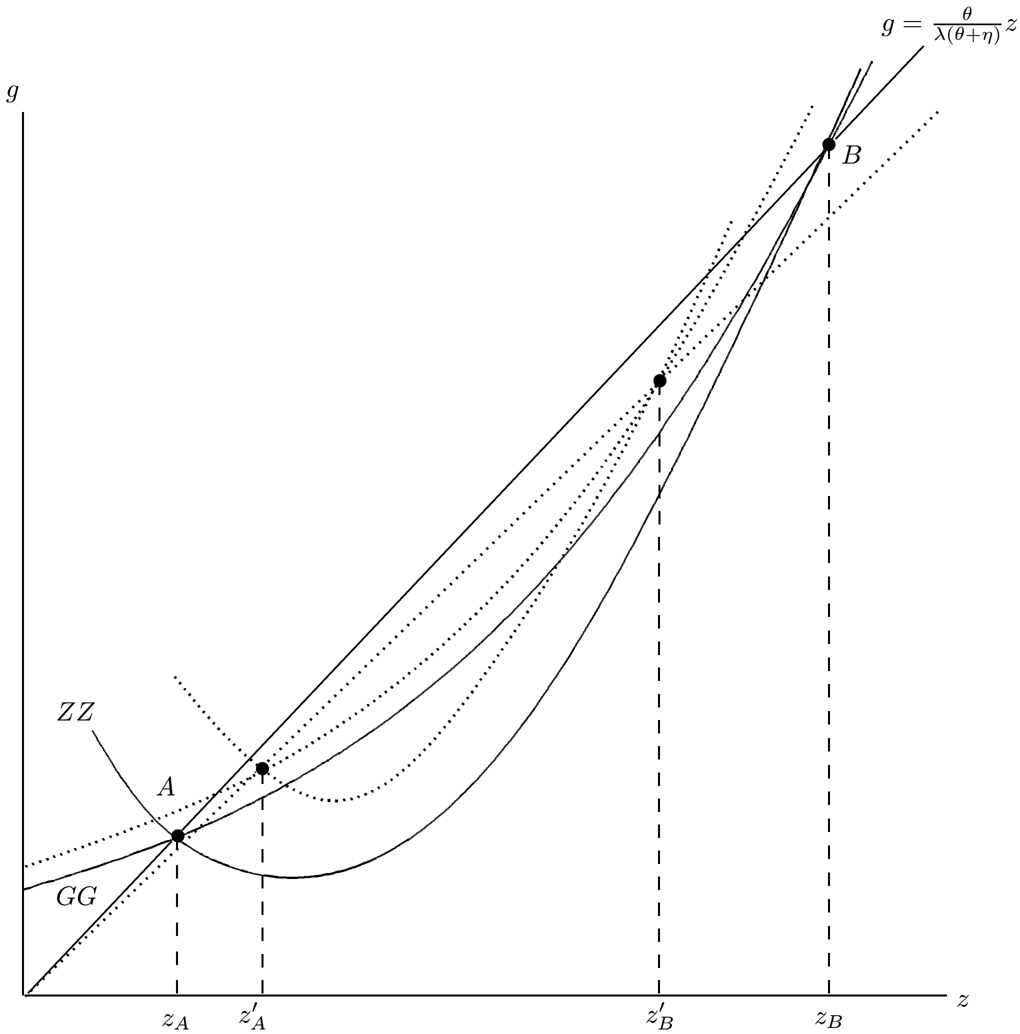

Figure 4(a). Case 1. $\delta < \frac{\lambda}{1-\frac{(1-\alpha)(1-\lambda)}{1+\alpha z_t}}$

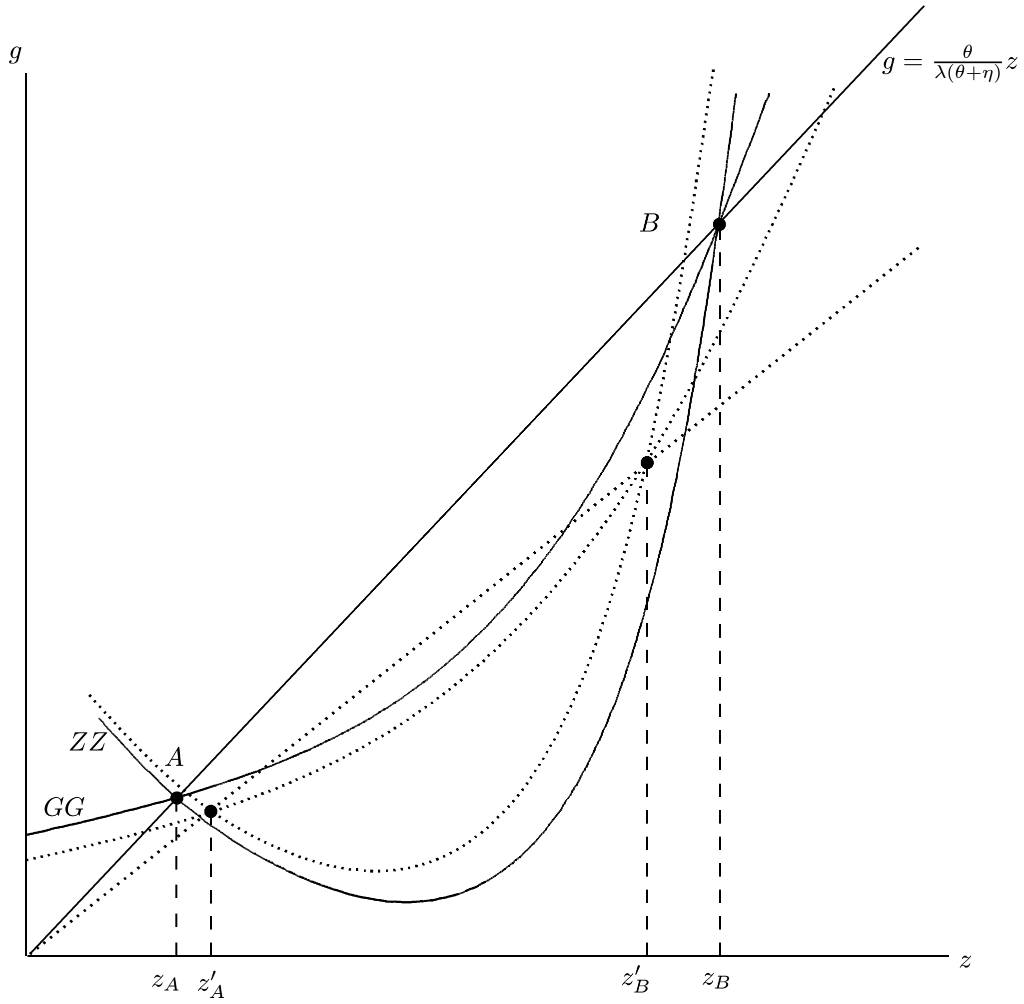

Figure 4(b). Case 2. $\frac{\lambda}{1-\frac{(1-\alpha)(1-\lambda)}{1+\alpha z_t}} < \delta < \frac{\lambda\left(1+\frac{1}{z_t}\right)}{1-\frac{(1-\alpha)(1-\lambda)}{1+\alpha z_t}}$

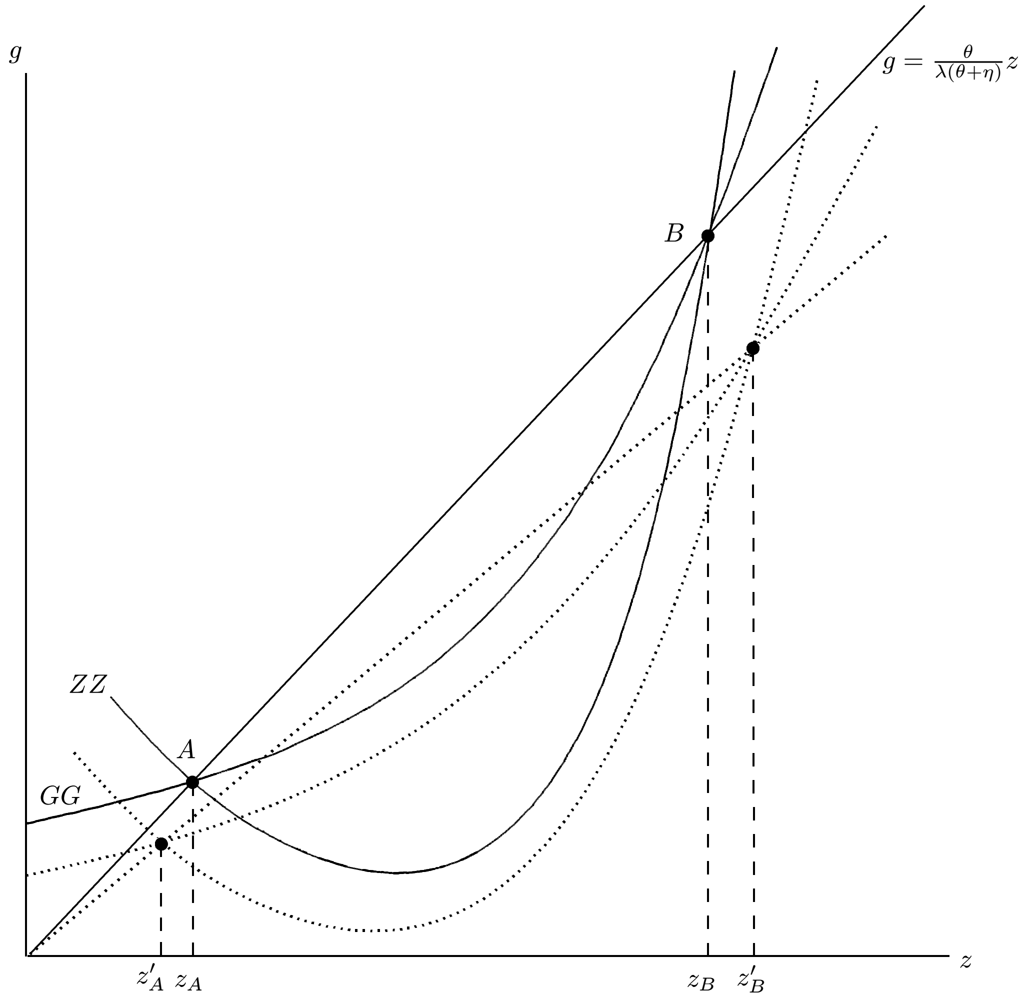

Figure 4(c). Case 3. $\frac{\lambda\left(1+\frac{1}{z_t}\right)}{1-\frac{(1-\alpha)(1-\lambda)}{1+\alpha z_t}} < \delta$