ECONOMIC RESEARCH CENTER
DISCUSSION PAPER

E-Series

No.E14-14

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December 2014

ECONOMIC RESEARCH CENTER
GRADUATE SCHOOL OF ECONOMICS
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The Welfare Effects of Oligopolistic Third-Degree Price Discrimination When Own and Cross Price Elasticities Are Constants*

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November 28, 2014

Abstract

This paper studies the welfare effects of oligopolistic third-degree price discrimination with constant own and cross price elasticities of demand under product differentiation. We verify the robustness of Adachi and Matsushima's (2014) result on social welfare under linear demands: price discrimination is more likely to improve social welfare for a higher value of the cross price elasticity in the "strong" market (where the discriminatory price is higher than the uniform price). In contrast to Aguirre and Cowan's (2013) results in the case of monopoly, social welfare can be higher with price discrimination even if the relative share of the strong market under uniform pricing is sufficiently small or the own elasticity difference between the two markets is also sufficiently small. Consumer surplus can also be higher with price discrimination if the cross price elasticities are sufficiently low. This suggests that Adachi and Matsushima's (2014) result on consumer surplus (price discrimination never improves social welfare) hinges on the linearity assumption.

JEL classification: D43; L11; L13.

*We are grateful to Takeshi Ikeda, Tamotsu Kadoda, Stephen King, Toshihiro Tsuchihashi and conference and seminar participants at the 41th Annual EARIE Conference, Daito Bunka University and the 2014 Autumn Meeting of the Japan Association for Applied Economics for helpful comments discussions. Adachi acknowledges a Grant-in-Aid for Young Scientists (B) from the Japanese Ministry of Education, Science, Sports, and Culture (24730205) as well as the Kikawada Foundation for their financial supports. Ebina acknowledges a Grant-in-Aid for Scientific Research (C) (24530264) and a Grant-in-Aid for Young Scientists (B) (24730224) from the Ministry. Any remaining errors are our own.

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1 Introduction

This paper studies oligopolistic third-degree price discrimination when demand in each discriminatory submarket has constant own and cross elasticities to examine an important question since Pigou (1920) and Robinson (1933): under which conditions third-degree price discrimination raises social welfare. In many applied studies of welfare effects of third-degree price discrimination, researchers typically assume that demand in each discriminatory submarket belongs to the same functional family. In fact, linear demand is often assumed.[1] In other theoretical studies, researchers consider nonlinear demands in a nonrestrictive way. For example, Aguirre, Cowan, and Vickers (2010) consider the curvatures of submarket demands to synthesize the existing studies[2] of output and welfare effects of *monopolistic* third-degree price discrimination. Notably, Aguirre, Cowan, and Vickers' (2010) Proposition 2 states that if the inverse demand in the weak market is more convex than the inverse demand in the strong market, then price discrimination raises social welfare. However, no comparable characterization has yet come to in the case of oligopoly with nonlinear demands.[3] Thus, this paper aims to take one step forward to the study of *oligopolistic* third-degree price discrimination by considering one of the most "popular" classes of nonlinear demand, namely, "log-linear" (Davis and Garcés (2010, p.447)) demands that have constant own and cross elasticities. It also studies the robustness of Adachi and Matsushima's (2014) study of oligopolistic third-degree price discrimination with linear demands.

To understand Adachi and Matsushima's (2014) necessary and sufficient condition for oligopolistic third-degree price discrimination to improve social welfare, consider the situation where there are two geographical markets: one is a hot resort area, and the other is a city area. There are two beverage companies, and they compete by selling their own (one) product (such as cola) in each market. Resale

[1] See, e.g., Layson (1988), Adachi (2002, 2004, 2005), and Bertoletti (2004).

[2] See, e.g., Schmalensee (1981), Varian (1985), Nahata, Ostaszewski, and Sahoo (1990), Schwartz (1990), Layson (1998), Cowan (2007), Cowan (2012), and Cowan (2013). Armstrong (2006), Stole (2007), and Liu and Serfes (2010) are excellent surveys on third-degree price discrimination.

[3] See, e.g., Holmes (1989), Corts (1998), Dastidar (2006), and Galera and Zaratiegui (2006) for studies of oligopolistic third-degree price discrimination.

by nonfirm agents for arbitrage is assumed impossible. The cost of production and sales is assumed common for both markets (for example, there are no differences in transportation costs between the markets). Although the firms are symmetric (i.e., homogeneous), consumers are not indifferent (except the marginal consumers) between the two products (given that the firms' prices are the same). In fact, horizontal product differentiation makes some consumers prefer one firm's product to the other's, and vice versa for the other consumers.[4]

Adachi and Matsushima (2014) find that price discrimination raises social welfare if and only if the degree of substitution between the two products in the "strong" market (where the discriminatory price is higher than the uniform price) is *sufficiently higher* than in the "weak" market (where it is lower).[5] If consumers care less about the firms' brands in the hot resort area (because when people are thirstier it is natural that they should be less concerned about which brand), then price discrimination may improve social welfare. One might think that substitution in the weak, not the strong, market should be sufficiently higher because a fiercer level of competition (due to a higher level of substitution) in the weak market increases the aggregate output more. However, weakening the *misallocation effect* caused by price discrimination is more important than this *output effect*: if substitution is sufficiently high in the strong market, then the price increase by price discrimination is smaller, which leads to a less number of consumers in the strong market give up consumption after price discrimination is introduced. This is beneficial for social welfare because, on average, those who consume a product in the strong market have higher willingness to pay than those in the weak market do *both* under uniform

[4]The source of horizontal product differentiation may vary. At the extreme, two similar products may be recognized as very different by consumers due to advertising by manufacturers. To quote Tremblay and Tremblay (2005, p.173), "[b]ecause their colas taste very much alike, Coke and Pepsi use advertising to segment the market by creating images that appeal to different consumers. Coke pursues an image of traditional family values, while Pepsi presents a more youthful and rebellious image. This strategy benefits both firms by strengthening brand loyalty and reducing price competition."

[5]The usage of "strong" and "weak" markets is in the tradition since Robinson (1933). A more precise statement for the necessity and sufficiency is that given the other parameter values, the substitution parameter in the strong market exceeds a threshold (see Adachi and Matsushima's (2014) Proposition 1).

pricing and under price discrimination.

Notice that the argument so far does not rely on the linearity of demands. We thus conjecture that this result and the intuition by Adachi and Matsushima (2014) on social welfare (their Proposition 1) are robust for nonlinear demands. In Adachi and Matsushima's (2014) formulation, the degree of substitution in each market is characterized by one constant parameter (because of linearity), which is separable from other variables and parameters. In equilibrium, this parameter, with normalization, coincides with the cross price elasticity, though it is not the elasticity *per se*. In particular, the normalization needs to take into account the slope of the linear demand, which itself is less relevant to the marginal conditions. Furthermore, the linearity restriction imposes the own price elasticity to be always one in equilibrium. Does the intuition above survive under nonlinear demands? By investigating constant own and cross price elasticities of demand, we provide a positive answer for this conjecture. In contrast, Adachi and Matsushima's (2014) result on aggregate consumer surplus (their Proposition 2) may depend on the linearity of demands. Adachi and Matsushima's (2014) Proposition 2 shows that price discrimination *always* lowers aggregate consumer surplus. This is true even if it raises social welfare. This result implies that firms 'squeeze' all extra surplus generated by welfare-improving price discrimination described as above. Is this still true if market demands are nonlinear?

In this paper, we extend Aguirre and Cowan's (2013) analysis on monopolistic third-degree price discrimination with constant elasticity demands[6] to the case of differentiated oligopoly.[7] As in Adachi and Matsushima (2014), we express the degree of product differentiation in a submarket by one parameter: in the present

[6] Aguirre and Cowan (2013) explain the reasons why the well-known results on third-degree price discrimination and welfare (such as Schmalensee (1981), Varian (1985), Aguirre, Cowan and Vickers (2010), and Cowan (2012)) are less applicable to the case of constant elasticity demand. Aguirre and Cowan (2013) also emphasize that Aguirre, Cowan and Vickers' (2010) sufficient condition for their Proposition 2 does not hold if submarket demands belong to the class of *constant elasticity demands* because they are all convex with respect to the own price.

[7] One thing to keep in mind in an analysis of third-degree price discrimination with linear demands (as in Adachi and Matsushima (2014)) is to guarantee conditions for all submarkets to be open under uniform pricing (the issue of 'market opening'). However, under constant elasticity demands, all submarkets are necessarily open under uniform pricing because there are no intercepts.

paper, in contrast to Adachi and Matsushima (2014), it is the cross price elasticity itself. Aguirre and Cowan (2013) show that price discrimination can raise social welfare if the output share of the strong market under uniform pricing (α in their and our notation) is sufficiently large and the (own) elasticity difference between the two markets is sufficiently large (θ in their and our notation, which is the elasticity difference between the two markets). Aguirre and Cowan (2013) also show that price discrimination can raise consumer surplus under stricter conditions. Notice the similarity between Aguirre and Cowan's (2013) sufficient condition and Adachi and Matsushima's (2014) necessary and sufficient condition. First, a fiercer level of competition in the strong market would push the amount of production in that market. Second, Adachi and Matsushima's (2014) Proposition 1 implies that the elasticity difference must be sufficiently high. In essence, the substitution parameter plays an important role to affect the equilibrium elasticity in each submarket. Thus, in equilibrium, oligopolistic firms can been seen as monopoly where strategic interactions are already incorporated: the intuition for welfare improvement in the case of oligopoly is similar to that in the case of monopoly once strategic interactions are taken into account. We also verify that consumer surplus can also be higher with price discrimination if he cross price elasticities are sufficiently low. This suggests that Adachi and Matsushima's (2014) result on consumer surplus hinges on the linearity assumption.[8]

The rest of the paper is organized as follows. The next section presents the model. We then provide welfare analysis in Section 3. Most of arguments are based on numerical analysis. Section 4 concludes the paper.

2 The Model

Consider J (≥ 2) oligopolistic firms producing (horizontally) differentiated products to compete in price to sell their products (directly) to consumers. Each firm produces

[8]Aguirre (2011) studies the situation where a multimarket firm and a local firm compete in one market while the multimarket is a monopolist in another market. As in Adachi and Matsushima (2014), Aguirre (2011) assumes the linearity of demands and shows that price discrimination can improve social welfare in this setting as well.

and sells only one type of product, which is interpreted as the firm's brand. The whole market can be segmented into independent M (≥ 2) separate sub-markets (hereafter, just called markets if there arises no ambiguity) according to identifiable signals (e.g., geography, age, and gender). If a firm implements uniform pricing, the firm's price that consumers face is common across all markets. On the other hand, if a firm price discriminates across markets, consumers may face different unit prices of the firm's product, depending on which market they belong to. We assume that resale of a product of the price discriminating firm across markets is not possible.

In this paper, we consider symmetric firms as in Holmes (1989), and thus assume the marginal cost is common for all firms. In addition, we assume that the marginal cost is constant, $c > 0$. For further simplicity, we mainly work on the case of two firms (A and B) and two markets in the following analysis. Specifically, indices $i, j \in \{A, B\}$ are used for firms, and index $m \in \{s, w\}$ is used for markets (s denotes (the set of) the strong markets and w the weak markets; these meanings will be clear below). Notice that, however, our main results can be extended to the case of J (≥ 3) firms and M (≥ 3) markets as long as the firms are symmetric, and the following s and w are considered as (arbitrary) two representatives of all markets.

The demand function of firm i in market m is given by

$$q_m^i(p_m^i, p_m^j) = a_m (p_m^i)^{-\varepsilon_m} (p_m^j)^{\sigma_m},$$

where $a_i > 0$ is a measure of market size, $\varepsilon_m > 1$ is the constant *own* price elasticity (notice that $(\partial q_m^i / \partial p_m^i)(p_m^i / q_m^i) = -\varepsilon_m$) and σ_m, which is assumed to be less than ε_m (the next paragraph explains the reason for this restriction) is the constant *cross* price elasticity (notice that $(\partial q_m^i / \partial p_m^j)(p_m^j / q_m^i) = \sigma_m$), which captures the degree of product differentiation (note that our demand form is equivalent to the following familiar form of log-linear demand: $\ln q_m^i = \ln a_m - \varepsilon_m \ln p_m^i + \sigma_m \ln p_m^j$). The assumption of identical firms requires that ε_m and σ_m are common for all firms.

Note here that ε_m *per se* indicates that how many percent of the customers leave the firm if it raises its price by one percent, but not that how many of them switch to the other firm. However, it is seen that the other firm gains σ_m percent more of the existing customers as new customers. Thus, as a response to the one percent increase of the firm's price, $(\varepsilon_m - \sigma_m)$ percent of the customers leave the market (i.e., purchase no products), and σ_m percent of them switch to the other firm (i.e., now purchase the other firm's product). As a natural restriction on the demand, $(\varepsilon_m - \sigma_m)$ should be positive (thus, $\varepsilon_m > \sigma_m$).[9]

If $\sigma_m = 0$, the products produced by two firms are *independent* in each market, in the sense that one firm's demand is not affected by the other firm's price. In other words, no marginal consumers are better off by switching to the other firm, and thus they leave the market if the price goes up. In this case, the two firms behave as an identical monopolist in each market: the demand function is identical (with rescaling) as the one in Aguirre and Cowan's (2013) analysis of monopoly. If $\sigma_m > 0$, the products are *substitutes.* As σ_m approaches to ε_m, competition in market m becomes fiercer. In the extreme case of $\sigma_m \approx \varepsilon_m$, the marginal consumers all switch to the rival: the two products are homogeneous and (almost) perfect substitutes. In the present paper, we consider the case of substitutes only (i.e., σ_m is positive): *complementarity* is assumed away as opposed to Adachi and Matsushima's (2014) analysis with linear demands. This is because consumer surplus defined in the next section assumes that consumers are segment into three groups: (i) those who purchase product A, (ii) those who purchase product B, and (iii) those who purchase nothing. If complementarity between two products is allowed, it is necessary to consider another type of consumers: those who purchase both products. As consumer surplus used in this paper is a 'naive' one, namely, the integral of the positive difference between the inverse demand and the price, it is less obvious to define proper consumer surplus in the case of complementarity. Thus, we simply assume $\sigma_m > 0$ for $m \in \{s, w\}$ throughout the paper.[10]

Following Aguirre and Cowan (2013), we assume that the elasticity in market w

[9] Holmes (1989) shows that with any symmetric price under oligopoly p, a firm's price elasticity is equal to the sum of the industry-demand elasticity and the cross-price elasticity. The industry demand in market m is given by $2q_m^i(p_m, p_m)$, and the industry-demand elasticity is $-(p_m/2q_m^i)(2dq_m^i/dp_m) = \varepsilon_m - \sigma_m$.

[10] Adachi and Ebina (2014c) argue that there is no repsentative consumer's utility that can be consistent with the log-linear demand system. Thus, in contrast to the case of linear demands, we cannot use the representative consumer's utility to derive consumer surplues under the log-linear demands,

is greater than that in market s: $\varepsilon_w = \varepsilon_s + \theta$ where $\theta > 0$ is the *own price elasticity difference* (more precisely, this condition that $\varepsilon_w > \varepsilon_s$ makes market w the weak market).

We consider two regimes, uniform pricing ($r = U$) and price discrimination ($r = D$): under uniform pricing, firms set a common unit price for all separate markets. Under price discrimination, they can set a different price in each market.[11] Furthermore, we impose the symmetry on demands $q_m^j(p', p'') = q_m^{-j}(p'', p')$ to focus on a symmetric equilibrium where all firms set the same price in one market. With little abuse of notation, let $q_m(p) = q_m^A(p, p)$.

2.1 Price Discrimination

First, suppose that firm i can discriminate a price p_m^i in each market m. The profit function of firm i from market m is given by

$$\pi_m^i(p_m^i, p_m^j) = (p_m^i - c)a_m(p_m^i)^{-\varepsilon_m}(p_m^j)^{\sigma_m}.$$

From the first-order condition of p_m^i, the equilibrium discriminatory price is obtained by

$$\begin{aligned} \frac{\partial \pi_m^i}{\partial p_m^i} &= a_m(p_m^i)^{-\varepsilon_m}(p_m^j)^{\sigma_m} + (p_m^i - c)a_m(-\varepsilon_m)(p_m^i)^{-\varepsilon_m - 1}(p_m^j)^{\sigma_m} = 0 \\ \Rightarrow (p_m^i)^* &= \frac{\varepsilon_m}{\varepsilon_m - 1}c, \end{aligned}$$

where superscript $*$ denotes the equilibrium under price discrimination.[12] Here, the discriminatory price in the symmetric equilibrium is (a bit surprisingly) *independent of the cross price elasticity, σ_m,* and *coincides with the monopolistic discriminatory*

[11] Note that all markets are served under both regimes: market opening is not an issue with constant elasticity demands because a positive number of consumers demands the product even if the price is tremendously high.

[12] Note that the second-order condition is satisfied at the equilibrium price because

$$\begin{aligned} & \frac{\partial^2 \pi_m^i}{\partial (p_m^i)^2} < 0 \\ \Leftrightarrow \quad & -2p_m^i + (\varepsilon_m + 1)(p_m^i - c) < 0 \Leftrightarrow p_m^i < \frac{\varepsilon_m + 1}{\varepsilon_m - 1}c, \end{aligned}$$

which implies that the first-order conditions attains the unique solution.

price in Aguirre and Cowan (2013). To understand why, the first-order condition in general is written by

$$\begin{aligned} q_m^i(p_m^i, p_m^j) + (p_m^i - c)\frac{\partial q_m^i}{\partial p_m^i}(p_m^i, p_m^j) &= 0 \\ \Leftrightarrow \frac{p_m^i - c}{p_m^i} = -\frac{q_m^i / p_m^i}{\partial q_m^i / \partial p_m^i}, \end{aligned}$$

which, known as the Lerner condition, essentially implies that the competing firms' problem can be seen as the *monopolist's* problem under the residual demand given the other firms' prices. The right hand side is the inverse of the firm's own price elasticity. It is in general a function of p_m^i and p_m^j. In our demand specification, however, firm i's own elasticity is independent of p_m^j (moreover, it is a constant, ε_m, which is also independent of p_m^i). This also makes firm i's optimal price independent of its belief about p_m^j.

Accordingly, firm i's output in market m, the aggregate output in market m, and the aggregate output in the industry are

$$\begin{aligned} (q_m^i)^* &= a_m \left(\frac{\varepsilon_m}{\varepsilon_m - 1}c\right)^{-(\varepsilon_m - \sigma_m)}, \\ (Q_m)^* &= 2a_m \left(\frac{\varepsilon_m}{\varepsilon_m - 1}c\right)^{-(\varepsilon_m - \sigma_m)}, \\ (Q)^* &= 2\sum_{m=s,w} a_m \left(\frac{\varepsilon_m}{\varepsilon_m - 1}c\right)^{-(\varepsilon_m - \sigma_m)}, \end{aligned}$$

respectively. The equilibrium profit of firm i from market m is

$$(\pi_m^i)^* \equiv \left[(p_m^i)^* - c\right](q_m^i)^* = a_m c^{1-(\varepsilon_m - \sigma_m)} \frac{(\varepsilon_m)^{-(\varepsilon_m - \sigma_m)}}{(\varepsilon_m - 1)^{1-(\varepsilon_m - \sigma_m)}},$$

and thus, the total profit of firm i is written by

$$(\Pi^i)^* \equiv \sum_{m=s,w} (\pi_m^i)^* = \sum_{m=s,w} a_m c^{1-(\varepsilon_m - \sigma_m)} \frac{(\varepsilon_m)^{-(\varepsilon_m - \sigma_m)}}{(\varepsilon_m - 1)^{1-(\varepsilon_m - \sigma_m)}}.$$

2.2 Uniform Pricing

Next, consider the case of uniform pricing. The profit function of firm i is given by

$$\pi_s^i(p^i, p^j) + \pi_w^i(p^i, p^j) = (p^i - c)\sum_{m=s,w} q_m^i(p^i, p^j) = (p^i - c)\sum_{m=s,w} a_m (p^i)^{-\varepsilon_m}(p^j)^{\sigma_m}.$$

By the first-order condition of p^i, we have[13]

$$\frac{\partial(\pi_s^i + \pi_w^i)}{\partial p^i} = \sum_{m=s,w} \left[a_m (p^i)^{-\varepsilon_m} (p^j)^{\sigma_m} + (p^i - c) a_m (-\varepsilon_m)(p^i)^{-\varepsilon_m - 1}(p^j)^{\sigma_m}\right] = 0.$$

Notice that the first-order condition is reformulated as the Lerner condition under uniform pricing:

$$\frac{p^i - c}{p^i} = \frac{1}{\frac{\sum_{m=s,w} \varepsilon_m q_m^i(p^i, p^j)}{\sum_{m=s,w} q_m^i(p^i, p^j)}},$$

where the right hand side is *no longer independent* of p^j. Here, the elasticity that firm i takes into account is the average sum of the own elasticities over markets *weighted by the outputs*. The symmetric equilibrium, where $p^i = p^j$, satisfies

$$\frac{p^0 - c}{p^0} = \frac{1}{\varepsilon(p^0)}, \tag{1}$$

where

$$\varepsilon(p^0) = \frac{\sum_{m=s,w} \varepsilon_m a_m (p^0)^{-(\varepsilon_m - \sigma_m)}}{\sum_{m=s,w} a_m (p^0)^{-(\varepsilon_m - \sigma_m)}},$$

and superscript 0 denotes the equilibrium outcome under uniform pricing. Therefore, firm i's output in market m, the aggregate output in market m, and the aggregate output in the industry are

$$\begin{aligned}
(q_m^i)^0 &= a_m q_m(p^0, p^0) = a_m (p^0)^{-(\varepsilon_m - \sigma_m)}, \\
(Q_m)^0 &= 2(q_m^i)^0 = 2a_m (p^0)^{-(\varepsilon_m - \sigma_m)}, \\
(Q^0) &= \sum_{m=s,w} (Q_m)^0 = 2 \sum_{m=s,w} a_m (p^0)^{-(\varepsilon_m - \sigma_m)},
\end{aligned}$$

[13] This profit function is not necessarily quasi-concave, so that it may have several peaks. Similar arguments for the optimal price with uniform pricing $(p^i)^0$ also hold as in Aguirre and Cowan (2013). Thus, we make almost the same assumptions and $(p^i)^0$ satisfying the first-order condition has maxima:

$$\begin{aligned}
\frac{\partial^2(\pi_s^i + \pi_w^i)}{\partial (p^i)^2} &= 2\left[\frac{\partial q_m^A}{\partial p^A}\right] + (p^i - c)\left[\frac{\partial^2 q_m^A}{\partial (p^A)^2}\right] \\
&= \sum_{m=s,w} \varepsilon_m a_m (p^i)^{-\varepsilon_m - 1}(p^j)^{\sigma_m}\left[-(1-\varepsilon_m) - \frac{\varepsilon_m + 1}{p^i} c\right] < 0.
\end{aligned}$$

respectively. The differences between the quantities under price discrimination and under uniform pricing are given by

$$\Delta q_m^i = (q_m^i)^* - (q_m^i)^0 = a_m \left[\left(\frac{\varepsilon_m}{\varepsilon_m - 1} c \right)^{-(\varepsilon_m - \sigma_m)} - (p^0)^{-(\varepsilon_m - \sigma_m)} \right],$$

$$\Delta Q_m = (Q_m)^* - (Q_m)^0 = 2a_m \left[\left(\frac{\varepsilon_m}{\varepsilon_m - 1} c \right)^{-(\varepsilon_m - \sigma_m)} - (p^0)^{-(\varepsilon_m - \sigma_m)} \right],$$

$$\Delta Q = 2 \sum_{m=s,w} a_m \left[\left(\frac{\varepsilon_m}{\varepsilon_m - 1} c \right)^{-(\varepsilon_m - \sigma_m)} - (p^0)^{-(\varepsilon_m - \sigma_m)} \right],$$

respectively.

Now, following Formby, Layson and Smith (1983) and Aguirre and Cowan (2013), we normalize c so that $p^0 = 1$. In this case, $(q_m^i)^0 = a_m$, $(Q_m)^0 = 2a_m$, and $Q^0 = 2(a_s + a_w)$. The averaged elasticity is also simplified as

$$\varepsilon(1) = \frac{\varepsilon_s a_s + \varepsilon_w a_w}{a_s + a_w}.$$

Given Equation (1) and $p^0 = 1$, the marginal cost satisfies

$$c = \frac{a_s(\varepsilon_s - 1) + a_w(\varepsilon_w - 1)}{a_s \varepsilon_s + a_w \varepsilon_w}.$$

Each firm's equilibrium aggregate output under uniform pricing is given by $a_s + a_w$. Let the share of the strong market under uniform pricing be defined by $\alpha \equiv a_s/(a_s + a_w)$. Analogously, the share of weak market is defined by $1 - \alpha = a_w/(a_s + a_w)$. As a further normalization, we also assume that $a_s + a_w = 1$. Thus, under this normalization a_m denotes the relative share of market m under uniform pricing. The above equation can be written as:

$$c = \frac{\varepsilon_s + (1-\alpha)\theta - 1}{\varepsilon_s + (1-\alpha)\theta},$$

which is less than $p^0 = 1$.

In the literature of third-degree market price discrimination, a market is called *strong* if the discriminatory price is higher than the uniform price, and it is called weak if the opposite is true. Now,

$$(p_s)^* = \frac{\varepsilon_s}{\varepsilon_s - 1} \cdot \frac{\varepsilon_s + (1-\alpha)\theta - 1}{\varepsilon_s + (1-\alpha)\theta}$$

$$
\begin{aligned}
&= \frac{\varepsilon_s^2 + (1-\alpha)\theta\varepsilon_s - \varepsilon_s}{\varepsilon_s^2 + (1-\alpha)\theta\varepsilon_s - \varepsilon_s - (1-\alpha)\theta} \\
&> 1 = p^0 \text{ (because } \theta > 0) \\
&> \frac{\varepsilon_s\varepsilon_w + (1-\alpha)\theta\varepsilon_w - \varepsilon_s - \theta}{\varepsilon_s\varepsilon_w + (1-\alpha)\theta\varepsilon_w - \varepsilon_s - \theta + \alpha\theta} = (p_w)^* > c.
\end{aligned}
$$

Thus, market s and market w are verified to satisfy the definition. Note here that as $\theta \to 0$, $(p_s)^* \to 1$ and $(p_w)^* \to 1$.

Finally, the equilibrium profit of firm i from market m is

$$
(\pi_m^i)^0 \equiv \left[(p_m^i)^0 - c\right](q_m^i)^0 = (1-c)a_m,
$$

and thus, the total profit of firm i is written by

$$
(\Pi^i)^0 \equiv \sum_{m=s,w} (\pi_m^i)^0 = (1-c)(a_s + a_w) = 1 - c.
$$

Thus, the profit changes between the two regimes is

$$
\begin{aligned}
\Delta\Pi &= (\Pi^i)^* - (\Pi^i)^0 \\
&= c^{1-(\varepsilon_m - \sigma_m)} \sum_{m=s,w} \frac{a_m(\varepsilon_m)^{-(\varepsilon_m-\sigma_m)}}{(\varepsilon_m - 1)^{1-(\varepsilon_m-\sigma_m)}} - 1 + c.
\end{aligned}
$$

3 Welfare Analysis

In the case of monopoly with constant elasticity submarket demands, Aguirre and Cowan (2013) show that price discrimination raises social welfare if *both* the share of the strong market under uniform pricing (α)[14] *and* the elasticity difference between markets (θ) are sufficiently high. If either parameter is further high, price discrimination raises consumer surplus as well. More precisely, Aguirre and Cowan (2013) show that if $\Delta W \geq 0$ then $\theta\alpha > 1$. Intuitively, it is necessary for θ to be sufficiently large for a nonnegative welfare change. If θ is large, it means that the price elasticity in the weak market is sufficiently large relative to that in the strong market. In other words, the strong market is sufficiently price inelastic relative to the weak

[14]Due to the normalization, the share of the strong market under uniform pricing, which is clearly an endogenous variable, can be expressed by one parameter, α.

market. Similar to the argument in Adachi and Matsushima's (2014) analysis of oligopoly, this is beneficial to weaken the distortion in the strong market (i.e., the output decrease in the strong market is kept small relative to the output increase in the weak market). In this section, we argue that $\theta\alpha > 1$ is not necessary for price discrimination to raise social welfare in the case of oligopoly. As expected, the two cross-elasticity parameters, σ_s and σ_w, play an important role.

First, note that under symmetric equilibrium, the equilibrium demand for firm i in market m is given by

$$\begin{aligned} q_m^i(p_m, p_m) &= a_m(p_m)^{-\varepsilon_m}(p_m)^{\sigma_m} \\ &= a_m(p_m)^{-(\varepsilon_m - \sigma_m)}, \end{aligned}$$

where p_m is the discriminatory or the uniform price. This provides the inverse demand function in symmetric equilibrium for firm i:

$$p_m(q_m) = a_m^{\frac{1}{\varepsilon_m - \sigma_m}} q_m^{-\frac{1}{\varepsilon_m - \sigma_m}}.$$

Following Aguirre and Cowan (2013), let SW_m^r be social welfare in market $m = s, w$ under regime $r = U, D$. Then, because of the symmetry,[15]

$$SW_m^r = 2\int_0^{q_m^r} \left(a_m^{\frac{1}{\varepsilon_m - \sigma_m}} q^{-\frac{1}{\varepsilon_m - \sigma_m}} - c \right) dq.$$

Therefore, a per-firm change in social welfare is written as

$$\begin{aligned} \frac{\Delta SW}{2} &= a_s^{\frac{1}{\varepsilon_s - \sigma_s}} \frac{\varepsilon_s - \sigma_s}{\varepsilon_s - \sigma_s - 1} [q_s^*]^{\frac{\varepsilon_s - \sigma_s - 1}{\varepsilon_s - \sigma_s}} - cq_s^* + a_w^{\frac{1}{\varepsilon_w - \sigma_w}} \frac{\varepsilon_w - \sigma_w}{\varepsilon_w - \sigma_w - 1} [q_w^*]^{\frac{\varepsilon_w - \sigma_w - 1}{\varepsilon_w - \sigma_w}} - cq_w^* \\ &\quad - a_s^{\frac{1}{\varepsilon_s - \sigma_s}} \frac{\varepsilon_s - \sigma_s}{\varepsilon_s - \sigma_s - 1} [q_s^0]^{\frac{\varepsilon_s - \sigma_s - 1}{\varepsilon_s - \sigma_s}} + cq_s^0 - a_w^{\frac{1}{\varepsilon_w - \sigma_w}} \frac{\varepsilon_w - \sigma_w}{\varepsilon_w - \sigma_w - 1} [q_w^0]^{\frac{\varepsilon_w - \sigma_w - 1}{\varepsilon_w - \sigma_w}} + cq_w^0 \\ &= \alpha \left\{ \frac{\varepsilon_s - \sigma_s}{\varepsilon_s - \sigma_s - 1} \left(\frac{\varepsilon_s - 1}{\varepsilon_s} \frac{1}{c} \right)^{\varepsilon_s - \sigma_s - 1} - \frac{\varepsilon_s - \sigma_s}{\varepsilon_s - \sigma_s - 1} - c \left(\frac{\varepsilon_s - 1}{\varepsilon_s} \frac{1}{c} \right)^{\varepsilon_s - \sigma_s} + c \right\} \\ &\quad + (1-\alpha) \left\{ \frac{\varepsilon_w - \sigma_w}{\varepsilon_w - \sigma_w - 1} \left(\frac{\varepsilon_w - 1}{\varepsilon_w} \frac{1}{c} \right)^{\varepsilon_w - \sigma_w - 1} - \frac{\varepsilon_w - \sigma_w}{\varepsilon_w - \sigma_w - 1} - c \left(\frac{\varepsilon_w - 1}{\varepsilon_w} \frac{1}{c} \right)^{\varepsilon_w - \sigma_w} + c \right\} \end{aligned}$$

[15] Adachi and Ebina (2014c) argue that the log-linear demand system as employed in this paper cannot be generated from the representative consumer's utility. Thus, consumer surplus used in this paer is not linked to consumers' utility.

$$
\begin{aligned}
&= \frac{\alpha}{\varepsilon_s - \sigma_s - 1}\left[\left(\frac{\varepsilon_s - 1}{\varepsilon_s}\frac{\varepsilon_s + (1-\alpha)\theta}{\varepsilon_s + (1-\alpha)\theta - 1}\right)^{\varepsilon_s - \sigma_s - 1}\frac{2\varepsilon_s - \sigma_s - 1}{\varepsilon_s}\right] \\
&\quad + \frac{1-\alpha}{\varepsilon_s + \theta - \sigma_w - 1}\left[\left(\frac{\varepsilon_s + \theta - 1}{\varepsilon_s + \theta}\frac{\varepsilon_s + (1-\alpha)\theta}{\varepsilon_s + (1-\alpha)\theta - 1}\right)^{\varepsilon_s + \theta - \sigma_w - 1}\frac{2\varepsilon_s + 2\theta - \sigma_w - 1}{\varepsilon_s + \theta}\right] \\
&\quad - \frac{1}{\varepsilon_s + (1-\alpha)\theta} - \frac{\alpha}{\varepsilon_s - \sigma_s - 1} - \frac{1-\alpha}{\varepsilon_s + \theta - \sigma_w - 1}.
\end{aligned}
$$

Similarly, per-firm changes in consumer surplus, profit, and output are

$$
\begin{aligned}
\frac{\Delta CS}{2} &= a_s^{\frac{1}{\varepsilon_s - \sigma_s}}\frac{\varepsilon_s - \sigma_s}{\varepsilon_s - \sigma_s - 1}\left[q_s^*\right]^{\frac{\varepsilon_s - \sigma_s - 1}{\varepsilon_s - \sigma_s}} - p_s^* q_s^* \\
&\quad + a_w^{\frac{1}{\varepsilon_w - \sigma_w}}\frac{\varepsilon_w - \sigma_w}{\varepsilon_w - \sigma_w - 1}\left[q_w^*\right]^{\frac{\varepsilon_w - \sigma_w - 1}{\varepsilon_w - \sigma_w}} - p_w^* q_w^* \\
&\quad - a_s^{\frac{1}{\varepsilon_s - \sigma_s}}\frac{\varepsilon_s - \sigma_s}{\varepsilon_s - \sigma_s - 1}\left[q_s^0\right]^{\frac{\varepsilon_s - \sigma_s - 1}{\varepsilon_s - \sigma_s}} + p^0 q_s^0 \\
&\quad - a_w^{\frac{1}{\varepsilon_w - \sigma_w}}\frac{\varepsilon_w - \sigma_w}{\varepsilon_w - \sigma_w - 1}\left[q_w^0\right]^{\frac{\varepsilon_w - \sigma_w - 1}{\varepsilon_w - \sigma_w}} + p^0 q_w^0 \\
&= \frac{\alpha}{\varepsilon_s - \sigma_s - 1}\left(\frac{\varepsilon_s - 1}{\varepsilon_s}\frac{\varepsilon_s + (1-\alpha)\theta}{\varepsilon_s + (1-\alpha)\theta - 1}\right)^{\varepsilon_s - \sigma_s - 1} \\
&\quad + \frac{1-\alpha}{\varepsilon_s + \theta - \sigma_w - 1}\left(\frac{\varepsilon_s + \theta - 1}{\varepsilon_s + \theta}\frac{\varepsilon_s + (1-\alpha)\theta}{\varepsilon_s + (1-\alpha)\theta - 1}\right)^{\varepsilon_s + \theta - \sigma_s - 1} \\
&\quad + 1 - \frac{\alpha(\varepsilon_s - \sigma_s)}{\varepsilon_s - \sigma_s - 1} - \frac{(1-\alpha)(\varepsilon_s + \theta - \sigma_w)}{\varepsilon_s + \theta - \sigma_w - 1},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\Delta \Pi}{2} &= \frac{\Delta W}{2} - \frac{\Delta CS}{2} \\
&= \frac{\alpha}{\varepsilon_s}\left(\frac{\varepsilon_s - 1}{\varepsilon_s}\frac{\varepsilon_s + (1-\alpha)\theta}{\varepsilon_s + (1-\alpha)\theta - 1}\right)^{\varepsilon_s - \sigma_s - 1} + \frac{1-\alpha}{\varepsilon_s + \theta}\left(\frac{\varepsilon_s + \theta - 1}{\varepsilon_s + \theta}\frac{\varepsilon_s + (1-\alpha)\theta}{\varepsilon_s + (1-\alpha)\theta - 1}\right)^{\varepsilon_s + \theta - \sigma_w - 1} \\
&\quad - \frac{1}{\varepsilon_s + (1-\alpha)\theta},
\end{aligned}
$$

and

$$
\frac{\Delta Q}{2} = \alpha\left(\frac{\varepsilon_s - 1}{\varepsilon_s}\frac{\varepsilon_s + (1-\alpha)\theta}{\varepsilon_s + (1-\alpha)\theta - 1}\right)^{\varepsilon_s - \sigma_s} + (1-\alpha)\left(\frac{\varepsilon_s + \theta - 1}{\varepsilon_s + \theta}\frac{\varepsilon_s + (1-\alpha)\theta}{\varepsilon_s + (1-\alpha)\theta - 1}\right)^{\varepsilon_s + \theta - \sigma_w} - 1,
$$

respectively. With a little abuse of notation, we consider these per firm measures below, and denote them as ΔSW, ΔCS, $\Delta \Pi$ and ΔQ.

We now conduct a number of numerical analyses of changes in social welfare, consumer surplus and output from uniform pricing to price discrimination More specifically, we investigate how the cross price elasticities, σ_s and σ_w, affect welfare changes from uniform pricing to price discrimination. In particular, our results below are in accordance with Adachi and Matsushima (2014): the greater σ_s and the less σ_w (i.e., competition is fiercer in the strong market than in the weak market) the more likely a positive change in social welfare (i.e., $\Delta SW > 0$).

Now, we consider the case of $\theta\alpha \leq 1$, where price discrimination never improves social welfare in the case of monopoly (see Aguirre and Cowan (2013)). The following examples show that in the case of oligopoly social welfare can be higher with price discrimination even if $\theta\alpha \leq 1$. Table 1 shows numerical values for Figures 1 and 2.

	ε_s	θ	α
Case 1	2	0.9	$\{0.05, 0.95\}$

Table 1: Parameter Values (for $\theta\alpha \leq 1$)

First, Figure 1 shows that social welfare under price discrimination is higher in the lower right area below the boundary (for $\alpha = 0.95$ and 0.05). This example shows that, in contrast to the case of monopoly analyzed by Aguirre and Cowan (2013), the elasticity difference, θ, does not always have to be large for price discrimination to be beneficial for social welfare under oligopoly. In line with Adachi and Matsushima (2014), the cross price elasticity in the strong market, σ_s, must be sufficiently high. Note also that the relative share of the strong market under uniform pricing, α, can be relatively small.

However, it is verified that a change in consumer surplus is negative for all (σ_s, σ_w) if α is equal to either 0.95 or 0.05. Figures 2 and 3 depict the area of $\Delta\Pi > 0$ and that of $\Delta Q > 0$ for $\alpha = 0.95$, 0.05, respectively. It seems that $\Delta Q > 0$ is necessary for $\Delta SW > 0$ in general.

Now, we consider numerical values in Table 2. Figure 4 is the corresponding figure.

	ε_s	θ	α
Case 2	2	$\{0.9.0.3\}$	0.95

Table 2: Parameter Values (for $\theta\alpha \leq 1$)

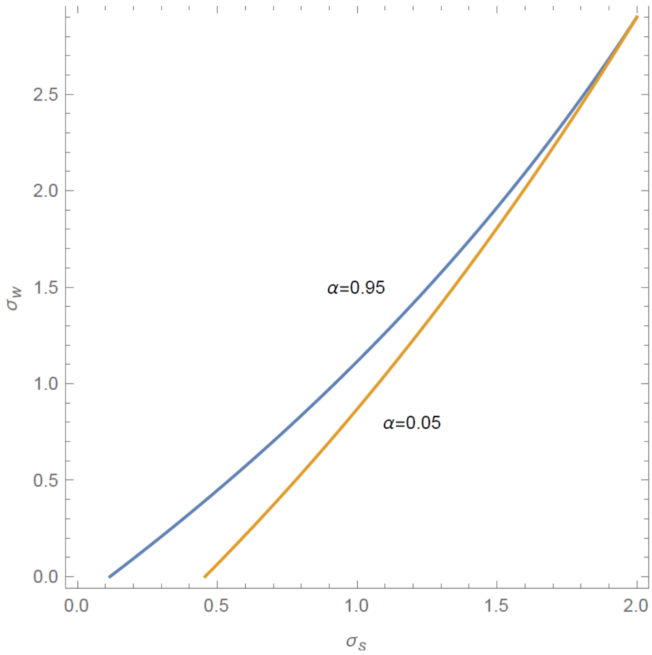

Figure 1: Area for $\Delta SW > 0$ (in the case of $\varepsilon_s = 2$ and $\theta = 0.9$).

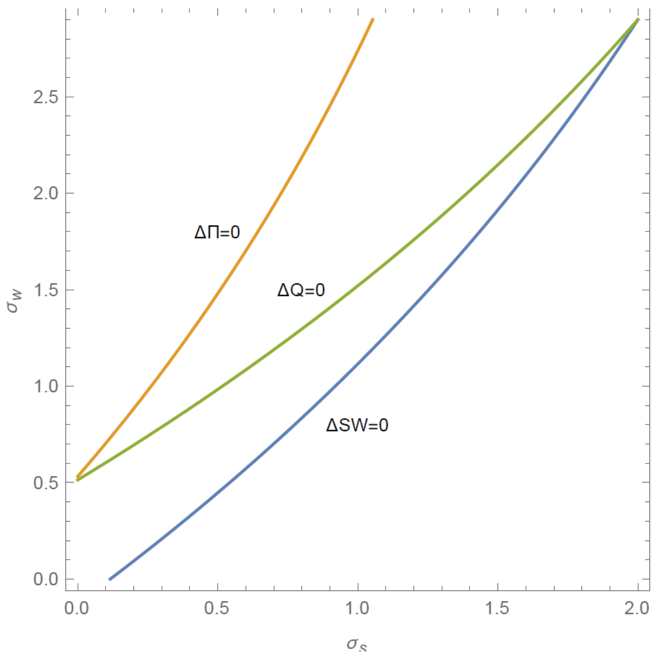

Figure 2: Areas for $\Delta SW > 0$, $\Delta\Pi > 0$ and $\Delta Q > 0$ (in the case of $\varepsilon_s = 2$ and $\theta = 0.9$ and $\alpha = 0.95$).

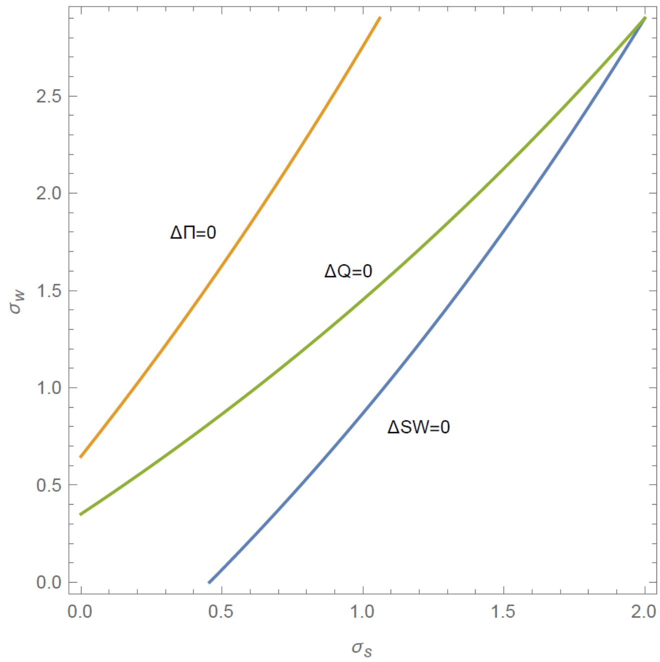

Figure 3: Areas for $\Delta SW > 0$, $\Delta \Pi > 0$ and $\Delta Q > 0$ (in the case of $\varepsilon_s = 2$ and $\theta = 0.9$ and $\alpha = 0.05$).

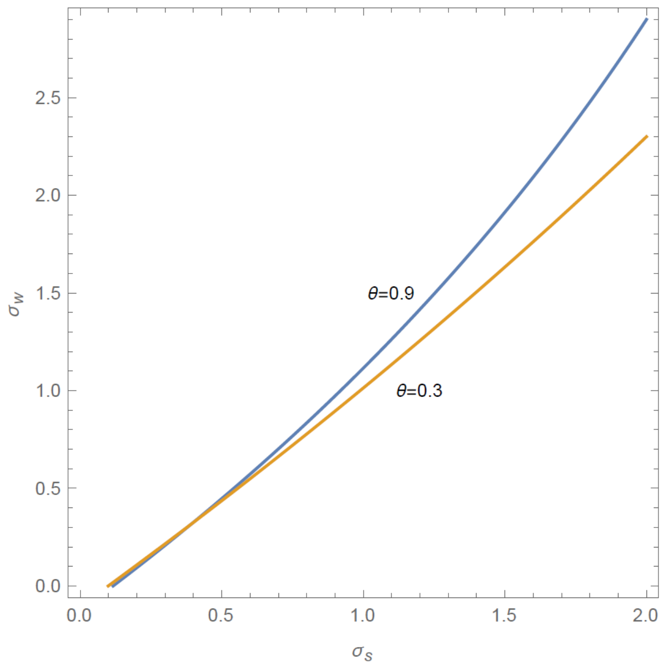

Figure 4: Area for $\Delta SW > 0$ (in the case of $\varepsilon_s = 2$ and $\alpha = 0.95$).

Based on these numerical examples above, we conjecture the following results on the condition for $\Delta W > 0$ hold in general: price discrimination is more likely to improve social welfare ($\Delta W > 0$)

1. as the equilibrium share of the strong market under uniform pricing (α) becomes *larger* (with other parameters held constant),
2. as the own price elasticity difference (θ) becomes *larger*,
3. as the cross price elasticity in the strong market (σ_s) becomes *larger*, and
4. as the cross price elasticity in the weak market (σ_w) becomes *smaller*.

In particular, the third and the fourth points correspond to Adachi and Matsushima's (2014) necessary and sufficient condition in their Proposition 1 which roughly states (in the notations of the present paper) that there exists a threshold for σ_s, and for a larger σ_s than the threshold price discrimination raises social welfare. Notice here that ΔW is not monotonically increasing in σ_s in either case. This should be further investigated to establish results similar to Adachi and Matsushima (2014).

Now, we analyze the case of $\theta\alpha > 1$ to see whether price discrimination can raise consumer surplus. First, numerical values in Table 3 consider different values of the own price elasticity in the strong market, ε_s. In Fugure 5, the area where price discrimination raises social welfare is right below the boundary for each ε_s. In contrast to the case of $\theta\alpha \leq 1$ above, price discrimination can raise social welfare as Figure 6 shows: a change in consumer surplus is positive in the area left below the boundary for each ε_s. It appears that *both* σ_s and σ_w must be sufficiently *small* for price discrimination to raise consumer. It is conjectured that if $\Delta CS > 0$ then $\Delta SW > 0$ (and $\Delta\Pi$ is necessarily positive). Notice that for a positive change in social welfare σ_w can be kept relatively small as long as σ_w is sufficiently large. That would be probably the reason why the region of (σ_s, σ_w) where price discrimination raises social welfare is contained in the region where price discrimination raises social welfare.

	ε_s	θ	α
Case 3	$\{1.05, 1.2, 1.35\}$	3	0.95

Table 3: Parameter Values (for $\theta\alpha > 1$)

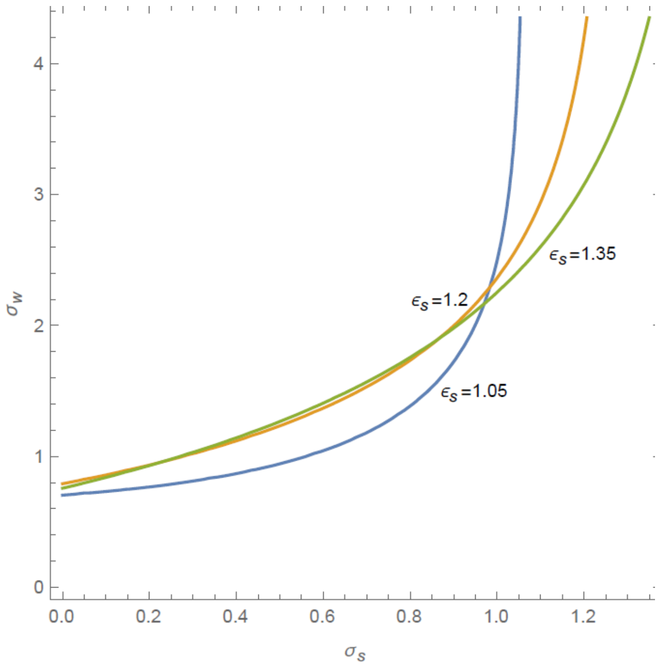

Figure 5: Area for $\Delta SW > 0$ (in the case of $\theta = 3$ and $\alpha = 0.95$).

Finally, fixing the value of ε_s, we consider the effects of different values of θ and α separately. First, Table 4 indicates different values for the differences in the own price elasticities. Interestingly, in Figure 7, the boundaries (corresponding to $\Delta SW = 0$) are quite homothetic to changes in θ. Figure 8 shows that the region of (σ_s, σ_w) where consumer surplus is higher under price discrimination is larger with $\theta = 4$ than with $\theta = 3$ (price discrimination never raises consumer surplus when $\theta = 2$). As Figure 9 (where $\Delta\Pi > 0$ for any (σ_s, σ_w)) shows in the case of the case most favorable for a positive ΔSW ($\theta = 4$), consumer surplus is higher under price discrimination for a wide range of σ_s as long as σ_w is kept *small* (i.e., *less* competition in the weak market). This would be probably because the own price elasticity in the weak market ($\varepsilon_s + \theta$) is already sufficiently large, suggesting that competition may work against consumer surplus (due to the lack of coordination) if

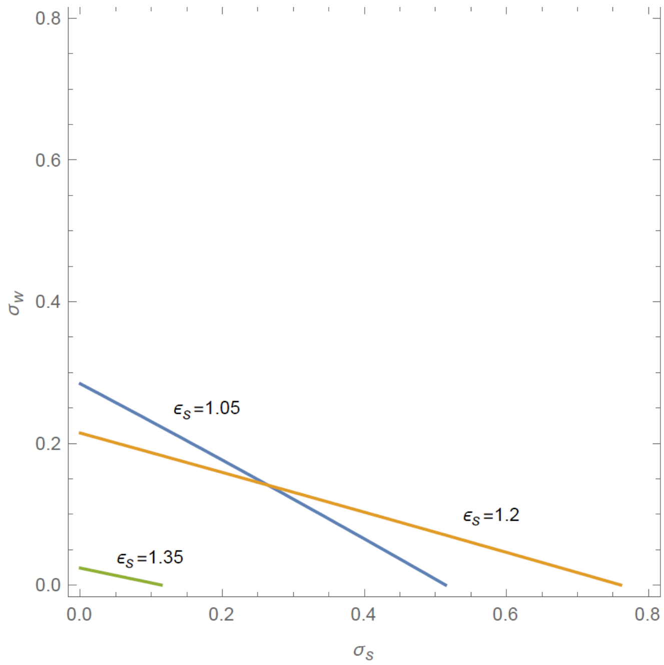

Figure 6: Area for $\Delta CS > 0$ (in the case of $\theta = 3$ and $\alpha = 0.95$).

the industry as a whole faces a sufficiently elastic demand in the weak market.

	ε_s	θ	α
Case 4	1.2	$\{2, 3, 4\}$	0.95

Table 4: Parameter Values (for $\theta\alpha > 1$)

Lastly, we consider the numerical values in Table 5 to see the effects of different shares of the strong market. Figure 10 depicts the areas where price discrimination raises social welfare. As expected, a higher α is favorable for a positive change in social welfare. However, Figure 11 shows that a change in consumer surplus is negative for any (σ_s, σ_w) if $\alpha = 0.5$ or 0.95. In particular, the case of $\alpha = 0.5$ satisfies $\theta\alpha > 1$. Thus, we predict that a higher value of the own elasticity difference (θ) is important then a higher share of the strong market for price discrimination to improve consumer surplus.

	ε_s	θ	α
Case 5	1.2	3	$\{0.05, 0.5, 0.95\}$

Table 5: Parameter Values (for $\theta\alpha > 1$)

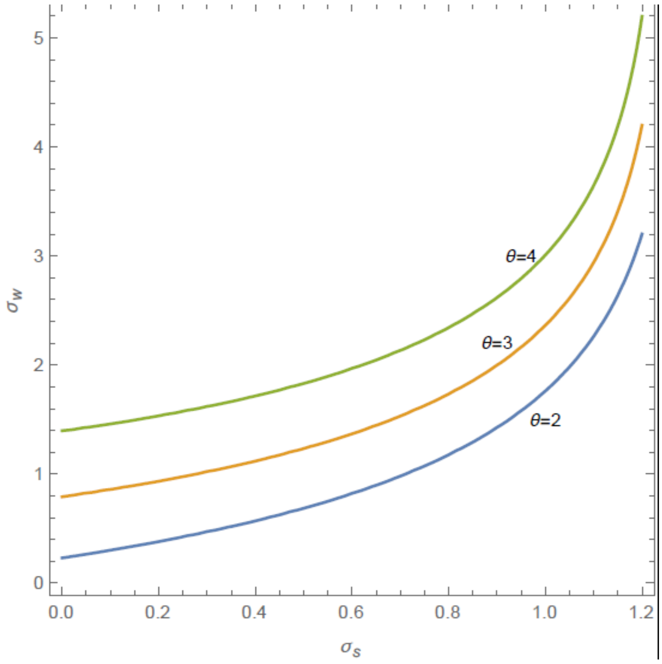

Figure 7: Area for $\Delta SW > 0$ (in the case of $\varepsilon_s = 1.2$ and $\alpha = 0.95$).

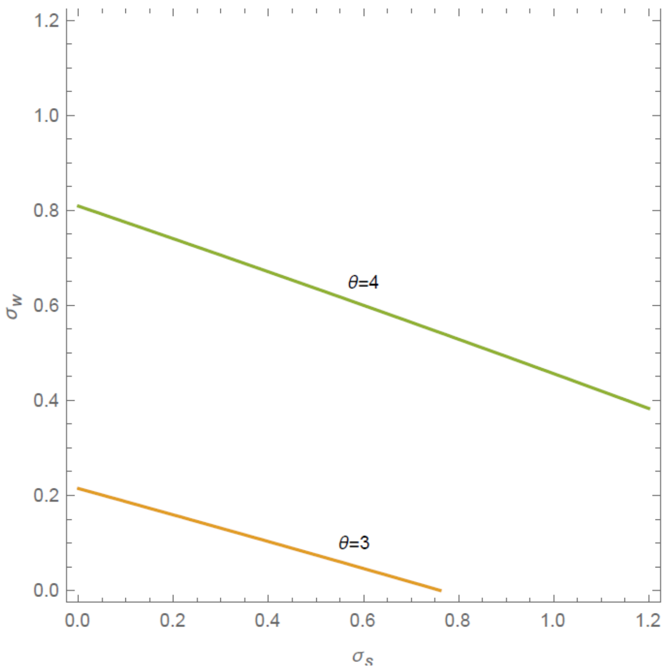

Figure 8: Area for $\Delta CS > 0$ (in the case of $\varepsilon_s = 1.2$ and $\alpha = 0.95$).

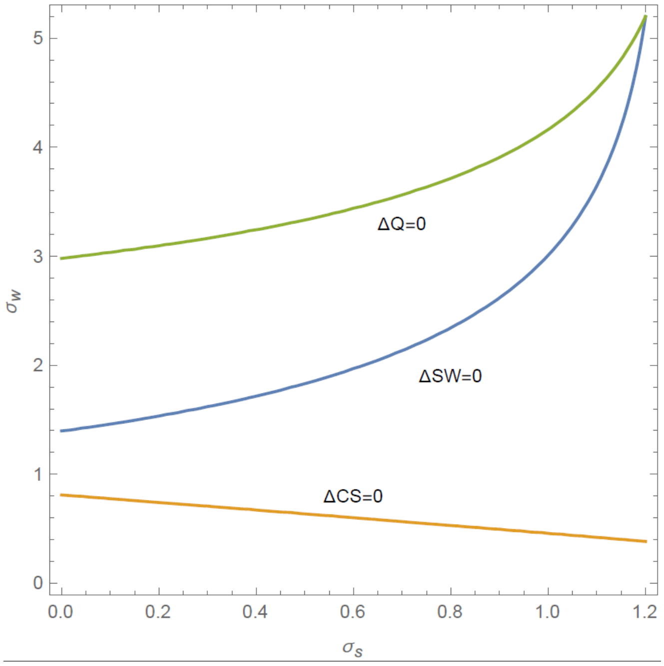

Figure 9: Areas for $\Delta SW > 0$, $\Delta\Pi > 0$ and $\Delta Q > 0$ (in the case of $\varepsilon_s = 1.2$, $\theta = 4$ and $\alpha = 0.95$).

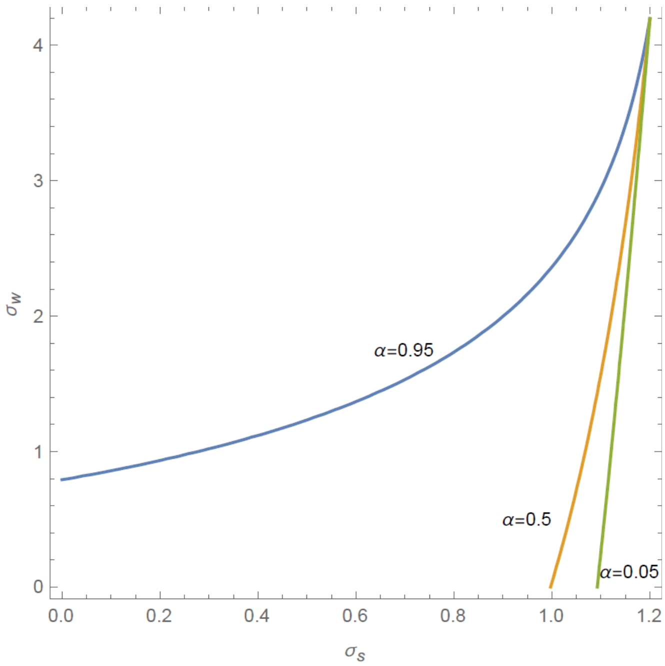

Figure 10: Area for $\Delta SW > 0$ (in the case of $\varepsilon_s = 1.2$ and $\theta = 3$).

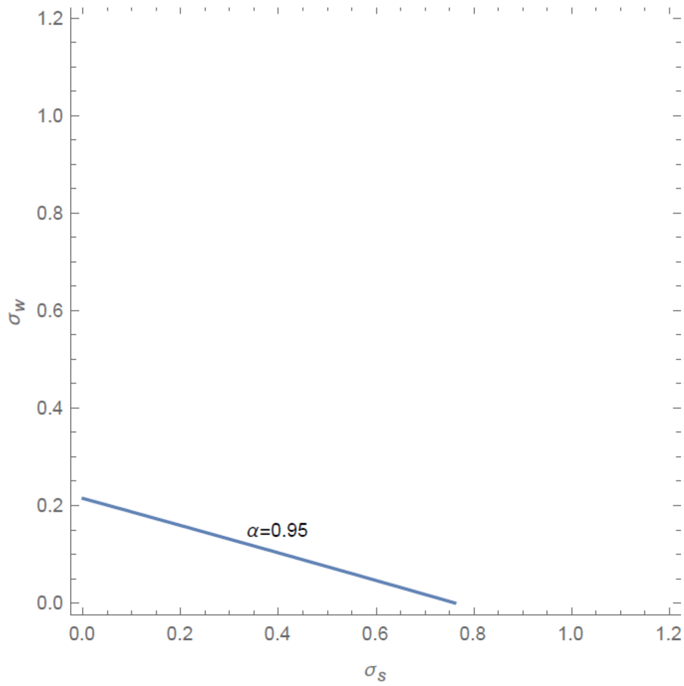

Figure 11: Area for $\Delta CS > 0$ (in the case of $\varepsilon_s = 1.2$ and $\theta = 3$).

4 Concluding Remarks

This paper has investigated the welfare consequences of oligopolistic third-degree price discrimination with constant own and cross price elasticities of demand. We find that the key parameters for price discrimination to improve social welfare and even consumer surplus are the cross price elasticities. If this parameter in the strong market is sufficiently large (i.e., competition in the strong market is fierce) with the corresponding parameter in the weak market kept sufficiently small, then price discrimination is more likely to be preferable. In comparison to the case of monopoly analyzed by Aguirre and Cowan (2014), price discrimination can improve social welfare even with the parameter values with which it does not under monopoly. This result is consistent with Adachi and Matsushima's (2014) on social welfare with linear demands. In addition, our result that consumer surplus can be higher with price discrimination shows that Adachi and Matsushima's (2014) result on consumer surplus (price discrimination never improves social welfare) hinges on the linearity assumption.

For future research, it is important to explore the conditions for price discrimination to improve social welfare and consumer surplus with general nonlinear demands. As in Adachi and Ebina (2014a,b) in the context of cost pass-through (see, e.g., Bulow and Klemperer (2012) and Weyl and Fabinger (2013)) in vertical relationships, one would be able to conduct welfare analysis with exponential demands, logistic demands, and type I extreme demands.[16]

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[16]By focusing on incidence properties, Fabinger and Weyl (2014) characterize the demand and supply system that allows closed-form solutions and yet flexibility to reflect the reality.

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